ON COHERENT STRUCTURES, FLOW-INDUCED VIBRATIONS, AND MIGRATORY FLOW in Liquid Metal Nuclear Reactors

FULVIO BERTOCCHI
With this you are warmly invited to attend the defence of my thesis with title

On Coherent Structures, Flow-Induced Vibrations, and Migratory Flow in Liquid Metal Nuclear Reactors

DAY, DD Month 2020
at hh:mm in the Senatzaal of the TU Delft, Mekelweg 5, Delft.

I will give a presentation before the defence, at hh:mm.

Fulvio Bertocchi
Propositions

accompanying the dissertation

ON COHERENT STRUCTURES, FLOW-INDUCED VIBRATIONS,
AND MIGRATORY FLOW
IN LIQUID METAL NUCLEAR REACTORS

by

Fulvio BERTOCCHI

1. A high speed camera provides the best compromise between accuracy and handiness to measure the frequency of vibration in a rod bundle. (This thesis, Chapter 4)

2. A 7-rod bundle is more difficult to build than larger ones. (This thesis)

3. Calculating the transversal pressure gradient with Equation 5.39 of this thesis is easier than with the Euler equation. (This thesis, Chapter 5)

4. If ionizing radiation would be visible to the eye, nuclear energy would be more acceptable to the public.¹

5. Starting the PhD with a thorough time planning weakens the ability of tackling unexpected setbacks.

6. Financial support to poor African countries will increase migration across the Mediterranean Sea.²

7. If one does not appreciate theatre, he/she cannot truly enjoy Caravaggio’s art.

8. In cage diving with white sharks, the cage protects the shark from humans, more than the other way around.

9. A good scientist must also be a good reader.

10. A stress reduction course for PhDs should also cover vacation time.

These propositions are regarded as opposable and defendable, and have been approved as such by promotors prof. dr. ir. J. L. Kloosterman and dr. ir. M. Rohde.

² M. A. Clemens and H. M. Postel, Deterring Emigration with Foreign Aid: An Overview of Evidence from Low-Income Countries, Center for Global Development, 2018.
Stellingen

behorende bij het proefschrift

ON COHERENT STRUCTURES, FLOW-INDUCED VIBRATIONS, AND MIGRATORY FLOW IN LIQUID METAL NUCLEAR REACTORS

door

Fulvio BERTOCCHI

1. Stelling 1.
2. Stelling 2.
5. Stelling 5.
7. Stelling 7.
10. Stelling 10.

Deze stellingen worden opponeerbaar en verdedigbaar geacht en zijn als zodanig goedgekeurd door de promotoren prof. dr. ir. J. L. Kloosterman en dr. ir. M. Rohde.
On Coherent Structures, Flow-Induced Vibrations, and Migratory Flow in Liquid Metal Nuclear Reactors
On Coherent Structures, Flow-Induced Vibrations, and Migratory Flow in Liquid Metal Nuclear Reactors

Proefschrift

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aan de Technische Universiteit Delft,
op gezag van de Rector Magnificus prof. dr. ir. ??,
voorzitter van het College voor Promoties,
in het openbaar te verdedigen op DATE om TIME

door

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“Those who have handled sciences have been either men of experiment or men of dogmas. The men of experiment are like the ant, they only collect and use; the reasoners resemble spiders, who make cobwebs out of their own substance. But the bee takes a middle course: it gathers its material from the flowers of the garden and of the field, but transforms and digests it by a power of its own.”

FRANCIS BACON, 1620
Summary

Flows in rod bundles are common to many industrial applications such as heat exchangers or some types of nuclear reactors. The core of many classes of nuclear reactors can be easily sketched as a bundle of rods, the fuel pins, immersed in an axial flow of coolant that removes the heat produced by the fission reaction. Coupling this geometry to an axial flow can trigger periodical vortices, known as large coherent structures or gap vortex streets, that move on both sides of the gaps between the rods. By crossing the gap (cross-flow), these vortices may enhance the heat removal mechanism, thus improving the performance of the reactor. However, coherent structures cause velocity oscillations in the flow that may induce vibrations of the fuel rods, leading to their long term damage. The length (or wavelength) of coherent structures is a key parameter for understanding the interplay between these vortices and the vibrations that may be triggered on the rods. Their wavelength determines the frequency of the velocity oscillations in the fluid, hence of the external force imposed on the rods.

One of the reactor designs belonging to the next generation (Gen-IV) of nuclear reactors is the Liquid Metal Fast Breeder Reactor (LMFBR). This reactor has the fuel rods in the core arranged in a hexagonal matrix. In this design, a wire is helicoidally wrapped around each fuel rod to keep them separated from each other. The presence of the wire diverts part of the more turbulent flow from the bulk towards the gap between the rods, where the flow would be otherwise less turbulent. This enhances the heat exchange and avoids hot spots on the fuel cladding.

A phenomenon known as migratory flow has been observed in rod bundles with wire spacers. In the presence of migratory flow, the fluid is diverted from the gap towards the main subchannel and it bends against the helicoid path of the wire, thus leading to a very complex flow, where part of the fluid follows the wire direction and part moves against it, away from the gap. Although this behaviour was first observed years ago, the governing mechanism is not clear yet. Explaining migratory flow is thus a fundamental step towards a general understanding of the mixing and mass transfer phenomena in rod bundles in the presence of helicoid wires.

This research considers several aspects of the flow inside rod bundle geometries. The aim is to study coherent structures to understand their role in inducing vi-
brations of the rods, and to provide a model that explains migratory flow caused by helicoid wire spacers.

To these purposes, three experimental facilities are designed and built that work in isothermal conditions, ambient pressure and with water as working fluid. The first is a rectangular channel hosting two half-rods whose distance can be adjusted. A second hexagonal bundle containing seven rods has the central rod made of flexible silicone to study vibrations induced by the flow. The third facility is a hexagonal bundle with seven rods, each of them equipped with a wire helicoidally wound around them in order to measure and explain migratory flow.

The measurements are carried out with non intrusive optical systems. Laser Doppler Anemometry (LDA) is used to measure the flow components in the rectangular channel with half-rods and in the hexagonal bundle without wire spacers. For the latter, a high speed camera is also used to measure the amplitude and frequency of the vibrations induced on the flexible rod by the flow. In the hexagonal bundle hosting wire spacers, planar (i.e. two dimensional) Particle Image Velocimetry (PIV) is used to measure the flow near to the wire of the central rod of the bundle. For all the three experiments, optical access is achieved by replacing part of the material of the rods with Fluorinated Ethylene Propylene (FEP), a Refractive Index-Matching (RIM) polymer that nearly matches the refractive index of plain water. In this way, optical distortion of the light rays is greatly reduced.

The measurements of the flow inside the channel hosting two half-rods show that cross-flow of coherent structures across the gap decreases in frequency if the distance between the rods is increased.

This research shows that the wavelength of coherent structures becomes independent of the flow rate above a certain value of the Reynolds number, and that it is affected solely by the geometry of the channel. Performing dimensional analysis of the problem, and supporting the reasoning with experimental evidence, has increased our understanding of what determines the length of the structures. This appears to be a function of only the hydraulic diameter of the gap region close to which they form. This finding is presented in the form of a novel correlation that predicts the length of coherent structures. This correlation is applicable to a number of different geometries, ranging from channels with few rods to full rod bundles.

The flow measurements in the hexagonal bundle (without wire spacers) show that the frequency of passage of coherent structures in the axial direction increases with the Reynolds number. If this frequency becomes equal to twice the first natural frequency of rod, a drastic increase in the amplitude of oscillation is observed, which can be ascribed to the synchronization between the rod and coherent structures carried by the flow.

It is shown that for hexagonal rod bundles with helicoid wires, the flow very close to the rod follows the helicoid path of the wire.

However, if the measurement region is shifted closer to the main subchannel surrounding the rod, the flow bends towards the wire, against the helicoid path,
suggesting the presence of migratory flow. A model to reconstruct the pressure gradient caused by the wire, and responsible for the bending of the flow, is derived from the steady-state, two-dimensional, inviscid Navier-Stokes equations applied to the flow streamlines. Following a theoretical approach, an equation is derived from this model to estimate the bending angle of the flow at any point inside the measured area, based on the time-averaged flow fields. This equation has the form of an integral evaluated along the path followed by a streamline. It shows how the bending of a streamline at an arbitrary point is the result of the interaction between the transversal pressure gradient, trying to bend the fluid, and the inertial forces, trying to straighten it towards the direction of the main flow.

A possible correlation for estimating the bending angle of the flow is obtained with dimensional analysis and with the support of the experimental results. Contrarily to the theoretical model, this correlation relies on macroscopic variables, being the bundle dimensions and the geometry of the subchannel. Finally, this thesis suggests possible topics of interest for future research.
Flows in rod bundles are common to many industrial applications such as heat exchangers or some types of nuclear reactors. The core of many classes of nuclear reactors can be easily sketched as a bundle of rods, the fuel pins, immersed in an axial flow of coolant that removes the heat produced by the fission reaction. Coupling this geometry to an axial flow can trigger periodical vortices, known as large coherent structures or gap vortex streets, that move on both sides of the gaps between the rods. By crossing the gap (cross-flow), these vortices may enhance the heat removal mechanism, thus improving the performance of the reactor. However, coherent structures cause velocity oscillations in the flow that may induce vibrations of the fuel rods, leading to their long term damage. The length of coherent structures is a key parameter for understanding the interplay between these vortices and the vibrations that may be triggered on the rods. Their length determines the frequency of the velocity oscillations in the fluid, hence of the external force imposed onto the rods.

One of the reactor designs belonging to the next generation (Gen-IV) of nuclear reactors is the Liquid Metal Fast Breeder Reactor (LMFBR). This reactor has its fuel rods in the core arranged in a hexagonal matrix. In this design, a wire is helicoidally wrapped around each fuel rod to keep them separated from each other. The presence of the wire diverts part of the more turbulent flow from the bulk towards the gap between the rods, where the flow would be otherwise less turbulent. This enhances the heat exchange and avoids hot spots on the fuel cladding. A phenomenon known as migratory flow has been observed in rod bundles with wire spacers. This phenomenon is responsible for the transfer of fluid from the gap region towards the main subchannel. In the presence of migratory flow, the fluid is diverted from the gap towards the main subchannel and bends against the helicoid path of the wire. The presence of the wire causes thus a very complex flow, where some fluid moves towards the gap region, and some is diverted towards the main subchannel. The mechanism responsible for migratory flow is not clear yet. Explaining migratory flow is thus a fundamental step for the understanding of the mixing and mass transfer phenomena of rod bundles with helicoid wires.

This research studies several aspects of the flow inside rod bundle geometries. The aim is to study coherent structures to understand their role in inducing vibrations
of the rods, and to provide a model that explains migratory flow caused by helicoid wire spacers.

To these purposes, three experimental facilities are designed and built that work in isothermal conditions, ambient pressure and with water as working fluid. The first is a rectangular channel hosting two half-rods whose distance can be adjusted. A second hexagonal bundle containing seven rods has the central rod made of flexible silicone to study vibrations induced by the flow. The third facility is a hexagonal bundle with seven rods, each of them equipped with a wire helicoidally wound around them in order to measure and explain migratory flow.

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The measurements of the flow inside the channel hosting two half-rods show that cross-flow of coherent structures across the gap decreases in frequency if the distance between the rods is increased.

This research shows that, above a certain value of the bulk Reynolds, the length of coherent structures becomes independent of the flow rate, and it is affected solely by the geometry of the channel. Relying on this experimental evidence, and applying dimensional analysis, a novel correlation is provided to predict the length of coherent structures. This correlation is applicable to a number of different geometries, ranging from channels with few rods to full rod bundles. According to the correlation, the length of the structures scales linearly with the hydraulic diameter of the gap region close to which they are formed.

The flow measurements in the hexagonal bundle (without wire spacers) show that the frequency of passage of coherent structures in the axial direction increases with the Reynolds number of the main subchannel (bulk Reynolds). If this frequency becomes equal to twice the first natural frequency of the flexible silicone rod, a drastic increase in the amplitude of oscillation is observed, which can be ascribed to the synchronization between the rod and the periodical vortices in the flow.

It is shown that for hexagonal rod bundles with helicoid wires, the flow very close to the rod follows the helicoid path downstream of the wire (ie. after the wire). However, if the measurement region is shifted closer to the main subchannel, the flow bends towards the wire, against the helicoid path, suggesting the presence of migratory flow. A model to reconstruct the pressure gradient caused by the wire, and responsible for the bending of the flow, is derived from the steady-state, two-dimensional, inviscid Navier-Stokes equations applied to the flow streamlines.
Following a theoretical approach, an equation is also derived from this model to estimate the bending angle of the flow at any point inside the measured region, based on the time-averaged flow fields. This equation has the form of an integral evaluated along the path followed by a streamline. It shows how the bending of a streamline at an arbitrary point is the result of the interaction between the transversal pressure gradient, trying to bend the fluid, and the inertial forces, trying to straighten it towards the direction of the main flow.

A possible correlation for estimating the bending angle of the flow based is obtained with dimensional analysis and with the support of the experimental results. Contrarily to the theoretical model, this correlation relies on macroscopic variables, being the bundle dimensions and the geometry of the subchannel.

Finally, this thesis suggests possible topics of interest for future research.
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Chapter 1

Introduction

1.1 Motivation of this research

As a result of the development of the developing countries, and the associated growth of the population, the electricity demand is expected to increase drastically in the next decades. The projections [1] of the International Atomic Energy Agency (IAEA) foresee an increase of the worldwide capacity of nuclear-generated electricity by 42% (reaching 554GW) within 2030, and doubling the actual capacity by 2050. However, relying on more conservative projections, the capacity might remain constant at nowadays’ levels. There is a big deal of uncertainty in the figures because it is yet not clear whether the nuclear power plants scheduled for decommissioning are going to be replaced by new nuclear capacity.

With the goal of pursuing safer and more sustainable nuclear energy, in the year 2000, the Generation IV International Forum identified six innovative designs of nuclear reactors planned to be commercially available by 2040. These reactors feature larger safety margins, pose an obstacle to nuclear proliferation, or are capable of using nuclear waste generated by conventional reactors as fuel. This generation of reactors could, thus, be part of a structured and long-term strategy for reducing emissions, which is currently a rather far-fetched vision in Europe [3, 4]. The underlying goal is to keep the average global temperature increase below the value agreed at the Paris Agreement of 2°C.

One of the proposed designs is the Liquid Metal Fast Breeder Reactor (LMFBR). The heat removed by the coolant, being sodium, lead, or an eutectic mixture of lead and bismuth (LBE) converts water into steam that, in turn, generates electricity through a spinning turbine (Figure 1.1). The core of a LMFBR typically contains a large number of fuel assemblies, each consisting of a bundle of rods hosting the fuel (fuel pins) arranged in a hexagonal lattice through which the liquid metal coolant flows, parallel to the fuel pins (Figure 1.2a). The lattice is characterised by the $P/D$ ratio (ratio between the pin pitch and pin diameter). Due to the tight assembly, one solution for keeping the rods from touching each other during normal operation is to wrap an helicoid wire spacer around each of them (Figure 1.2b). The coupling between an axial flow of fluid and such a geometry gives origin to a
1.1. MOTIVATION OF THIS RESEARCH

Figure 1.1: Schematic of the Liquid Metal Fast Breeder Reactor. Adopted from [2].

Figure 1.2: a) Sketch of one of the many hexagonal fuel assemblies hosted inside the core. Adopted from [5]. b) Fuel rod with helicoid wrapped-wire. Public domain, US DOE.

number of phenomena that need to be studied for the sake of a safe operation of the reactor.

Streaks of relatively large periodical vortices may occur that, on one hand, are beneficial for the safe operation of the core because they can enhance the mixing, hence lowering the temperature of the fuel cladding. On the other hand, they can induce vibrations of the fuel rods leading to damage by fretting and, eventually, fatigue.

Moreover, the presence of the helicoid wire makes the flow even more complex to understand and to model due to the additional motion of fluid across the gaps between the rods.

*This experimental work aims at studying large coherent structures and their role in inducing vibrations in rod bundles, as well as the effect of helicoid wires on the flow.*
1.2 Vortex streets in a rod bundle geometry

Rod bundle flows occur in industrial applications, such as heat exchangers or LWR and Generation IV (Gen-IV) nuclear reactors. When geometries such as the core of a nuclear reactor are coupled with an axial flow we refer to it as rod bundle flow.

The presence of an axial flow leads to velocity differences between the low-speed region of the gap between two adjacent rods, and the high-speed region of the main sub-channels. This velocity difference produces a shear layer between the two flow regions, leading to streaks of vortices carried by the current. Generally these vortices (or flow structures) occur on both sides of the gap enclosed by two neighbouring rods, identifying the so-called gap vortex streets [6], also known as coherent structures. They look similar to the well-known Von Karman streets shedding in the wake of a bluff body in the presence of a transversal flow (Figure 1.3) immersed in a current of fluid. The mechanism responsible for their formation inside a rod bundle can be compared to the Kelvin-Helmholtz instability that occurs between two parallel layers of fluid moving with different speeds [8] (Figure 1.4). An inflection point in the stream-wise velocity profile is a necessary condition (although not sufficient) to have these coherent structures, as predicted by the Rayleigh’s instability criterion [9]. Unlike free mixing layers, gap vortex streets are stable along the flow, hence the adjective coherent.

Research has widely covered the topic of coherent structures in rod bundles, both experimentally and numerically. Rowe measured coherent flow structures through

![Figure 1.3: Photograph of the Von Karman streets; the camera is moving with the main flow speed. Adopted from [7].](image1)

![Figure 1.4: Kelvin-Helmholtz instability arising from two layers of water with different velocities, being the upper one the fastest. Adopted from [7].](image2)
a gap where the $P/D$ was adjustable to 1.125 and 1.250 [10]. Rehme investigated the axial flow of air inside a rectangular channel with four rods [11]. He detected coherent structures in the flow responsible for the lateral mass and momentum transfer across the subchannels that were affected by the $P/D$ ratio. Rehme proposed a static pressure instability mechanism to account for the formation of these structures. Möller conducted experiments in a similar setup to study the formation of coherent structures [12]. Velocity and wall-pressure fluctuations were measured and coherent structures were found to occur near the gap between two rods. He adopted the term *metastable equilibrium* to describe the instantaneous velocity and vorticity difference caused by these structures. Möller also pointed out that vibrations of the fuel elements might occur if the natural frequency of the pins is close to the oscillating pressure field’s frequency due to the periodic structures in the flow. Later, Choueiri and Tavoularis made use of flow visualization techniques: the structures formed near to the gap region and the boundary between the low-speed and high-speed fluid oscillated as they moved downstream [13].

A lateral (span-wise) movement of coherent structures across the gap between the rods may also occur, which is often referred to as cross-flow. In a nuclear reactor, cross-flow is the mechanism responsible of enhancing the lateral mixing between subchannels, decreasing the temperature on the fuel rod’s outer cladding, and improving the overall safety of the plant. Gosset [14] and Piot [15] investigated such lateral mass transfer across an eccentric annular gap with flow visualization techniques. The instability mechanism responsible for cross-flow was found to be dependent on a critical Reynolds number, strongly affected by the geometry of the gap. Baratto investigated the air flow inside a 5-rod model of a CANDU (CANadian Deuterium Uranium) fuel assembly [16] finding that the frequency of passage of the coherent structures decreases with the gap size, along the circumferential direction around the rod.

Mahmood studied coherent structures and the inter-channel mixing in a square rod bundle over a range of Reynolds numbers [17]. One of his main findings was that the length (or wavelength) of coherent structures does not depend on the Reynolds number for values higher than 2000. The wavelength is affected merely by the geometry of the channel. These findings were consistent with those of Meyer [18] and Guellouz [19]. Later, Choueiri and Tavoularis studied the flow instability through the gap of an eccentric rod inside a circular channel [13, 20]. They found that the velocity fluctuations along the span-wise direction in the centre of the gap were varying in time with a rate twice as low compared to those in the axial direction. This was consistent with the model previously proposed by Meyer [18] of coherent structures regarded as counter-rotating vortices on either side of a gap.

Parallel numerical efforts have been made by Chang and Tavoularis with Unsteady Reynolds-averaged Navier Stokes (URANS) [21] and by Merzari and Ninokata with Large Eddy Simulations (LES) [22] to reproduce the complex flow inside such a geometry. However, the effects that gap geometry, in particular the $P/D$ ratio, has on cross-flow has been debated long since and yet, a generally accepted conclusion is still seeked.
1.3 Fluid-structure interaction

An axial flow inside a rod bundle (typical of a nuclear reactor's core) induces oscillations of the rods due to the action of turbulence. This type of Fluid-Structure Interaction (FSI) is known as flow-induced vibration (FIV) and can be further classified in instability-induced excitations (IIE) or extraneous-induced excitations (EIE), following the Naudascher & Rockwell classification [23]. EIE are caused by pressure or velocity fluctuations in the main flow (not caused by movements of components of the bundle or by flow instabilities). Instability-induced excitations are due to local flow instabilities such as vortex shedding in the wake of a cylinder. There is also a third category termed movement-induced excitations (MIE) caused by the movement of the object itself, which drives the phenomenon (i.e. the flutering wing of an airplane). Obviously the border between the three classes is not strict, especially between EIE and IIE, since these categories share some common aspects [24].

In order to summarise the behaviour of the rods in the presence of an axial flow, the description provided by De Ridder et al. may help [25]. It describes the case of a solitary cylinder placed inside an outer cylindrical channel. For very low flow velocities, the rod stays in the center of the channel. As the velocity increases, the rod exhibits small oscillations induced by the turbulent flow. When the velocity is increased, the amplitude will also increase and, at a specific critical velocity, the rod will buckle to one side. Simultaneously, its natural frequency will drop to zero. For flow velocities lower than the critical value, the observed vibrations are called sub-critical. Beyond the critical point, the system enters the so-called divergence regime: the rod positions itself back at the center of the channel. If the velocity is increased even further, the rod starts to flutter around the central position with much larger oscillations (analogously to a fluttering hose for watering the garden). The aforementioned process is shown in Figure 1.5. The sub-critical vibrations are generally characterised by a narrow frequency spectrum close to the ground mode value of the rod [26].

In a rod bundle the situation is more complex because of the presence of the surrounding rods. Their effect is to confine even more the considered rod, in-

![Figure 1.5: Natural frequency and maximum displacement of a rod inside a concentric channel depending on the flow speed. Adopted from [25].](image-url)
creasing the damping and reducing the rod’s natural frequency. FIV have been widely studied [26, 27] and, recently, Paidoussis wrote a two-volumes handbook collecting most of the knowledge and literature on the subject [24, 28]. However, little has been done to address the specific issue of vibrations induced by coherent structures.

When periodical structures occur in the flow, and their frequency (i.e. rate of passage) is close to the natural frequency of the rod, the amplitude of oscillation is expected to increase drastically. If coherent structures have a length comparable in magnitude with the axial dimension of the rod assembly, they may cause resonance in the first and most energetic mode. Conversely, the presence of shorter, multiple coherent structures on either side of the rod could diminish their effect on the most energetic mode (although they might still cause oscillations at higher, less energetic modes of vibration). When the shedding frequency of vortices (coherent structures in the case of this work) synchronises with the natural frequency of the rod, this flow-induced instability is referred to as lock-in condition, also known as synchronization. The condition of lock-in can also occur without necessarily having the frequency of the coherent structures matching the rod’s natural frequency [30]. An example is the collapse of the Tacoma Narrows bridge: there the shedding frequency of the vortices formed in the current downstream the bridge locked in with the frequency of motion of the bridge (Figure 1.6), even though the latter was far from the natural frequency [31].

Another kind of flow-induced instability is the so-called parametric oscillation in which the system is made oscillate by modulating one of its parameters: in case of rod bundles, this parameter can be the frequency of oscillation of the axial flow around the rods. Parametric resonance in the most energetic mode occurs if the frequency matches twice the natural frequency of the rod, thereby increasing the amplitude of oscillation. A child standing up and crouching while playing on a

![Figure 1.6: The bridge over the Tacoma Narrows before its collapse: the frequency with which vortices of air were being shed downstream matched the oscillation frequency of the bridge. Adopted from [29].](image-url)
swing with twice the frequency of oscillation of the swing itself is a clear (although involuntary) example of a parametric resonator.

1.4 Influence of the helicoid wire wrapper

Due to the tight arrangement of fuel rods inside the core of a LMFBR, either spacing grids positioned at a specified distance or wrapping wires are used to ensure space among the fuel elements. When the latter solution is pursued, a wire is wrapped helicoidally around each rod to prevent the fuel rods from touching each other due to swelling or buckling of the fuel elements during the reactor operation. If the rods come into contact with each other, the flow speed is locally decreased and the heat transfer deteriorates, leading to hot spots and possible damage on the fuel cladding.

Over the years, considerable efforts have been dedicated to investigating wire-wrapped rod bundles to understand how the wire spacers affect the flow of coolant through the core. A comprehensive review of both experiments and numerical studies was provided by Moorthi et al. [32]. Sato et al. [33] performed Particle Image Velocimetry (PIV) measurements inside a 7-rod, wire-wrapped hexagonal bundle observing that the flow follows the wrapping wire’s direction. The flow inside a 61-rod, wire-wrapped hexagonal bundle was extensively studied at Texas A&M University for code validation [34], and for studying vortical structures that may occur in the flow [35, 36]. Computational Fluid Dynamics (CFD) has focused on modelling the flow around a single wrapped-wire pin [37, 38], and on simulating the entire bundle for validation purposes [39, 40, 41, 42]. Recent CFD calculations revealed a very complex flow where the main flow follows the helical path of the wire wraps [43, 44].

Although the flow is mostly found to follow the direction of the wrapped wire, Ohtake et al. observed a flow bent against the direction of the wire [45]. This was called “migratory flow” and it was found to depend on the position of the wire around the rod. Similar findings were described in the numerical work of Song et al. for a 37-rod, wire-wrapped hexagonal bundle [46]. This interesting phenomenon was ascribed to the hydraulic resistance caused by the relative position of the wire inside the subchannel of the bundle. However, a physical explanation of migratory flow was not provided and it is still missing.

1.5 Scope of this work and thesis outline

The main aspects covered by this research are:

1. The effects of Reynolds number and $P/D$ ratio on frequency and size of coherent structures are investigated in a rectangular channel hosting two half-rods whose distance can be adjusted.

2. Predicting the wavelength of the coherent structures occurring in a rod bundle flow is also of primary importance for both designing experiments, and
safely operating a nuclear power plant. Therefore, a new correlation for estimating the size of the coherent structures has been derived and validated against the experiments of this work and performed on different geometries available in literature.

3. The role of coherent structures in inducing vibrations of the fuel rods is of primary interest for the safety of the reactor. Hence the role of coherent structures in flow-induced vibrations is studied in a 7-rod hexagonal bundle.

4. The complex, yet unexplained, migratory flow observed in rod bundle with helicoid wire spacer has been investigated and modelled in a 7-rod wire-wrapped, hexagonal bundle with the help of both experiments and a theoretical approach.

The outline of the thesis is as follows:

**Chapter 2** describes the experimental study carried out with a setup consisting of two half-rods facing each other inside a rectangular channel. The flow inside the gap between the rods is studied with Laser Doppler Anemometry (LDA) to assess the effects of Reynolds number and gap spacing on the size of coherent structures as well as on cross-flow. Several Reynolds numbers with three \( P/D \) ratios are investigated, being 1.07, 1.13, and 1.20, are studied.

**Chapter 3** is about the design and the making process of the two hexagonal rod bundles employed for the experiments described in the following chapters of the thesis. Both the bundles have \( P/D=1.11 \). One setup hosts a flexible section in the central rod to study FSI, whereas the second has a helicoid wire wrapped around each rod to study migratory flow.

**Chapter 4** describes the experimental campaign carried out with LDA to measure the size of coherent structures occurring in the flow in order to validate a newly derived correlation to predict their wavelength. A high speed camera is employed to measure vibrations induced on the central rod of the bundle (i.e. frequency and amplitude) at several Reynolds numbers.

**Chapter 5** describes the experiments performed with PIV in a 7-rod, wire-wrapped hexagonal bundle. The bending of the flow due to the action of the wire is measured near the central rod. A model for predicting the pressure gradient induced by the wire and for estimating the bending angle of the flow is derived and validated against the experiments.

**Chapter 6** presents the main findings of this work, and provides recommendations for future studies.
### 1.6 Nomenclature

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Description</th>
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<tbody>
<tr>
<td>$P/D$</td>
<td>Ratio between the pitch and the diameter of the half rods</td>
</tr>
<tr>
<td>CANDU</td>
<td>Canadian Deuterium Uranium</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>DOE</td>
<td>Department Of Energy</td>
</tr>
<tr>
<td>EIE</td>
<td>Extraneously-induced excitation</td>
</tr>
<tr>
<td>FIV</td>
<td>Flow-induced vibration</td>
</tr>
<tr>
<td>FSI</td>
<td>Fluid-structure interaction</td>
</tr>
<tr>
<td>Gen-IV</td>
<td>Generation IV</td>
</tr>
<tr>
<td>IAEA</td>
<td>International Atomic Energy Agency</td>
</tr>
<tr>
<td>IIE</td>
<td>Instability-induced excitation</td>
</tr>
<tr>
<td>LBE</td>
<td>Lead-bismuth eutectic</td>
</tr>
<tr>
<td>LDA</td>
<td>Laser Doppler Anemometry</td>
</tr>
<tr>
<td>LES</td>
<td>Large Eddy Simulations</td>
</tr>
<tr>
<td>LMFBR</td>
<td>Liquid Metal Fast Breeder Reactor</td>
</tr>
<tr>
<td>LWR</td>
<td>Light Water Reactor</td>
</tr>
<tr>
<td>MIE</td>
<td>Movement-induced excitation</td>
</tr>
<tr>
<td>PIV</td>
<td>Particle Image Velocimetry</td>
</tr>
<tr>
<td>URANS</td>
<td>Unsteady Reynolds-averaged Navier Stokes</td>
</tr>
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</table>

#### Non-dimensional groups

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re$</td>
<td>Reynolds number</td>
</tr>
</tbody>
</table>
Chapter 2

Vortex streets in a two half-rods compound channel

2.1 Introduction and motivation

Rod bundle flows characterise the core of LMFBR, PWR, BWR or CANDU reactors, as well as steam generators employed in the nuclear industry. In the presence of an axial flow of a coolant, this geometry leads to velocity differences between the low-speed region of the gap between two rods and the high-speed region of the main sub-channels. Moreover, a transversal flow of coherent structures across the gap between two rods can also occur. In a nuclear reactor cross-flow is important as it enhances the heat exchange between the nuclear fuel and the coolant. As a result, the fuel temperature decreases improving the safety performance of the reactor. Much research has been done in studying periodic coherent structures and gap instability phenomena in rod bundles resembling the core of LMFBRs, PWRs, BWRs and CANDUs [10, 11, 12, 16, 14, 15]. However, the effects that the gap spacing and the Reynolds number have on coherent structures and cross-flow is still unclear.

Therefore, this chapter presents the experimental work carried out in order to detect coherent structures and cross-flow inside a single gap between two half-rods facing each other. The distance between the rods, defined by the pitch-to-diameter ratio $P/D$, can be adjusted to three values. Near-wall measurements in water are performed with the non-intrusive Laser Doppler Anemometry (LDA) measurement system. The optical access is achieved by using Fluorinated Ethylene Propylene (FEP), whose refractive index matches the water’s in order to minimise distortion of light.

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2.2 Experimental apparatus

The CAMEL experimental loop consists of the test setup and the Laser Doppler Anemometry (LDA) system. The test section is a rectangular channel with two half-rods facing each other (Figure 2.1a), whose distance \( P/D \) can be adjusted. The water enters the facility from two inlets at the bottom and flows inside the lateral subchannels and through the gap defined by the rods. The flow rate is manually adjusted by two valves at the inlet lines and monitored by two pairs of magnetic flow-meters (Rosemount, USA) for inlet and outlet lines. At the measurement section, one of the two half-rods is made of FEP (Adtech Polymer Engineering), shown in Figure 2.1b, to match the refractive index of the water. A scheme of the loop is sketched in Figure 2.2a. FEP has the same refractive index of water at 20° \( (\eta_{\text{FEP}}=1.338 [17]; \eta=1.333 [47] \text{ with } 532\text{nm wavelength}) \), so it can be used to minimise the refraction of the laser light when the measurements are performed through the half-rod (Figure 2.2b). To reduce the distortion of light even more, the FEP half-rod is filled with water. The spacing between the rods can be adjusted to \( P/D \) ratios of 1.07, 1.13 and 1.20. The measured quantities are the stream-wise and span-wise velocity components and their fluctuations. The dimensions of the test section are given in table 2.1.

![Figure 2.1](image)

**Figure 2.1**: a) Main subchannels and central gap defined by the two half-rods. b) Hollow half-rod of FEP seen from the outside of the transparent test section: of the two half-rods the top grey one is the rod hosting the FEP section.
Figure 2.2: a) CAMEL test loop: a centrifugal pump provides the flow regulated by two valves operated manually at the inlet branches and monitored by 4 magnetic flow-recorders (FR). The water flows out from the top of the test section and it is collected inside a vessel. b) Part of the half-rod is replaced by FEP to let the laser light through during the measurements.

Table 2.1: CAMEL main dimensions. $D_r$: half-rod diameter, $L$: Perspex box long side, $H$: Perspex box short side, $t_{\text{PMMA}}$: Perspex wall thickness, $t_{\text{FEP}}$: FEP half-rod wall thickness, $\delta$: gap spacing, $P/D$: pitch-to-diameter ratio.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_r$</td>
<td>15 mm</td>
<td>mm</td>
</tr>
<tr>
<td>L</td>
<td>58.2  mm</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>26 mm</td>
<td></td>
</tr>
<tr>
<td>$t_{\text{PMMA}}$</td>
<td>8 mm</td>
<td></td>
</tr>
<tr>
<td>$t_{\text{FEP}}$</td>
<td>0.3 mm</td>
<td></td>
</tr>
<tr>
<td>$\delta(P/D=1.07)$</td>
<td>1 mm</td>
<td></td>
</tr>
<tr>
<td>$\delta(P/D=1.13)$</td>
<td>2 mm</td>
<td></td>
</tr>
<tr>
<td>$\delta(P/D=1.20)$</td>
<td>3 mm</td>
<td></td>
</tr>
</tbody>
</table>
2.3 Laser Doppler Anemometry system

The measurement system is a 2-components LDA system (DANTEC, Germany): a green laser beam pair (wavelength of 561 nm) measures the stream-wise velocity component and a yellow laser beam pair (wavelength of 532 nm) the lateral component with a maximum power of 300 mW. The measurement settings are chosen through the BSA Flow Software. The flow is seeded with particles to scatter the light and allow their detection inside the measurement volume. Borosilicate glass hollow spheres with an average density of $1.1 \text{ gm}^{-3}$ and a diameter of $9 - 12 \mu m$ are employed. The measurement region is formed by the two overlapping laser beams, forming an ellipsoidal measurement volume (or probe). In this region, the two beams form fringes whose distance $d_f$ is known through the following relation:

$$d_f = \frac{\lambda}{2 \sin \alpha}$$

(2.1)

where $\lambda$ is the wavelength of the laser light and $\alpha$ is the half-beam angle. If a particle goes through the measurement volume, the light that is scattered back has a different wavelength than the one hitting the particle. This difference depends on the particle’s velocity according to the Doppler effect (see Figure 2.3). The reflected light is collected by the receiving optics and converted into an electric current, called Doppler burst. This is a signal oscillating in time with frequency $f_D$. Finally, the velocity of the particle in a plane perpendicular to plane of the laser beam is calculated as

$$v = d_f \cdot f_D.$$  

(2.2)

A schematic of the LDA system is shown in Figure 2.3. The frequency of the light of one laser per pair is shifted to detect also the direction of motion of the particle. The LDA is moved by a traverse system and, to provide a dark background, the whole apparatus is enclosed by a black curtain.

2.3.1 LDA uncertainty

The uncertainty on the measurements has different expressions for mean velocities and root mean square values. They are defined as [49]

$$\sigma_T = \frac{\sigma}{\sqrt{N}} = \sqrt{\frac{1}{N(N-1)} \sum_{i=1}^{N} (v_i - \bar{v})^2}$$

and

$$\sigma_{\text{rms}} = \frac{\sigma_T}{\sqrt{2N}},$$

(2.3)

where $\sigma_T$ and $\sigma_{\text{rms}}$ are the uncertainty on the mean value and on the root mean square of the velocity components, $\sigma$ is the standard deviation of the measured samples, $N$ is the number of samples, and $\bar{v}$ is the mean velocity value.

Each measurements point is measured for a time window long enough to achieve sufficiently narrow confidence intervals. At high flow rates the recording time is set to 30 s whereas, for low flow rates, the recording time is as long as 120 s. $\sigma_{\text{rms}}$ is determined by the number of collected samples only. The most critical conditions are encountered at very low Reynolds numbers and in the centre of the gap because the laser beams must pass the FEP half-rod (see “path A”, Figure 2.4a ). Here,
Figure 2.3: Schematic of LDA measurement system. Image modified from [48]. Doppler effect: the frequency of light reflected by a moving particle differs from the one impinging on the target.

The maximum $\sigma_{\text{rms}}$ is 1.5%. $\sigma_T$ depends also on the mean velocity value $\bar{v}$ as well, thus the requirement are even more strict than for $\sigma_{\text{rms}}$. The lower the Reynolds number, the more samples are required and the longer the acquisition time is. With a $P/D$ of 1.20 (i.e. 3 mm gap spacing, see table 2.1), for example, $\sigma_T=0.8\%$ for the stream-wise component and becomes $\sigma_T=0.5\%$ when measuring from the side (path B). The span-wise velocity exhibits even more significant uncertainties since it is always characterised by near-zero values. $\sigma_T$ increases when the measurement volume approaches the wall (lower data rate) and if the gap width is reduced (see Figure 2.4a ). In the latter case, the issue of the light reflected into the photodetector can be tackled to some extent (see section 2.5.5).

2.3.2 Slotting technique

One of the most common techniques for computing the frequency spectrum of a signal is the Fast Fourier Transform (FFT), which requires regularly-sampled data. However, with LDA measurements, the inter-arrival time (time between two detected particles) is not constant as it depends on when the particle is carried through the measuring laser probe by the flow. This issue can sometimes be overcome by applying the sample-and-hold algorithm to the original velocity signal. This means that the velocity of each sample is kept constant (“hold”) until the next particle is detected. The entire signal is then resampled with constant time intervals, and then the FFT is applied. However the sample-and-hold technique followed by FFT works well only with
2.4 Measurement campaign

The measurements are conducted along two paths: along the symmetry line of the gap, from one sub-channel to the other, and at the centre of the gap along the rod-to-rod direction. For each $P/D$ ratio, different flow rates, hence different Reynolds numbers, are studied. The first series of measurements is done by shooting the laser through the FEP half-rod (Figure 2.4a) and by mapping the symmetry line through the gap.

high data rate. If the data rate is low or if the inter-arrival times differ by much, the effect of the so-called white noise on the spectrum becomes more important [50]. The longer the “hold” phase, the more the re-sampled signal is biased with respect to the real one.

Alternatively, one can calculate the discretized autocorrelation function of the velocity by means of the Slotting technique [51], and then use the Fourier transform to compute the spectrum. Sample pairs with inter-arrival time falling within a certain time interval (lag time) are allocated into the same time slot of width $\Delta \tau$, hence the name of the method. Then the ensemble average of the cross-products falling within each slot is calculated as

$$\rho_{ac}(k\Delta \tau) = \frac{\sum v_i(t_i) v_j(t_j)}{N_{\text{slot}}} \quad k=0,1,2,\ldots M-1,$$  \hspace{1cm} (2.4)

where $\rho_{ac}(k\Delta \tau)$ is the autocorrelation coefficient for the time slot $k\Delta \tau$, $v_i(t_i) v_j(t_j)$ are the cross-products whose arrival time differences falls within the considered lag time. In mathematical terms that is

$$\Delta \tau \left( k-\frac{1}{2} \right) \leq t_i - t_j \leq \Delta \tau \left( k+\frac{1}{2} \right).$$

The frequency spectrum is then calculated as

$$S = \frac{\Delta \tau}{\pi} \left[ \frac{1}{2} \rho_{ac}(0) + \sum_{k=1}^{M-1} \rho_{ac}(k\Delta \tau) \cos(2\pi f k\Delta \tau) \right].$$  \hspace{1cm} (2.5)

The randomness of the sampling process increases the variance of the spectrum. This can be reduced by increasing the mean seeding data rate through the probe. However, this is not always possible, especially in regions characterised by low velocity (i.e. the centre of the gap) or with low flow rates. Therefore, the so-called Fuzzy algorithm is implemented in the Slotting technique. Cross-products with inter-arrival time closer to the centre of a slot contribute more to the autocorrelation estimation [52, 53].

Generally the spectrum can also be biased towards higher velocities (i.e. higher frequencies) since the amount of high speed particles going through the measurement probe is larger than for low speed particles [50]. Hence, their contribution to the spectrum would be higher than in reality. The Slotting technique hereby implemented adopts the transit time algorithm to weight the velocity samples by their residence time within the measurement probe to decrease the velocity bias.
Figure 2.4: a) Top view of the measurement crossing the FEP. The ellipsoidal measurement volume is represented as well; the solid green line represents the laser beam (sketch not drawn to scale). b) Top view of the measurement without crossing the FEP. The measurement paths are the dashed lines.

The second series of experiments is done with the light entering the setup through the short Perspex side (Figure 2.4b), without crossing the FEP. In this case the measurements are performed along both the symmetry line through the gap and normal to the rods at the centre. The Reynolds number of the bulk flow, $Re$, is calculated based on the bulk velocity:

$$Re = \frac{\rho \cdot V_b \cdot D_h}{\mu}, \quad (2.6)$$

where $\rho$ is the water density, $\mu$ is the water dynamic viscosity, $V_b$ is the bulk velocity calculated as $V_b = Q/A$ where $Q$ is the total flow rate and $A$ is the total flow area, $D_h = 4A/P_w$ is the hydraulic diameter of the test section, being $P_w$ the wetted perimeter. The Reynolds number of the gap region, $Re_{gap}$, is calculated as

$$Re_{gap} = \frac{\rho \cdot v_{gap} \cdot D_h^*}{\mu}, \quad (2.7)$$

where $D_h^*$ is the hydraulic diameter of the gap region defined by the flow area bounded by the two half-rod walls and closed by the gap borders at the rod ends. $v_{gap}$ is the average stream-wise velocity through the gap region, which is measured over the area $A$ shown in Figure 2.5. The average stream-wise gap velocity $v_{gap}$ is calculated as

$$v_{gap} = \frac{1}{A_{gap}} \int_{z_1}^{z_2} \int_{x_L}^{x_R} v(x,z) \, dx \, dz, \quad (2.8)$$

where $x_L$, $x_R$, $z_1$, $z_2$ are the coordinates defining the area $A$.

$Q$, $Re$, $V_b$ and $Re_{gap}$ for the three $P/D$ ratios are reported in table 2.2.
2.5. Results and discussion

### 2.5.1 Stream-wise velocity r.m.s. through the gap

The stream-wise velocity component $v$ and its root mean square $v'_{\text{rms}}$ are measured along “path A, No-FEP” (Figure 2.4b). The data are then corrected for the refraction of light due to the Perspex wall (later discussed in section 2.5.6). The main subchannels are located at $|x/D_r|=1$, where the stream-wise velocity profile reaches the highest value, whereas the gap borders are at $|x/D_r|=0.5$. The centre of the gap is at $x/D_r=0$, where the minimum in the velocity profile is measured. The normalised difference between the velocity in the bulk and in the gap increases if either the Reynolds number or the $P/D$ decrease.

Figure 2.7 compares the results obtained with this geometry and the geometry.
used by Mahmood [17] at similar values of $Re$. The normalised velocity difference between the bulk region and the gap centre is larger with the two half-rods geometry ($\square$) than with only one half-rod ($\times$); this is more evident with the lower Reynolds number. The $v'_{\text{rms}}$ profiles shown Figure 2.8 correspond to $P/D=1.07$; the horizontal coordinate is normalised by the rod diameter $D_r$. The $v'_{\text{rms}}$ profile of Figure 2.8a has two peaks at the borders of the gap ($|x/D_r|=0.5$) and a dip region in the centre. As the measurement approaches the walls of the Perspex encasing ($|x/D_r|>1$) the $v'_{\text{rms}}$ increases like in common wall-bounded pipe flows. The water enters the facility via two bent rubber pipes next to each other. The bend causes a non-zero lateral momentum transfer among the subchannels that is responsible for the asymmetry of the $v'_{\text{rms}}$ profile at $|x/D_r|\approx 0.5$ (Figure 2.8a). The lower $Re$ case reported in Figure 2.8a exhibits a symmetric $v'_{\text{rms}}$ profile with respect to the gap centre. If the $P/D$ ratio is increased to 1.13 and 1.20, the $v'_{\text{rms}}$ profile appears symmetric (Figure 2.9). The gap region acts as a damping region for flow oscillations [14], especially with smaller gaps where the confinement of lateral momentum within the sub-channel is more dominant. If the gap size is increased, such transversal components may redistribute more freely among the subchannels, which may explain why the $v'_{\text{rms}}$ profile is more symmetric with larger gaps.

2.5.2 Stream-wise velocity r.m.s. from rod to rod

The $v'_{\text{rms}}$ is measured for each $P/D$ ratio and flow rate at the centre of the gap, from rod to rod. The results for each $Re$ value are plotted against the rod-to-rod coordinate $z$, normalized by the gap spacing $\delta$. The flow shows some analogy
2.5. RESULTS AND DISCUSSION

**Figure 2.7:** Comparison between the stream-wise velocity profile with \( P/D = 1.13 \) and experiments from Mahmood [17]. a) Stream-wise velocity normalised by the bulk velocity at \( Re = 3600 \) compared with data obtained at \( Re = 3440 \). b) Stream-wise velocity normalised by the bulk velocity at \( Re = 12000 \) compared with data obtained at \( Re = 15400 \). Data from [17] are obtained from a similar geometry consisting of one half-rod.

**Figure 2.8:** a): \( v'_{rms} \) profile along the gap; \( P/D = 1.07, Re = 29000 \). The asymmetry is due to the lateral momentum component of the flow in the main sub-channels. b): \( v'_{rms} \) profile along the gap; \( P/D = 1.07, Re = 3600 \).

with common wall-bounded flows in that it features two near-wall peaks where the viscous stresses equal the Reynolds shear stresses [54] and the turbulent production is the highest. A dip occurs in the centre for \( Re = 29000, Re = 20000 \) and \( Re = 12000 \) (Figure 2.10). \( v'_{rms} \) decreases approaching the rod walls due to the viscous sub-layer: velocity fluctuations can still be detected inside this region but they are produced in the outer log-layer region [55]. With \( Re = 12000 \) and \( P/D = 1.07 \), a weak third peak in the \( v'_{rms} \) appears between the rod walls. As \( Re \) is decreased to 6500, this additional peak becomes clearer and dominant over the near-wall peaks. The \( v'_{rms} \) profiles for \( P/D = 1.13 \) and \( P/D = 1.20 \) do not display such a peak, although the near-wall peaks become less sharp. The results of the measurements performed with the four lower Reynolds numbers are reported in Figure 2.11. The \( v'_{rms} \) profile with \( Re = 3600 \) increases towards the centre of the gap for \( P/D = 1.07 \) and \( P/D = 1.13 \), whereas the case with \( P/D = 1.20 \) still displays a weak dip. If \( Re \) is further decreased...
Figure 2.9: $v'_{\text{rms}}$ profile along the gap; $P/D=1.20$, $Re=29000$.

Figure 2.10: $v'_{\text{rms}}$ at the centre of the gap (path B; No-FEP), between the half-rod walls. a) $Re=29000$, b) $Re=20000$, c) $Re=12000$, d) $Re=6500$; $P/D=1.20$ (black), $P/D=1.13$ (red) and $P/D=1.07$ (blue). As $Re$ decreases, a weak peak appears first with $P/D=1.07$ ($Re=12000$), which is also found with $P/D=1.13$ and $Re=6500$.

to 2400 the $P/D$ ratios have all the same increasing trend towards the centre. With $Re=1200$ and $Re=600$, the $v'_{\text{rms}}$ profile varies significantly depending on the $P/D$ ratio. The central $v'_{\text{rms}}$ peak can be originated by the transport of turbulence from the borders of the gap region (where the production is higher) towards the centre by means of cross-flow. The analysis of the frequency spectrum of the spanwise velocity component can confirm this assumption: the periodical transversal flow would appear as a peak in the frequency spectrum of the measured velocity component [12, 16].

2.5.3 Cross-flow pulsations through the gap

The study of the autocorrelation function and the frequency spectrum of the spanwise (lateral) velocity component $u$ is a powerful method to detect and characterize
Figure 2.11: $v'_{\text{rms}}$ at the centre of the gap (path B; No-FEP), between the half-rod walls. a) $Re=3600$, b) $Re=2400$, c) $Re=1200$, d) $Re=600$; $P/D$ ratio of 1.2 (black), 1.13 (red) and 1.07 (blue). As the $Re$ is further decreased, the $P/D=1.20$ also leads to an increase of turbulence between the rod walls, in the centre of the gap.

periodicities in the flow. The frequency spectrum of $u$ is calculated with the Slotting technique [56, 51, 57]. The span-wise velocity is measured across the FEP half-rod (“path A, FEP”), as shown in Figure 2.4a, at several points between the bulk region of the flow and the centre of the gap; the spectrum is then calculated. The spectra exhibit a peak at different measurement locations near the centre for $Re \leq 6500$, as shown in Figure 2.12. The peak in the power spectra proves that there is a low-frequency periodic behaviour in the span-wise velocity component of the flow near the centre of the gap. This can be the result of large coherent flow structures forming near the rod edges and periodically crossing the gap. The spectral peaks are fitted with a Gaussian bell so that the mean and the bell’s width are estimated. The cross-flow frequency decreases with increasing gap spacing (Figure 2.13a) within the Reynolds number range $2400 - 6500$, as found also by Wu and Trupp [58]. The frequency values are also expressed as a non dimensional quantity via the Strouhal number (Figure 2.13b), defined as

$$Str = \frac{f_x \sqrt{D_r \delta}}{v_{in}}$$

(2.9)

where $f_x$ is the average frequency at which the structures cross the gap, $D_r$ is the half-rod diameter, $\delta$ is the gap spacing and $v_{in}$ is the value of $v$ estimated at the inflection point ($\partial^2 v / \partial x^2 = 0$) of the velocity profile measured along path A (“path A, No-FEP” Figure 2.4b): here the velocity gradient is the largest [59]. The rod diameter $D_r$ and the gap spacing $\delta$ are important parameters in experiments with rod bundles. The characteristic length scale of the Strouhal number includes both,
Figure 2.12: Frequency spectrum of the span-wise velocity component $u$ for $Re=6500$ and $P/D=1.07$; three locations near the centre of the gap are shown. A peak is evident near $3.8\text{Hz}$. The fit with a Gaussian bell yields a mean standard deviation of $1.47\text{Hz}$.

Equation 2.10 describes the overall trend of the experimental points measured for three $P/D$ values in the range $2400 \leq Re \leq 6500$. It has to be pointed out that this correlation is an estimation of the global trend. However, if the data series corresponding to the three $P/D$ ratios are considered separately, the dependence between $1/Str$ and $P/D$ is not necessarily linear.

The lateral frequency of the structures seems to be independent of the Reynolds number and to be affected only by the geometry of the channel, for $Re \geq 2400$ (Figure 2.14a). However, at low Reynolds numbers, $Str$ decreases with the Reynolds number. These results and those of Möller [12] are compared in Figure 2.14b. He defined the Strouhal number in a different way, being

$$Str = \frac{f_x \cdot D}{u_\tau},$$

where $u_\tau$ is the friction velocity defined as

$$u_\tau = \sqrt{\frac{\mu}{\rho} \left( \frac{\partial v}{\partial z} \right)_{\text{wall}}},$$

where $\left( \frac{\partial v}{\partial z} \right)_{\text{wall}}$ is the velocity gradient near the wall of the rod.
2.5. RESULTS AND DISCUSSION

Figure 2.13: a) Average span-wise frequency against $P/D$ for three values of $Re$. b) $1/Str$ against $P/D$: experimental results and proposed correlation.

Figure 2.14: a) Average non-dimensional span-wise frequency versus $Re$ for three $P/D$ ratios. b) $1/Str$ against the Reynolds number for the three $P/D$ values compared with [12].

2.5.4 Gap vortex streets

The stream-wise velocity component $v$ measured in the left part of the gap (“path A, No-FEP” Figure 2.4b) is used to calculate the frequency spectrum. The average frequency of the spectral peak $f_y$ is plotted at the location where the coherent structures are detected within the gap (Figure 2.15). This represents the spatial distributions of the coherent structures depending on the considered Reynolds number. Peaks in the frequency sectrum are found for all the $P/D$ ratios at different locations within the gap and inside the main sub-channel close to the gap borders, suggesting gap vortex streets moving along with the stream. The case with $P/D=1.07$ (Figure 2.15a) shows that $f_y$ increases with $Re$. For $Re=600$ the flow structures extend well within the main sub-channel whereas, as the Reynolds increases, the structures are localised within the gap and at the gap border. The case with $P/D=1.13$ (Figure 2.15b) shows once more that the frequency increases with $Re$. However, the spatial distribution is more scattered at high Reynolds.
This result indicates that the structures tend to move closer to the gap centre as the Reynolds increases. Taylor’s hypothesis (coherent structures in the flow are regarded as frozen entities moving with the stream at velocity \( v_{\text{in}} \)) enables to estimate the average length of the vortices. Although this assumption may become inaccurate with very long structures [61], experiments in bundles show that these vortices move with a convection velocity which is independent of the position inside the gap [8]. The structure length is calculated as

\[
\lambda = \frac{v_{\text{in}}}{f_y},
\]  

(2.13)

where \( v_{\text{in}} \) is the stream-wise convection velocity measured at the inflection point of the velocity profile through the gap (“path A, No-FEP” Figure 2.4b). The spectral peak is fitted with a Gaussian bell to retrieve the mean frequency value and the bell’s width \( \sigma_y \). This gives a frequency interval \( f_y \pm \sigma_y \) to estimate also the lower and upper limit around the mean structure length \( \lambda \):

\[
\lambda_{\text{min}} = \frac{v_{\text{in}}}{f_y + \sigma_y} \quad \lambda_{\text{Max}} = \frac{v_{\text{in}}}{f_y - \sigma_y}
\]  

(2.14)

The mean wavelength of the structures is shown in Figure 2.16 as a function of \( Re \). Coherent structures become longer with \( Re \leq 2400 \) consistently with results presented in [17, 62] for compound channels. With increasing \( Re \), \( \lambda \) tends to reach the asymptotic value of

\[
\frac{\lambda}{D_h} \approx 14,
\]  

(2.15)

as observed in [14].

If \( P/D \) increases (i.e. larger gap spacing), the structures become longer, within the Reynolds range 2400 – 29000. From Figure 2.16a it appears that with \( Re \geq 2400 \)
the length of the periodical structures is merely affected by geometrical parameters such as the gap spacing, confirming, thus, one of the findings reported in [18, 19]. The velocity difference between the bulk flow and the centre of the gap, normalised by the bulk velocity, is plotted in Figure 2.17 for the three gap sizes. Its trend versus the Reynolds number is similar to the what is observed with the wavelength of the structures. For $Re>6500$ the velocity difference becomes almost constant for increasing values of Reynolds, whereas for $Re\leq 6500$ it increases as the Reynolds number decreases. Moreover, it is evident from the figure that a smaller gap induces a higher velocity differences between the gap region and the main subchannel.

Since the driving force of coherent structures is the velocity difference between the bulk flow and the centre of the gap, the almost-constant profile that this assumes for high Reynolds numbers could be related the constant length of coherent structures in the same range of Reynolds. However, for $Re\leq 2400$ the Reynolds

Figure 2.16: a) Absolute and b) normalised stream-wise coherent structures size as a function of $Re$ for three $P/D$ ratios. The experiments are compared with data from [17]. The normalised data are scaled with the hydraulic diameter of the gap region.

Figure 2.17: Normalised velocity difference between the bulk of the flow and the centre of the gap for three $P/D$ ratios.
The lengthening of the structures at low flow rates, and the widening of the region where they are found (Figures 2.15a, 2.15b) seem to indicate that these structures grow both in length and in width as the Reynolds decreases, as hypothesized in [63].

An almost-constant wavelength of the coherent structures allows to compare the turbulent dissipation rate of the flow at different Reynolds numbers, as explained below.

According to Kolmogorov’s length scale, the ratio between the largest and smallest vortices occurring in the flow, \(d_{\text{Max}}\) and \(d_{\text{min}}\) respectively, is proportional to \(Re^{3/4}\) [64]:

\[
\frac{d_{\text{Max}}}{d_{\text{min}}} \approx Re^{3/4}.
\] (2.16)

Assuming that the largest eddies occurring in the flow correspond to the measured coherent structures, that is \(d_{\text{Max}}=\lambda\), the previous expression becomes

\[
\frac{\lambda}{d_{\text{min}}} \approx Re^{3/4}.
\] (2.17)

The wavelength \(\lambda\) can be expressed as

\[
\lambda \approx d_{\text{min}}Re^{3/4}.
\] (2.18)

From the experiments it follows that, for \(Re \geq 2400\), \(\lambda\) is weakly dependent on the Reynolds number (Figure 2.16a), so the following approximation is done (accurate within 15% of the mean value):

\[
[\lambda]_{Re=2400} \approx [\lambda]_{Re=29000}.
\] (2.19)

From Equation 2.18 follows that

\[
\left[ d_{\text{min}} \cdot Re^{3/4} \right]_{Re=2400} \approx \left[ d_{\text{min}} \cdot Re^{3/4} \right]_{Re=29000}.
\] (2.20)

\(d_{\text{min}}\) is defined by the Kolmogorov microscale as \(d_{\text{min}}=v^{3/4}\varepsilon_{\text{d}}^{-1/4}\) where \(v\) is the kinematic viscosity and \(\varepsilon_{\text{d}}\) is the energy dissipation rate. Substituting this expression into the previous equation leads to

\[
\left[ v^{3/4}\varepsilon_{\text{d}}^{-1/4} Re^{3/4} \right]_{Re=2400} \approx \left[ v^{3/4}\varepsilon_{\text{d}}^{-1/4} Re^{3/4} \right]_{Re=29000}.
\] (2.21)

Since \(v\) is constant, the equation is simplified into

\[
\left[ \varepsilon_{\text{d}}^{-1/4} Re^{3/4} \right]_{Re=2400} \approx \left[ \varepsilon_{\text{d}}^{-1/4} Re^{3/4} \right]_{Re=29000}.
\] (2.22)

This relation states that the quantity \(Re^3/\varepsilon_{\text{d}}\) is conserved if the wavelength of coherent structures does not change within the considered range of the Reynolds number:

\[
\frac{Re^3}{\varepsilon_{\text{d}}} \approx \mathcal{K}_1,
\] (2.23)
where $\mathcal{K}_1$ is a constant and $\text{Re} \in [2400; 29000]$. This equation is useful to compare the turbulent dissipation rate $\varepsilon_d$ measured at two different Reynolds numbers. Equation 2.23 can be applied to the cases with $\text{Re} = 2400$ and $\text{Re} = 29000$:

$$
\left[ \frac{\text{Re}^3}{\varepsilon_d} \right]_{\text{Re}=2400} = \left[ \frac{\text{Re}^3}{\varepsilon_d} \right]_{\text{Re}=29000},
$$

which yields

$$
\left[ \frac{\varepsilon_d}{\varepsilon_d} \right]_{\text{Re}=29000} = \left( \frac{29000}{2400} \right)^3 \approx 1780. \quad (2.25)
$$

An almost-constant wavelength of coherent structures implies that the dissipation rate with $\text{Re} = 29000$ is approximately three orders of magnitude larger than with $\text{Re} = 2400$.

The remainder of the chapter compares the $v'_{\text{rms}}$ measured along path A for the “FEP” and “No-FEP” cases (see Figures 2.4a and 2.4b) to assess the effects of the refraction and reflection of light.

### 2.5.5 Light reflection effects

When measuring along “path A, FEP” (see Figure 2.4a) the measurement is disturbed by the light reflected from the second rod behind the ellipsoidal laser probe. As the probe is moved further towards the centre of the gap the reflection becomes increasingly stronger, especially with $P/D = 1.07$. This issue is tackled by filtering out the near-zero velocity samples caused by the reflective surface. This improves the results as long as the ellipsoidal volume fits the gap and the flow speed is not too close to zero. The cases where the filter is successfully applied are shown in Figure 2.18. Although the filtered $v'_{\text{rms}}$ shows yet some dispersion close to the left border of the gap ($x/D_r = -0.5$ in Figure 2.18a), it decreases the overall scattering of the experimental values. As the probe approaches the FEP borders, the signal drops because these are the regions where the light attenuation is the highest.

### 2.5.6 Light refraction effects

In one case (Figure 2.4a) the refraction occurs when the laser crosses the FEP rod and in the other case (Figure 2.4b) refraction is caused by the Perspex wall as the probe volume moves further inside the test section. The results of the measurements for both cases are corrected to compensate for this issue. Referring to Figure 2.20a, the horizontal distortion of the light ray due to FEP is estimated as follows:

$$
\Delta x = AB \sin(\alpha_1 - \alpha_3). \quad (2.26)
$$

For a complete derivation of the previous equation, the reader is directed to Appendix A.

The refraction of the laser beam through the Perspex wall is discussed with the aid of Figure 2.20b. The position of the probe volume inside the setup, corrected by the refraction due to the Perspex wall, is given by
Figure 2.18: Comparison between the $v'_{\text{rms}}$ affected by light reflection from the wall (red) and filtered (blue). a) $P/D=1.13$ a) $Re=6500$, b) $Re=3600$. c) $P/D=1.20$, $Re=3600$.

\[ \Delta x = t_{\text{PMMA}} - X_0 + \frac{X_0 \tan \alpha_5 - t_{\text{PMMA}} \tan \alpha_6}{\tan \alpha_7}, \]

where $t_{\text{PMMA}}$ is the Perspex wall thickness, $X_0$ is the position of the probe volume without refraction and $\alpha_5=5.711^\circ$ is the half-beam aperture of the laser; $\alpha_6$ and $\alpha_7$ are calculated through the Snell’s law. The $v'_{\text{rms}}$ measured through the FEP half-rod (“path A, FEP”) and from the short side (“path A, No-FEP”) are shown in Figure 2.19. The two $v'_{\text{rms}}$ profiles are yet slightly shifted with respect to each other after the refraction correction is applied: Equations 2.26 and 2.27 depend on $t_{\text{PMMA}}$, $t_{\text{FEP}}$ and on $R_{i,FEP}$ which vary due to the dimensional tolerance of the material. This introduces a source of uncertainty in the refraction calculation. Moreover, when the laser reaches the FEP borders ($|x/D_r|=0.5$), the light is not transmitted anymore and the signal drops to zero.

### 2.5.7 Laser probe size effects

The refraction of the laser beam pair affects the size of the probe volume as well [65]. The length of its long axis, in air, is calculated as

\[ l_{p,a} = \frac{d_0}{\sin \alpha_5} = 0.9 \text{ mm}, \]
2.5. RESULTS AND DISCUSSION

Figure 2.19: $v'_{\text{rms}}$ measured along “path A, FEP” and “path A, No-FEP” against the position along the gap normalized by the half-rod diameter. $P/D=1.20$, $Re=29000$.

Figure 2.20: a) Top view of the refraction of the green laser beam pair due to FEP: the light ray goes through the FEP half-rod, filled with water, and it is refracted as it crosses its wall. b) Refraction of the laser beam as it crosses the FEP half-rod.

where $d_0$ is the laser beam diameter at the focal point [66]. Applying the same relation to the laser beam in water yields a longer probe whose length is

$$l_{p,w} = \frac{d_0}{\sin \alpha_7} = 1.2 \text{ mm},$$

where $\alpha_7$ is the half-beam angle of the laser in water (see Figure 2.20b). If the measurement is performed along “path A, FEP” (Figure 2.4a), the ellipsoidal probe volume is oriented with the longest axis normal to the rods. The longer probe is more difficult to fit in the centre of the gap with $P/D=1.07$, where $\delta=1 \text{ mm}$. This also implies an increased reflection of light from the rod wall. The $v'_{\text{rms}}$ measured with the laser light going through the FEP and from the free side of the setup are compared to assess the influence of a longer probe. Figure 2.21 shows that the $v'_{\text{rms}}$ measured through the FEP rod has a peak at the centre of the gap ($\times$). The light reflected by the rod behind the probe is regarded by the software as particles with near-zero velocity, which increase the deviation around the mean value (the root
mean square). The $v'_{\text{rms}}$ profiles measured at lower flow rates are shown in Figure 2.22. In these cases the $v'_{\text{rms}}$ profile does not show the central peak found at higher Reynolds numbers. Although the light reflection and the elongated probe volume still contribute with near-zero velocity signals, the $v'_{\text{rms}}$ is not peaked because the flow velocity closer to zero reduces the statistical deviation from the mean value. With a larger gap (Figure 2.23), reflections are weaker and the probe fits better inside the gap, improving the results.
2.5. RESULTS AND DISCUSSION

Figure 2.22: $v'_{\text{rms}}$ along “path A, FEP” ($\times$) and along “path A, No-FEP” from the second side (□). a) $Re=3600$, b) $Re=2400$, c) $Re=1200$, d) $Re=600$; $P/D=1.07$.

Figure 2.23: $v'_{\text{rms}}$ along “path A, FEP” ($\times$) and along “path A, No-FEP” from the second side (□). a) $Re=29000$, b) $Re=20000$, c) $Re=12000$, d) $Re=6500$; $P/D=1.20$. 
2.6 Summary

The flow between two half-rods in a square channel was measured by Laser Doppler Anemometry with three $P/D$ ratios and at several Reynolds numbers, ranging from 600 to 29000. As the flow rate decreased, an additional peak in the root mean square of the stream-wise velocity was found at the centre of the gap. It became clearer and occurred at higher Reynolds values as the gap spacing was reduced. This peak can be ascribed to coherent structures moving through the gap that enhance cross-flow. The power spectrum of the span-wise velocity exhibited a peak near the gap centre revealing the presence of such periodical structures in the transversal direction.

The study of the stream-wise velocity component found coherent structures near the gap border whose wavelength is affected by the geometry, and by the Reynolds only when this reaches low values due to the increased velocity difference between the bulk of the flow and the gap region. Moreover, as the Reynolds was decreased, these structures were found also further away from the border inside the main sub-channel, suggesting that coherent structures may grow not only in length, but also in width.

The refraction of the laser beam pairs in water led to an elongation of the laser probe that intensified the light reflection when measuring through the FEP normal to the rods, especially in the middle of the gap. With $P/D$ of 1.20 and 1.13 reflections could be filtered out. However, a $P/D$ of 1.07 led to biased measurements in the centre since the measurement probe came in contact with the rod walls. Nevertheless, the central region of the gap could be properly studied by measuring through the short side of the Perspex casing (“path A-B, No-FEP”).
## 2.7 Nomenclature

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Flow area</td>
<td>$\text{m}^2$</td>
</tr>
<tr>
<td>$A_{\text{gap}}$</td>
<td>Flow area considered to estimate the gap Reynolds</td>
<td>$\text{m}^2$</td>
</tr>
<tr>
<td>$D_r$</td>
<td>Diameter of the half rods</td>
<td>m</td>
</tr>
<tr>
<td>$D_h$</td>
<td>Hydraulic diameter of the channel</td>
<td>m</td>
</tr>
<tr>
<td>$D_h^*$</td>
<td>Hydraulic diameter of the gap region</td>
<td>m</td>
</tr>
<tr>
<td>$d_f$</td>
<td>Fringe distance</td>
<td>nm</td>
</tr>
<tr>
<td>$d_{\text{Max}}, d_{\text{min}}$</td>
<td>Characteristic length scale of the largest and smallest vortices based on Kolmogorov theory</td>
<td>m</td>
</tr>
<tr>
<td>$d_0$</td>
<td>Laser beam diameter</td>
<td>mm</td>
</tr>
<tr>
<td>$f_x, f_y$</td>
<td>Span-wise (transversal) and stream-wise (vertical) frequency of coherent structures in the flow</td>
<td>Hz</td>
</tr>
<tr>
<td>$f_D$</td>
<td>Frequency of the Doppler burst</td>
<td>Hz</td>
</tr>
<tr>
<td>$H, L$</td>
<td>Dimension of the rectangular channel</td>
<td>m</td>
</tr>
<tr>
<td>$l_{p,a}, l_{p,w}$</td>
<td>Length of the laser probe volume in air and water, respectively</td>
<td>mm</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of collected samples per LDA measurement point</td>
<td>–</td>
</tr>
<tr>
<td>$N_{\text{slot}}$</td>
<td>Number of cross-product allocated within each slot of the autocorrelation function</td>
<td>–</td>
</tr>
<tr>
<td>$P/D$</td>
<td>Ratio between the pitch and the diameter of the half rods</td>
<td>–</td>
</tr>
<tr>
<td>$P_w$</td>
<td>Wetted perimeter</td>
<td>m</td>
</tr>
<tr>
<td>$Q$</td>
<td>Volumetric flow rate</td>
<td>$\text{m}^3\text{s}^{-1}$</td>
</tr>
<tr>
<td>$R_i, R_{\text{FEP}}$</td>
<td>Inner radius of FEP half-rod</td>
<td>mm</td>
</tr>
<tr>
<td>$S$</td>
<td>Spectrum of the measured time-dependent velocity signal, based on the autocorrelation function</td>
<td>s</td>
</tr>
<tr>
<td>$t_{\text{FEP}}, t_{\text{PMMA}}$</td>
<td>Thickness of FEP and Perspex wall, respectively</td>
<td>m</td>
</tr>
<tr>
<td>$u$</td>
<td>Span-wise (transversal) velocity component</td>
<td>$\text{ms}^{-1}$</td>
</tr>
<tr>
<td>$u_f$</td>
<td>Friction velocity</td>
<td>$\text{ms}^{-1}$</td>
</tr>
<tr>
<td>$v$</td>
<td>Stream-wise (Vertical) velocity component</td>
<td>$\text{ms}^{-1}$</td>
</tr>
<tr>
<td>$v_{i(t_i)}, v_{j(t_j)}$</td>
<td>Velocity samples used in the Slotting technique</td>
<td>$\text{ms}^{-1}$</td>
</tr>
<tr>
<td>$V_b$</td>
<td>Bulk velocity</td>
<td>$\text{ms}^{-1}$</td>
</tr>
<tr>
<td>$V_c$</td>
<td>Stream-wise (vertical) velocity component in the centre of the gap</td>
<td>$\text{ms}^{-1}$</td>
</tr>
<tr>
<td>$v_{\text{gap}}$</td>
<td>Stream-wise (vertical) velocity component in the gap region</td>
<td>$\text{ms}^{-1}$</td>
</tr>
<tr>
<td>$v_{\text{in}}$</td>
<td>Value at the inflection point in the profile of the stream-wise (vertical) velocity component</td>
<td>$\text{ms}^{-1}$</td>
</tr>
<tr>
<td>$v'_{\text{rms}}$</td>
<td>Root mean square of the stream-wise (vertical) velocity component</td>
<td>$\text{ms}^{-1}$</td>
</tr>
<tr>
<td>$x, y, z$</td>
<td>Span-wise (transversal), stream-wise (vertical), and normal directions</td>
<td>m</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>-------</td>
</tr>
<tr>
<td>( x_L, x_R )</td>
<td>Borders of the flow area where the gap Reynolds is evaluated, along the span-wise (transversal) direction</td>
<td>m</td>
</tr>
<tr>
<td>( \Delta x )</td>
<td>Lateral refraction of the laser beam</td>
<td>m</td>
</tr>
<tr>
<td>( X_0 )</td>
<td>Position of the laser probe volume without refraction</td>
<td>m</td>
</tr>
<tr>
<td>( z_1, z_2 )</td>
<td>Borders of the flow area where the gap Reynolds is evaluated, along the normal direction</td>
<td>m</td>
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</table>

**Greek letters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7 )</td>
<td>Main angles for estimating the refraction of the laser</td>
<td>°</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Gap width between the half-rods</td>
<td>mm</td>
</tr>
<tr>
<td>( \varepsilon_d )</td>
<td>Dissipation rate based on Kolmogorov theory</td>
<td>ms(^{-3})</td>
</tr>
<tr>
<td>( \eta_{\text{FEP}}, \eta_{\text{PMMA}}, \eta )</td>
<td>Refractive indices of FEP, Perspex, and water</td>
<td>–</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Average stream-wise (vertical) wavelength of coherent structures</td>
<td>m</td>
</tr>
<tr>
<td>( \lambda_{\text{Max}}, \lambda_{\text{min}} )</td>
<td>Maximum and minimum estimated stream-wise wavelength of coherent structures</td>
<td>m</td>
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<tr>
<td>( \lambda_l )</td>
<td>Wavelength of the laser</td>
<td>nm</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Dynamic viscosity of water</td>
<td>Pas</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Kinematic viscosity</td>
<td>ms(^{-1})</td>
</tr>
<tr>
<td>( \rho_{\text{ac}} )</td>
<td>Autocorrelation coefficient</td>
<td>–</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Density of water</td>
<td>kg m(^{-3})</td>
</tr>
<tr>
<td>( \sigma_\tau )</td>
<td>95% confidence interval of the mean velocity</td>
<td>ms(^{-1})</td>
</tr>
<tr>
<td>( \sigma_{\text{rms}} )</td>
<td>95% confidence interval of the root mean square velocity</td>
<td>ms(^{-1})</td>
</tr>
<tr>
<td>( \sigma_y )</td>
<td>Width of the Gaussian bell fitting the frequency spectra</td>
<td>Hz</td>
</tr>
<tr>
<td>( \Delta \tau )</td>
<td>Time slot of the Slotting technique</td>
<td>s</td>
</tr>
<tr>
<td>( \left( \frac{\partial v}{\partial z} \right)_{\text{wall}} )</td>
<td>Velocity gradient near the half-rod surface</td>
<td>s(^{-1})</td>
</tr>
</tbody>
</table>
### Acronyms

<table>
<thead>
<tr>
<th>Acronyms</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>BWR</td>
<td>Boiling Water Reactor</td>
</tr>
<tr>
<td>CANDU</td>
<td>CAnadian Deuterium Uranium</td>
</tr>
<tr>
<td>FEP</td>
<td>Fluorinated Ethylene Propylene</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>LDA</td>
<td>Laser Doppler Anemometry</td>
</tr>
<tr>
<td>LMFBR</td>
<td>Liquid Metal Fast Breeder Reactor</td>
</tr>
<tr>
<td>PWR</td>
<td>Pressurized Water Reactor</td>
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### Non-dimensional groups

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re$</td>
</tr>
<tr>
<td>$Re_{\text{gap}}$</td>
</tr>
<tr>
<td>$Str$</td>
</tr>
<tr>
<td>$Str_{\tau}$</td>
</tr>
</tbody>
</table>
Chapter 3

Rod bundle designs and fabrication

3.1 Introduction and motivation

Chapter 2 focuses on a rectangular channel with two half-rods as a preliminary study. The next step, covered by this chapter, is to design an experimental setup resembling more closely the hexagonal rod bundle geometry of the core of a LMFBR. The core of conventional and Gen-IV LMFBR reactors consists of a bundle of fuel rods immersed in an axial flow of coolant removing the generated heat. Depending on how tight the coupling of the rods is, such a geometry leads to a velocity difference, and thus to a shear layer between the fluid in the gap and in the main subchannels of the bundle, causing coherent structures in the flow (vortex streets). As mentioned in the introduction to this thesis, these large coherent structures can lead to vibrations of the internal components, especially the rods, due to the pressure fluctuations they cause. These vibrations can damage the rods by fretting [67] and fatigue. This phenomenon is of even higher importance in a LMFBR whose fuel rods are more tightly packed than those of a conventional light water reactor. SEEDS-1 (SEven rods Experiments in Delft for Sesame) will serve as the experimental facility dedicated to study large coherent structures and the related vibrations that they may induce inside a hexagonal rod bundle. The optical measurement techniques used in the SEEDS-1 experiment will be LDA for the flow measurements and a high-speed camera for the vibrations of the rods. A second point of concern in an LMFBR is that the fuel rods do not obstruct the flow locally by touching each other by bending. This is achieved by winding a wire spacer around each rod following a helicoid path. On top of the the spacer function, the wire partially leads the coolant through the gaps between the fuel pins, enhancing the mixing. In this way hot spots can be avoided on the cladding of the fuel pins, improving the safety of the reactor. The flow between neighbouring subchannels strongly depends on the position of the wire, as shown in [33]. Although the main effect of the wire is to direct the flow towards the gaps, a transversal flow
moving against the wrapping path can also occur, which is known as *migratory flow* [45]. The physics of such a flow behaviour is not yet fully understood. Therefore, a second facility, SEEDS-2, will be dedicated to optical measurements of the flow inside a wrapped-wire hexagonal bundle by means of PIV. Although the two facilities serve different functions, the two designs have, to some extent, similar requirements. Both bundles consists of seven rods and water is the working fluid since it is cheap and transparent to light. The $P/D$ ratio should be small enough to cause enough shear between the gap and the subchannel regions to have coherent structures to occur in the flow. The flow of both the facilities should also reach a fully developed, turbulent condition before entering the measurement section, so sufficient development length has to be taken into account. The measurements will be carried out with LDA, a high-speed camera, and PIV, which are optical measurement techniques. The location of the measurements has also to be be optically accessible and the light refraction due to the different media should be minimised as much as possible. The next sections of this chapter will discuss the main aspects of the SEEDS experimental designs, detailing the technical expedients adopted to overcome possible issues during the fabrication of some key components, such as the optical window and the wire.
Figure 3.1: SEEDS experimental loop: the water flows from the upper vessel, enters the hexagonal bundle via a flow distributor (detail A) to reach the measurement location (detail B).
3.2 Design requirements

3.2.1 Pitch-to-diameter ratio

The \( \frac{P}{D} \) ratio defines the hexagonal lattice of the bundle and dictates the shape of the velocity profile throughout the gap region which, in turn, characterises the coherent structures in the flow. The velocity difference between the low-speed region of the gap and the bulk flow inside one of the main subchannels produces a shear layer that may induce coherent structures along the borders of the gap between the rods [17]. A lower \( \frac{P}{D} \) ratio corresponds to a larger velocity difference between the gap and the bulk region, which can trigger coherent structures in the flow. Their interaction with some components of the bundle is the purpose of the SEEDS-1 experiment. As the gap between the rods become smaller, the wire spacers cover a relatively larger area of the subchannels, affecting the surrounding flow. This is the subject of study for the SEEDS-2 experiment. However, a low \( \frac{P}{D} \) means a more tight lattice, making the optical access for the measurements more difficult inside the inner subchannels. The low value of the \( \frac{P}{D} \) ratio is, thus, a trade-off between a geometry that fosters coherent structure and the need for optical access inside the setup. CFD has shown that coherent structures are expected to occur inside a hexagonal rod bundle with a \( \frac{P}{D} \) of 1.1 [68, 69], so the \( \frac{P}{D} \) of both the facilities is set accordingly to 1.11.

3.2.2 Flow development length

Both SEEDS-1 and SEEDS-2 test sections are installed in a loop where water is the working fluid, at ambient pressure. The flow is gravity-driven from an upper vessel to avoid any vibration induced by the pump. The water enters the test section through a 3D-printed flow distributor (Figure 3.1, detail A). This component has a dual function: it distributes the water among the subchannels of the bundle and it breaks the large vortices that may have formed in the downcomer pipe by means of an internal framework of ribs. Flow detachment from the walls of the distributor is avoided adopting a divergent angle of 4° [70]. Reaching fully turbulent flow conditions requires some development length between the flow distributor and the measurement section of the bundle. This length is evaluated considering the hydraulic diameter of a near wall subchannel (edge subchannel) since this, of all the three types of subchannel, has the largest hydraulic diameter. The development length is estimated as [71]

\[
L_{\text{dev}} = 4.4D_{h,e}Re_e^{1/6} \approx 0.3 \text{ m}
\]  

where \( Re_e = 40000 \) and \( D_{h,e} = 12.7 \text{ mm} \) is the hydraulic diameter of an edge subchannel, evaluated as \( 4A/\pi w \). On top of fully turbulent conditions, coherent structures also need to develop before reaching the measurement region (Figure 3.1, detail B). Therefore, the development length is increased by a factor five, based on CFD evidence [68], to \( L_{\text{dev}} = 1.5 \text{ m} \).
3.2.3 Optical access

The employed optical measurement techniques require optical access inside the bundle near the central rod, so the components of the bundle must have the same refractive index as water to minimize distortion of the light. For this purposes the refractive index-matching (RIM) technique is applied. This has become a widely used solution for performing optical measurements in rod bundles. Dominguez followed such a method for his measurements inside a $3 \times 3$ and $5 \times 5$ square rod bundle [72, 73]. More recently experiments performed at Texas University made use of the RIM technique with a larger 61-pins hexagonal bundle [36, 35]. Two possible applications of such a technique are investigated. The first has the rods of the bundle made of Perspex and the water’s refractive index is adjusted to the Perspex’s by adding some salt (F2S for conciseness [74]). This option was discarded as too much salt should be added, leading to corrosion of the metal parts of the loop such as the pump. The study leading to this conclusion is provided in Appendix B. Other chemical compounds can also be employed to adjust the refractive index of water, such as para-cymene (1-methyl-4-(propan-2-yl)benzene) [73, 75] to match the refractive index of the structural components of the bundle. The second option is to use FEP similarly to what is described in Chapter 2 (S2F for conciseness [74]). Part of the metal of the outer rods is removed and a FEP tube is used instead. FEP is one of the refractive-index matching materials, together with Mexflon-DC employed by Sato [33], commonly used for this kind of applications [76, 77]. The FEP tube is slid over each of the outer rods and heat-shrunk in order to adhere firmly (Figure 3.2a). However, FEP would not retain the cylindrical shape during the heat treatment; in fact it would form wrinkles. So, some disposable molds of a water-soluble resin, namely PVA (Polyvinyl alcohol), are placed inside the space left by the metal of the rod (Figure 3.2b) before heating. As a last step, the molds are dissolved by rinsing the inside of the rod with water (Figure 3.2c). The outer radius of the metal rods is reduced by the FEP wall thickness, hence there is no step in the transition between the stainless steel and FEP. The rods are filled with water to further decrease the refraction and to balance the external pressure.

3.2.4 Monitoring systems

The read-out signals of the loop are the temperature of the fluid and the flow rate. An in-house heat exchanger removes the heat generated by the pump (the one that pumps the water from the lower reservoir to the upper reservoir) keeping the water temperature constant. This heat exchanger consists of two concentric tubes in which the water of the loop and of the external cooling system flow separately. The temperature is monitored by a thermocouple (Labfacility, type K) in the upper reservoir. The water falls by gravity and a centrifugal pump (Duijvelaar Pompen, type DPVF 40/1 B) circulates the fluid from the lower to the upper reservoir. The flow rate is read out by three flowmeters: one with a floating element, a magnetic flow meter (ABB MagMaster), and an ultrasonic sensor (B. M. Tecn. Industriali, type TTFM100 NG). The flow is regulated by a valve with linear response (Ebora) placed downstream the upper reservoir. A feedback mechanism implemented in LabVIEW (National Instruments, 2012) adjusts the pump’s rotation speed so that
Figure 3.2: a) Heat-shrinking of FEP: quite often experiments require a generous amount of craftsmanship. b) PVA mold to keep the cylindrical shape of the FEP layer upon heat treatment. c) Final product: the PVA mold is dissolved by rinsing the inner rod with water.

the water level is kept constant in the lower vessel.
3.3 SEEDS-1 rod bundle for FSI experiments

The purpose of SEEDS-1 is to study coherent structures forming in the flow through a tightly-packed hexagonal rod bundle, and to investigate the fluid-structure interaction (FSI) between such periodical structures and the central rod of the bundle. Part of the central rod consists of a cylinder of silicone rubber, fairly common as material in FSI experiments [78, 79], where the interaction between the fluid and the rod is to be studied. Resonance, and its resulting increased vibration amplitude, can be induced if the large vortices occurring in the flow have a size comparable to the dimension of the pin [80]. Hence, the length of the silicone rod has to be comparable with the expected wavelength of coherent structures. CFD has shown that coherent structures with a length of 7cm are expected in a rod bundle with the same $P/D$ ratio of SEEDS [68], so the length of the silicone rod is set to 10cm. The main geometrical parameters of the bundle are listed in the following table. The measurement section of SEEDS-1 hexagonal bundle is sketched in Figure 3.3a where the outer rods are partially removed to make the central Silicone rod visible.

Figure 3.3: Measurement section for SEEDS-1 (a) and SEEDS-2 (b) experiments: four of the outer rods are partly made of FEP to provide optical access around the central rod. For clarity, the outer rods are removed to show the central rod.
3.4 SEEDS-2 wrapped-wire rod bundle

The goal of SEEDS-2 is to generate experimental results by means of PIV and to provide an explanation of the flow behaviour near to the wire. SEEDS-2 is a 7-rods hexagonal bundle with helicoid wire spacers wrapped around the rods (Figure 3.3b). The length of the optical Perspex window of FEP is half of the wrapping pitch $h$: the measurements can cover half of the circumference. At the same time, the FEP window is kept as short as possible to keep the FEP as stiff as possible. The main geometrical parameters of the bundle are listed in the following table.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{r}$</td>
<td>$30.0 \pm 0.1$</td>
<td>mm</td>
</tr>
<tr>
<td>$P/D$</td>
<td>1.11</td>
<td></td>
</tr>
<tr>
<td>$W/D$</td>
<td>1.11</td>
<td></td>
</tr>
<tr>
<td>$D_{r,\text{in}}$</td>
<td>20</td>
<td>mm</td>
</tr>
<tr>
<td>$D_{\text{sil,o}}$</td>
<td>30</td>
<td>mm</td>
</tr>
<tr>
<td>$t_{\text{sil}}$</td>
<td>$1.5 \pm 1.0$</td>
<td>mm</td>
</tr>
<tr>
<td>$L_{\text{sil}}$</td>
<td>$100 \pm 5$</td>
<td>mm</td>
</tr>
</tbody>
</table>

Table 3.1: SEEDS-1 dimensions. $D_{r}$: outer rod diameter, $D_{r,\text{in}}$: inner rod diameter; $P/D$: pitch-to-rod diameter ratio, $W/D$: nearest wall distance-to-rod diameter ratio, $D_{\text{sil,o}}$: outer Silicone rod diameter, $t_{\text{sil}}$: Silicone rod wall thickness, $L_{\text{sil}}$: Silicone rod length.

Table 3.2: SEEDS-2 dimensions. $D_{r}$: outer rod diameter, $D_{r,\text{in}}$: inner rod diameter; $P/D$: pitch-to-rod diameter ratio, $W/D$: nearest wall distance-to-rod diameter ratio, $D_{w}$: wire diameter, $h$: wire wrapping pitch, $\gamma$: helicoid pitch angle with the vertical axis.
3.4.1 Positioning of the wire

The positioning of the wire is critical for a successful experiment as any asymmetry in its position would deeply bias the experimental results. The wire is a stainless steel tube with a diameter of 3 mm wound around each of the seven rods of the bundle. There are two issues concerning its positioning: the first is that the wire cannot be glued nor welded along the FEP sleeve of the outer rods because glue will not attach to FEP, and welding will deteriorate it. Secondly, welding the wire along the entire length of the rod would produce large heat-affected zones (HAZ), leading eventually to strong deformations or even rupture of the wire itself. Moreover, the amount of welding beads must be as little as possible to not affect the flow behaviour too much. The adopted solution is to 3D print several cylindrical plastic guides with a notch on the lateral surface that follows the winding path of the wire (Figure 3.4a). These guides are assembled in series and slid over the rod; then the wire is positioned inside the notch and welded at few points along the path (Figure 3.4b). In this way the nominal position of the wire is followed avoiding to weld the whole wire. Since the wire is very thin, it can break rather easily while welding. However, if a hollow wire is used, less heat is required, resulting in a faster welding and smaller HAZ.

![Figure 3.4: a) Sample of a 3D-printed guiding tube with a notch to keep the wire in position during the welding. b) Multiple guides holding in position the wires around two rods.](image)

3.5 Summary

Two hexagonal rod bundles were designed and assembled, applying the refractive-index matching technique to provide optical access during the measurements. The two bundles are almost identical, but have few differences: SEEDS-1 has a section of the central rod made of flexible silicone rubber to detect flow-induced vibrations, whereas SEEDS-2 has wires wrapped around each rod to more closely mimick the core of a liquid metal reactor and to measure the effect on the surrounding fluid. Both the designs can work within the fluid temperature range 20 – 40°C and the maximum achievable Reynolds number (based on the total bundle flow area) is 28000 (SEEDS-1) and 24000 (SEEDS-2).

Laser Doppler Anemometry and high speed camera measurements will be performed in SEEDS-1 to study flow structures and vibrations, respectively. Particle Image Velocimetry will be used in SEEDS-2 to study the wire’s effect on the flow.
### 3.6 Nomenclature

#### Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Flow area</td>
<td>$m^2$</td>
</tr>
<tr>
<td>$D_{hc}$</td>
<td>Hydraulic diameter of the edge subchannel</td>
<td>$m$</td>
</tr>
<tr>
<td>$D_r$</td>
<td>Rod diameter</td>
<td>$m$</td>
</tr>
<tr>
<td>$D_{r,in}$</td>
<td>Inner rod diameter</td>
<td>$m$</td>
</tr>
<tr>
<td>$D_{sil,o}$</td>
<td>Outer diameter of the silicone rod</td>
<td>$m$</td>
</tr>
<tr>
<td>$D_w$</td>
<td>Wire diameter</td>
<td>$m$</td>
</tr>
<tr>
<td>$h$</td>
<td>Helicoid wire pitch</td>
<td>$m$</td>
</tr>
<tr>
<td>$L_{dev}$</td>
<td>Development length</td>
<td>$m$</td>
</tr>
<tr>
<td>$L_{sil}$</td>
<td>Silicone rod length</td>
<td>$m$</td>
</tr>
<tr>
<td>$P/D$</td>
<td>Ratio between the pitch and the diameter of the half rods</td>
<td>–</td>
</tr>
<tr>
<td>$P_w$</td>
<td>Wetted perimeter</td>
<td>$m$</td>
</tr>
<tr>
<td>$Q$</td>
<td>Volumetric flow rate</td>
<td>$m^3,s^{-1}$</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature of the water</td>
<td>°C</td>
</tr>
<tr>
<td>$t_{sil}$</td>
<td>Silicone wall thickness</td>
<td>$mm$</td>
</tr>
<tr>
<td>$W/D$</td>
<td>Nearest wall distance -to-rod diameter ratio</td>
<td>–</td>
</tr>
</tbody>
</table>

#### Greek letters

<table>
<thead>
<tr>
<th>Greek Letter</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>Helicoid pitch angle with the vertical axis</td>
<td>°</td>
</tr>
<tr>
<td>$\eta_{PMMA}$, $\eta_{sol}$</td>
<td>Refractive indices of Perspex, and of a generic solution</td>
<td>–</td>
</tr>
<tr>
<td>$\eta_{sol,L}$</td>
<td>Refractive index of a solution based on Looyenga expression</td>
<td>–</td>
</tr>
<tr>
<td>$\eta_{sol,LL}$</td>
<td>Refractive index of a solution based on the Lorentz-Lorenz expression</td>
<td>–</td>
</tr>
<tr>
<td>$\phi_{i}$, $\phi_{sol}$</td>
<td>Molar volume</td>
<td>$m^3,mol^{-1}$</td>
</tr>
</tbody>
</table>

#### Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>CsI</td>
<td>Caesium Iodide</td>
</tr>
<tr>
<td>CsCl</td>
<td>Caesium Chloride</td>
</tr>
<tr>
<td>FEP</td>
<td>Fluorinated Ethylene Propylene</td>
</tr>
<tr>
<td>FSI</td>
<td>Fluid-structure interaction</td>
</tr>
<tr>
<td>F2S</td>
<td>Matching fluid’s to solid’s refractive index</td>
</tr>
<tr>
<td>Gen-IV</td>
<td>Fourth generation of nuclear reactors</td>
</tr>
<tr>
<td>HAZ</td>
<td>Heat-affected zone</td>
</tr>
<tr>
<td>KI</td>
<td>Potassium Iodide</td>
</tr>
<tr>
<td>KSCN</td>
<td>Potassium Thiocyanate</td>
</tr>
<tr>
<td>LDA</td>
<td>Laser Doppler Anemometry</td>
</tr>
<tr>
<td>LMFBR</td>
<td>Liquid Metal Fast Breeder Reactor</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>-----------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>PIV</td>
<td>Particle Image Velocimetry</td>
</tr>
<tr>
<td>PVA</td>
<td>Polyvinyl alcohol</td>
</tr>
<tr>
<td>RIM</td>
<td>Refractive Index Matching</td>
</tr>
<tr>
<td>SEEDS-1</td>
<td>SEven rods Experiments in Delft for Sesame-1</td>
</tr>
<tr>
<td>SEEDS-2</td>
<td>SEven rods Experiments in Delft for Sesame-2</td>
</tr>
<tr>
<td>S2F</td>
<td>Matching solid’s to fluid’s refractive index</td>
</tr>
</tbody>
</table>

**Non-dimensional groups**

<table>
<thead>
<tr>
<th>$Re_e$</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Reynolds number of the edge subchannel</td>
</tr>
</tbody>
</table>
Chapter 4

Fluid-structure interaction in a 7-rods hexagonal bundle

4.1 Introduction and motivation

The rod bundle geometry of the fuel elements forming the core of a LWR or a Gen-IV LMFBR may trigger large coherent structures in the axial flow of coolant [77, 17, 16]. These periodical vortices interact with the surrounding structural components and may cause flow-induced vibrations (FIV) on the fuel rods, by imposing a fluctuating pressure field on these elements. There are reports of FIV-related accidents in nuclear reactors that happened in the past. In his review, Blevins [81] describes the accident occurred to a gas-cooled Magnox reactor in UK where some tubes of the heat exchanger experienced extreme fatigue due to flow-induced vibrations. Another example is the case of the TRIGA (Training Research Isotopes General Atomic) reactor in Korea: vibrations induced on the core by the flowing coolant could even be heard by the operators. A third case that is mentioned regards the sodium-cooled Fermi reactor, where a piece of a conical guide, broken due to vibrations, obstructed the flow into some subchannels leading to a partial meltdown of two fuel elements.

FIV should therefore be avoided because they may lead to damage by fretting and fatigue of the internal components of the reactor core [67]. The fluctuating external force depends on the flow velocity and on the size of these coherent structures. Hence, predicting their wavelength with sufficient accuracy is important for a safer design of the reactor. Currently, there is but one empirical correlation to estimate the stream-wise length $\lambda$ of these flow structures provided by Guellouz and Tavoularis [19] for a rod inside a rectangular channel and immersed in an axial

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flow:

\[ \frac{\lambda}{D_r} = 18.7 \frac{D_r + \delta}{D_r} - 16.3, \]  

(4.1)

where \( \delta \) is the gap width between the rod and the outer channel’s wall. However, its applicability to more complex geometries with different shape of the gap region, such as rod bundles, is rather limited.

There is abundance of studies regarding flow-induced vibrations either on solitary cylinders or rod bundles [27, 82, 67, 78], involving a axial or transversal flows. In the latter case, vibrations induced by periodical vortices have also been investigated due to the implications at industrial level [83]. However, an experimental study of the role that coherent structures play in Fluid-Structure Interactions (FSI) inside rod bundles is missing. Furthermore, a correlation for estimating the length of the coherent structures applicable to different geometries would contribute to designing safer components not subject to resonance.

The aim of the work described in this chapter is twofold. It proves a new general correlation to estimate the size of the structures in different channel geometries and it characterises the response frequency of the flexible, central rod of a rod bundle as a function of the frequency of the detected large coherent structures.

The employed measurement systems are Laser Doppler Anemometry (LDA) and a high-speed camera for measuring the flow and the vibrations of the flexible rod, respectively. The experimental setup is SEEDS-1, which is a 7-rod hexagonal bundle where part of the central rod is made of silicone to be flexible. A description of the setup is provided in Chapter 3. LDA measurements of the flow field are done to characterise the vortex streets in the considered geometry, followed by a measurement campaign with the high-speed camera to detect flow-induced vibration of the rod. The small size of this work’s bundle allows for an easier optical access around the central rod, which is crucial for measuring vibrations. The optical access around the central rod is achieved through the refractive-index matching technique (RIM): the hexagonal casing is made of Perspex and part of the steel of the rods is replaced with FEP, which matches the water’s refractive index.

### 4.2 Theory

Evaluating the natural frequency of the central flexible rod of the bundle is needed for interpreting part of the results of the measurements. This section provides the theoretical background that makes this possible. The second part of the section describes how a new correlation is derived to estimate the wavelength of coherent structures occurring in the flow.

#### 4.2.1 Solitary cylinder case

Estimating the natural frequency of the silicone rod is required to interpret the results of the FSI measurement campaign. The Euler-Bernoulli beam theory for a single cylinder clamped at both ends, immersed in a steady, axial flow, and
surrounded by an outer circular channel, gives the equation originally derived by Paidoussis [27]:

\[
\begin{align*}
\text{EI} \frac{\partial^4 x}{\partial y^4} + m_a \left( v^2 \frac{\partial^2 x}{\partial y^2} + \frac{\partial^2 x}{\partial t^2} \right) - \frac{1}{2} C_T \frac{m_a v^2}{D_r} \left( \frac{1}{2} L_{\text{sil}} - y \right) \frac{\partial^2 x}{\partial y^2} + 2m_a v \frac{\partial^2 x}{\partial y \partial t} + \\
+ \frac{1}{2} C_N \frac{m_a v}{D_{\text{sil,o}}} \left( v \frac{\partial x}{\partial y} + \frac{\partial x}{\partial t} \right) + C_V \frac{\partial x}{\partial t} + m_{\text{sil}} \frac{\partial^2 x}{\partial t^2} = 0
\end{align*}
\]  

(4.2)

where \( E=1 \text{MPa} \) is the Young's modulus of the silicone, \( I=\pi \left( \frac{D_{\text{sil,o}}^2}{4} - \frac{D_{\text{sil,i}}^2}{4} \right) \) is the moment of inertia of the silicone rod, being \( D_{\text{sil,o}} \) and \( D_{\text{sil,i}} \) the outer and inner diameter of the silicone rod. \( x \) is the rod radial displacement, \( z \) is the axial coordinate along the rod, \( m_a \) is the added mass accounting for the additional force exerted by the fluid on the rod while it moves, \( v \) is the mean axial flow velocity, \( C_T \) is the longitudinal viscous force coefficient whose definition is provided by Hoerner [84], \( L_{\text{sil}} \) is the rod length, \( C_N=C_T \) is the lateral drag force coefficient, \( C_V \) is the viscous damping coefficient [85], and \( m_{\text{sil}} \) is the rod mass.

The added mass \( m_a \) deserves a closer look since it accounts for the confinement effect given by the proximity of other bodies (i.e. walls, rods) around the silicone rod. The added mass is defined as

\[
m_a=C_m \cdot \rho \pi \frac{D_{\text{sil,o}}^2}{4},
\]  

(4.3)

where the \( C_m \) is the added mass coefficient which multiplies the weight of the fluid displaced by the rod in the flow. It represents the confinement effect of an outer channel surrounding the single rod [85, 86, 24]. \( C_m \) is a function of the outer diameter of the tube \( D_o \) that bounds the flow around the rod. The natural frequency \( f_{\text{sil,n}} \) of the rod is calculated by following the method described in by Chen [87] and Paidoussis [24]: Equation 4.2 is first rendered non dimensional and then solved through the Galerkin method.

4.2.2 Rod bundle case

Considering a rod bundle, the effect of the rods surrounding the central one is to confine more the flow: displacing the fluid while the rod oscillates becomes more difficult due to the proximity of the surrounding cylinders. This is taken into account by a virtual additional mass that affects the dynamic of the deformation by changing the rod’s natural frequencies. The effect of the surrounding rods is to increase the added mass via the added mass coefficient \( C_m \), thereby decreasing the natural frequency. In a rod bundle there are multiple surrounding rods in place of an outer channel, therefore \( D_o \) has to be adjusted to account for their effect. \( D_o \) is evaluated as the hydraulic diameter of the hexagonal duct whose corners are at a distance \( P \) (rod pitch) from the centre:

\[
D_o = \frac{4A}{P_w} = \frac{3\sqrt{3}P^2 - \pi D_o^2}{\pi D_r}.
\]  

(4.4)
The added mass coefficient is now evaluated following the expression of Pettigrew [86]

\[ C_m = \frac{D_{\text{sil,o}}^2 + D_o^2}{D_{\text{sil,o}}^2 - D_o^2}. \] (4.5)

The natural frequency of the first mode of vibration of the silicone rod is evaluated based on the hexagonal lattice of SEEDS-1 (see table 3.1); the values are plotted in Figure 4.1 as non dimensional quantities. Frequency and velocity are rendered non dimensional with the following substitution [24]:

\[ f_{\text{sil,n}}^* = f_{\text{sil,n}} L_{\text{sil}}^2 \sqrt{\frac{m_{\text{sil}} + m_a}{EI}} \quad v^* = \nu L_{\text{sil}} \sqrt{\frac{m_a}{EI}}. \] (4.6)

The non dimensional natural frequency decreases to zero when the flow velocity reaches the a critical value. For a clamped-clamped cylinder, the frequency goes to zero at \( v^* = 2\pi \).

![Figure 4.1: Non dimensional 1st mode natural frequency of the silicone rod used for this study as a function of the non dimensional flow velocity. For a clamped-clamped cylinder, the frequency goes to zero at \( v^* = 2\pi \).](image)

4.2.3 Empirical correlation derivation

Estimating the wavelength of coherent structures is important for designing experiments that aim at studying specific sizes of flow structures in rod bundles. Ideally, the required correlation should be applicable to as many geometries of the subchannel as possible. Therefore, an empirical correlation for estimating the coherent structure wavelength \( \lambda \) is derived based on dimensional analysis. \( \lambda \) is assumed to depend on the local channel geometry (i.e. hydraulic diameter of the main subchannel and gap region) and on the fluid properties. The flow velocity
in the gap region and in the main subchannel is also considered. In mathematical terms,

\[ \lambda = \mathcal{K}_2 \cdot (D_h^*)^{a_1} \cdot \rho^{a_2} \cdot \mu^{a_3} \cdot v^{a_4} \cdot v_{gap}^{a_5} \cdot D_h^{a_6}, \]  

(4.7)

where \( \mathcal{K}_2 \) is an arbitrary constant, \( D_h^* \) is the hydraulic diameter of the gap region (defined in figure 4.2), \( \rho \) is the fluid density, \( \mu \) is the dynamic viscosity, and \( v \) is the fluid velocity in the main subchannel, \( v_{gap} \) is the fluid velocity in the gap region, and \( D_h \) is the hydraulic diameter of the main subchannel. Based on Buckingham’s \( \pi \) theorem [89], the wavelength \( \lambda \) can be expressed by means of three non dimensional groups. From dimensional analysis, it follows that

\[ [m] = [m]^{a_1} \cdot [kgm^{-3}]^{a_2} \cdot [kgm^{-1}s^{-1}]^{a_3} \cdot [ms^{-1}]^{a_4} \cdot [ms^{-1}]^{a_5} \cdot [m]^{a_6}. \]  

(4.8)

The associated system of equations

\[ \begin{cases} a_1 - 3a_2 - a_3 + a_4 + a_5 + a_6 = 1 \\ a_2 + a_3 = 0 \\ -a_3 - a_4 - a_5 = 0 \end{cases} \]  

(4.9)

leads to the following expression:

\[ \frac{\lambda}{D_h} = \mathcal{K}_2 \cdot \left( \frac{D_h^*}{D_h} \right)^{a_1} Re^{a_2} \left( \frac{v_{gap}}{v} \right)^{a_5}. \]  

(4.10)

At high Reynolds numbers it is reasonable to assume that the pressure drops through the gap region \([\Delta p]_{gap}\) and through the main subchannel \([\Delta p]_{ch}\), across the length \( L \) of the bundle, are equal. Expressing the pressure drops by means of the Darcy-Weisbach equation, it follows that

\[ [\Delta p]_{gap} = \frac{D}{2} \frac{v_{gap}^2}{D_h^*} f_{gap}(Re_{gap}) = [\Delta p]_{ch} = \frac{D}{2} \frac{v^2}{D_h} f_{ch}(Re), \]  

(4.11)

where the friction coefficients are expressed as \( f_{ch}(Re) = C_1 Re^{-\chi_1} \) and \( f_{gap}(Re_{gap}) = C_2 Re_{gap}^{-\chi_2} \) [90]. From the previous equation, the ratio \( v_{gap}/v \) can be expressed as

\[ \frac{v_{gap}}{v} = \left[ \frac{f_{gap}(Re_{gap})}{f_{ch}(Re)} \frac{D_h^*}{D_h} \right]^{1/2}. \]  

(4.12)

Substituting Equation 4.12 into Equation 4.10 leads to

\[ \frac{\lambda}{D_h^*} = \mathcal{K}' \cdot \left( \frac{D_h^*}{D_h} \right)^{\xi} Re^{a_2} f_{ch}(Re) f_{gap}(Re_{gap}). \]  

(4.13)

Experimental evidence [60, 19, 17, 77] has shown that the wavelength is independent of the Reynolds number of the main subchannel \( Re \), so \( a_2 = 0 \) meaning

\[ \frac{\lambda}{D_h^*} = \mathcal{K}' \cdot \left( \frac{D_h^*}{D_h} \right)^{\xi} Re_{gap}^{\zeta}. \]  

(4.14)
This is the most general form of the correlation. It will be tested against the experiments performed in simple geometries such as rectangular channels hosting respectively one or two half-rods [17, 77], and an eccentric rod hosted in a rectangular channel [19]. Furthermore, two rod bundle geometries are considered: the hexagonal bundle of this work and a sector of a circular bundle [88]. The results of the validation with the experiments is discussed in section 4.5.2.
Figure 4.2: Definition of the hydraulic diameters of the gap region for the bundle geometries on which the correlation is to be tested. (a) Adopted from [17]. (b) Adopted from [77]. (c) Adopted from [19]. (d) Adopted from [88]. (e-f) This work. Horizontal hatching: gap region. Vertical hatching: main subchannel. For clarity, the main subchannel and the gap region are drawn as two separate regions whereas, in reality, they partly overlap.
4.3 Experimental apparatus

The measurement systems used to carry out the work described in this chapter are a Laser Doppler Anemometry system (described in section 2.3) for measuring the flow, and a high-speed camera to capture the vibrations occurring on the flexible part of the central rod. Both systems are mounted on and moved by the same traverse. The face of the hexagonal casing through which the measurements are performed is rotated in order to be perpendicular to the measurement systems. The alignment is done with a portable laser installed on the traverse and a reflective metal plate used as target on the casing. The LDA system and the post-processing of the measurements are discussed in Chapter 2, whereas the high-speed camera system is described in the following section.

4.3.1 High-speed camera

The equipment to measure flow-induced vibrations of the silicone rod consists of a Complementary Metal-Oxide Semiconductor (CMOS) camera Imager MX 4M (LaVision, Germany) capable of recording at 180 fps with full resolution (4 MP) with a reduced field of view. The camera cannot have both borders of the rod in focus without losing much spatial resolution: the camera should be placed too far from the target. Thus, only one border of the silicone rod is in focus and 15000 frames at 300Hz are recorded in each measurement. The Nyquist frequency, being the highest frequency of a signal that can be captured with a given sampling rate,
is defined as half of the sampling frequency of a signal. This high-speed camera allows for a Nyquist frequency of 150Hz. The frequency of the vibrating silicone rod is expected to be of the same order of the coherent structures’ frequency, which is ≈10Hz based on preliminary LDA measurements. Hence, a recording rate of 300Hz is considered high enough to measure vibrations induced on the silicone rod.

The silicone appears white in the images and the background contrast with the surrounding fluid is enhanced with a flash light to illuminate the target area, and by keeping the setup in the dark. The recorded frames are post-processed through the Matlab Image Toolbox. The intensity values of the light in each image are converted into ones or zeros with a binary filter that uses a threshold level determined by the Otsu algorithm [91]. The position of the edge of the silicone rod appears, in each frame, as the border between the regions of ones and zeros obtained through the binary filter.

The recorded positions of the silicone rod’s edge gives the instantaneous displacement, and its root mean square, on a plane orthogonal to the line of sight of the camera, calculated as follows:

$$\bar{\varepsilon} = \frac{1}{N} \sum_{i=1}^{N} |\varepsilon(t_i)|; \quad \varepsilon_{\text{rms}} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} [\varepsilon(t_i) - \bar{\varepsilon}]^2},$$

where $\varepsilon$ is the mean displacement, $N$ is the number of recorded frames, $\varepsilon(t_i)$ is the displacement of the $i$-th frame and $\varepsilon_{\text{rms}}$ is the displacement root mean square. The recorded time signal of the displacement is filtered with the Henderson’s 23 points moving average to reduce the noise at high frequency, following the approach of Cioncolini et al. in a similar work [92]. The time signal of the displacement is used as input to calculate the frequency spectrum. The first method to calculate the spectrum relies on the autocorrelation function of the displacement (Figure 4.4a), and the dominant peak in the frequency spectrum is fitted with a Gaussian bell (Figure 4.4b). The frequency range where the spectral peak is fitted is determined by the two points where the first derivative of the spectrum goes to zero. The fitting error is expressed as the Normalised Root Mean Square Error (NRMSE):

$$\text{NRMSE} = \sqrt{\frac{\sum_{k=1}^{N_g} [S_{\text{fit}}(k) - S(k)]^2}{N_g \bar{S}}},$$

where $N_g$ is the number of fitted points of the peak, $S_{\text{fit}}$ and $S$ are the fitted and the measured value of the spectrum, respectively, and $\bar{S}$ is the mean value of $S$. The accuracy with which the average frequency is determined from the fitting is lower than 2%.

A second way to estimate the spectrum is through the discrete Fourier transform (DFT) of the displacement signal as

$$S = \sum_{j=0}^{M-1} \varepsilon(t_j) \exp(2\pi i j f_{\text{sil}}),$$
where $\varepsilon(t_j)$ is the j-th recorded displacement sample, $f_{\text{sil}}$ is the vibration frequency of the silicone rod, $t_j$ is the j-th time instant in the signal, and $i$ is the imaginary unit. The noise in the spectrum is reduced by means of the Bartlett’s method [93] (Figure 4.4c). The noise in the signal, estimated through a no-flow record-

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4_4}
\caption{a) Autocorrelation function and b) Spectral peak characterising the vibration of the silicone rod fitted with a Gaussian function (\(\bigcirc\)). c) DFT of the vibrating silicone rod.}
\end{figure}

ing, corresponds to an equivalent displacement of $3 \mu m$ (the minimum measurable displacement is $9 \mu m$).
4.4 Measurement campaign

The LDA measurements are carried out in the edge and central subchannel, so it is more accurate to refer to the Reynolds number of the edge ($Re_e$) and of the central subchannel ($Re_c$). $Re_e$ is estimated as

$$Re_e = \frac{\rho \cdot v_e \cdot D_{h,e}}{\mu},$$

where $\rho$ and $\mu$ are the density and dynamic viscosity of water, respectively; $D_{h,e}$ is the hydraulic diameter of the edge subchannel, and $v_e$ is the average stream-wise velocity measured inside the edge subchannel. The latter is evaluated through

$$v_e = \frac{1}{A} \int v(x,z) dA = \frac{1}{A} \sum_i \sum_j v(x_i,z_j) dA_{ij},$$

where $dA_{ij}$ differs per position.

An example of the results from the LDA measurement of the velocity inside the edge and part of the central subchannel is shown in Figure 4.5. The two regions ($x \approx 160\,\text{mm}; 360\,\text{mm} \leq y \leq 370\,\text{mm}$) and ($x \approx 160\,\text{mm}; y \approx 395\,\text{mm}$) are the most affected by the reflecting metal surface of the rods, hence no velocity value is available. The Reynolds number of the central subchannel, $Re_c$, is determined based on $Re_e$. $Re_c$ requires the values of the average stream-wise velocity $v_c$ in the central subchannel.

**Figure 4.5:** Contour plot of the stream-wise velocity component measured with LDA around the two rods close to the outer wall.

LDA measurements of the flow area $A$ belonging to the edge subchannel (Figure 4.3b). The average is calculated as

$$v_e = \frac{1}{A} \int v(x,z) dA = \frac{1}{A} \sum_i \sum_j v(x_i,z_j) dA_{ij},$$

where $dA_{ij}$ differs per position.

An example of the results from the LDA measurement of the velocity inside the edge and part of the central subchannel is shown in Figure 4.5. The two regions ($x \approx 160\,\text{mm}; 360\,\text{mm} \leq y \leq 370\,\text{mm}$) and ($x \approx 160\,\text{mm}; y \approx 395\,\text{mm}$) are the most affected by the reflecting metal surface of the rods, hence no velocity value is available. The Reynolds number of the central subchannel, $Re_c$, is determined based on $Re_e$. $Re_c$ requires the values of the average stream-wise velocity $v_c$ in the central subchannel.
The pressure drops throughout all the subchannels may be considered the same \cite{90}; in particular the pressure drops through the edge $[\Delta p]_e$ and the central $[\Delta p]_c$ subchannels may be equated and expressed via the Darcy-Weisbach equation \cite{94}:

$$\frac{f_c \rho v^2_c}{D_{h,c}} = f_e \rho v^2_e \frac{1}{D_{h,e}},$$  \hspace{1cm} (4.20)

where $D_{h,c}$ is the hydraulic diameter of the central subchannel, and $f_c$ and $f_e$ are the friction factors of central and edge subchannels, respectively. $v_c$ can be obtained from the previous equation. For a bare rod bundle (no spacers) with turbulent flow, $f_c$ and $f_e$ can be expressed as \cite{90}

$$f_c = \frac{C_{T,c}'}{Re^{n}}, \quad f_e = \frac{C_{T,e}'}{Re^{n}},$$  \hspace{1cm} (4.21)

where $n=0.18$, and $C_{T,c}'$ and $C_{T,e}'$ are coefficients depending on the hexagonal lattice. This correlation is valid for rod bundles within the pin number range $7 - 217$ and the associated mean error is as low as $9\%$ \cite{95}. Substituting Equation 4.21 into Equation 4.20 leads to

$$v_c = v_e \cdot \left[ \frac{C_{T,e}'}{C_{T,c}'} \right] \left[ \frac{D_{h,c}}{D_{h,e}} \right]^{\frac{1}{2-n}}.$$  \hspace{1cm} (4.22)

$Re_c$ is finally evaluated as

$$Re_c = \frac{\rho \cdot v_c \cdot D_{h,c}}{\mu}.$$  \hspace{1cm} (4.23)

The values of $Re_c$ and $Re_e$ are reported in table 4.1.
Table 4.1: $Q$: flow rate, $Re$: Reynolds number based on the bundle’s total flow area, $Re_e$: edge subchannel Reynolds number, $Re_c$: central subchannel Reynolds number.

<table>
<thead>
<tr>
<th>$Q$ [m$^3$s$^{-1}$]</th>
<th>$Re$</th>
<th>$Re_e$</th>
<th>$Re_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4.78 \times 10^{-3}$</td>
<td>27200</td>
<td>48630</td>
<td>36530</td>
</tr>
<tr>
<td>$3.48 \times 10^{-3}$</td>
<td>20620</td>
<td>30540</td>
<td>22940</td>
</tr>
<tr>
<td>$3.28 \times 10^{-3}$</td>
<td>17600</td>
<td>28200</td>
<td>21180</td>
</tr>
<tr>
<td>$2.92 \times 10^{-3}$</td>
<td>15040</td>
<td>26240</td>
<td>19700</td>
</tr>
<tr>
<td>$2.68 \times 10^{-3}$</td>
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<td>25130</td>
<td>18800</td>
</tr>
<tr>
<td>$2.42 \times 10^{-3}$</td>
<td>11830</td>
<td>22660</td>
<td>17000</td>
</tr>
<tr>
<td>$2.13 \times 10^{-3}$</td>
<td>10300</td>
<td>20310</td>
<td>15260</td>
</tr>
<tr>
<td>$1.94 \times 10^{-3}$</td>
<td>10100</td>
<td>16620</td>
<td>12490</td>
</tr>
<tr>
<td>$1.59 \times 10^{-3}$</td>
<td>8530</td>
<td>14950</td>
<td>11230</td>
</tr>
<tr>
<td>$1.31 \times 10^{-3}$</td>
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<td>9560</td>
</tr>
<tr>
<td>$1.05 \times 10^{-3}$</td>
<td>6470</td>
<td>10100</td>
<td>7580</td>
</tr>
</tbody>
</table>

4.5 Results and discussion

This section presents the experimental results. First, the LDA measurements of the coherent structures occurring in the flow are discussed. The results are used to validate the correlation derived in section 4.2.3 for predicting the wavelength of the flow structures.

The second part focuses on the vibrations measured with the high speed camera on the silicone rod surface and on the influence of coherent structures on the oscillation of the rod wall is discussed.

4.5.1 Coherent structures in the flow

The LDA measurements are carried out at mid-length in the hexagonal Perspex casing sampling several locations in the flow along a straight line going from the outer wall to the central silicone rod. The frequency of passage of the coherent structures through the measurement region is extracted from the frequency spectrum of the stream-wise velocity component. Two additional measurements at the beginning and at the end of the transparent section are performed to check that the coherent structures are fully developed in the flow (i.e. their wavelength does not change through the measurement section).

Figure 4.6 shows the LDA measurement along the straight line throughout the gap between two front rods, with $Re_e=30540$ ($Re_c=22940$). The normalised stream-wise velocity component is reported in Figure 4.6a. The velocity root mean square is shown in Figure 4.6b, which features two maxima located at the outer wall and at close to the central rod, where turbulence increases due to the shear produced
by the viscous sublayer, similarly to common wall-bounded flows [54]. The relative maxima closer to the centre gap are due to the shear between the high-velocity region in the bulk and the low-velocity fluid inside the narrow gap.

Figure 4.6: Stream-wise velocity profile (a) and root mean square (b) measured with LDA through the gap between two front rods. The abscissa is the distance from the gap centre, normalized to the rod diameter.

Turbulence inside the gap between the two rods closer to the wall is examined based on the turbulence spectrum. The slope of the spectrum carries information on the nature of turbulence, i.e. whether this is two-dimensional or three-dimensional (2D, 3D for short) within the inertial subrange of the spectrum. This section will focus on the wavelength and the frequency of the coherent structures, which is used to validate an empirical correlation as proposed in section 4.5.2. The measured frequency of passage of the coherent structures is compared with the structural response frequency of vibration of the rod wall, as discussed in section 4.5.3.

For a 3D homogeneous turbulent flow, only the conservation of energy applies and the inertial subrange of the turbulent spectrum usually shows the well-known slope of $-5/3$. The vorticity equation for 3D turbulence and incompressible flow is

$$\frac{D\vec{\omega}}{Dt} = (\vec{\omega} \cdot \nabla) \vec{v} + \nu \nabla^2 \vec{\omega},$$

(4.24)

where $\frac{D}{Dt}$ is the Lagrangian (or substantial) derivative, $\vec{\omega}$ is the vorticity vector and $\nu$ is the kinematic viscosity. The first term at the right hand side of the equation accounts for the vortex-stretching, which is the largest if the velocity gradient is parallel to the vorticity vector. In 2D turbulence the vortex-stretching effect is absent [96], hence the general vorticity equation, for incompressible and inviscid fluids, takes the form
\[
\frac{D\vec{\omega}}{Dt} = 0, \quad (4.25)
\]

which states that the vorticity must also be conserved. This additional condition results in a slope of the spectrum equal to \(-3\), within the inertial subrange. The energy cascade is towards larger scales (lower wavenumber), whereas vorticity cascade is towards the smallest scales in the viscous subrange, contrarily to 3D turbulent flows \[97\]. The slope of the inertial subrange gives thus an indication of the type of turbulence.

The frequency spectrum of the stream-wise velocity is evaluated in the middle of the gap between the edge and central subchannel. The frequency spectrum is then multiplied by \(f_3^3\) (or \(f_3^{5/3}\)): following the approach of Romano \[98\] the resulting function \(S \cdot f_3^3\) (or \(S \cdot f_3^{5/3}\)) should have a flat plateau within the frequency range where turbulence is 2D (or 3D). Figure 4.7a shows the frequency spectrum and Figure 4.7b shows both \(S \cdot f_3^3\) and \(S \cdot f_3^{5/3}\) for a Reynolds corresponding to \(Re_e = 12730\) (\(Re_c = 9560\)). A low-frequency peak is found, which is characteristic of coherent structures that affect periodically the velocity field while moving with the mean flow. Although the spectrum exhibits a \(-3\) slope over a short frequency decade, the overall slope appears to be close to \(-5/3\), as shown by the almost flat plateau of the \(S \cdot f_3^{5/3}\) plot.

\[
\begin{align*}
\text{(a)} & \\
\text{(b)} & 
\end{align*}
\]

Figure 4.7: a) Frequency spectrum measured at the centre of the gap. b) \(S \cdot f_3^{5/3}\) (square), \(S \cdot f_3^3\) (triangle). \(Re_e = 12730\). The black line highlights a plateau, indicating three-dimensional turbulence.

Figure 4.8 reports the case with \(Re_e = 14950\) (\(Re_c = 11230\)), where the peak in the spectrum is at 5.3 Hz. The slope is close to \(-5/3\), as shown by the constant trend of \(S \cdot f_3^{5/3}\) in the same frequency range. Figure 4.9 refers to the case with \(Re_e = 48630\)
Figure 4.8: a) Frequency spectrum measured at the centre of the gap. b) $S \cdot f_y^{5/3}$ (square), $S \cdot f_y^3$ (triangle). $Re_e=14950$. The black line highlights a plateau, indicating three-dimensional turbulence.

($Re_e=36530$), where coherent structures occur at a higher frequency, being 17Hz (Figure 4.9a). The corresponding plots of $S \cdot f_y^3$ and $S \cdot f_y^{5/3}$ are shown in Figure 4.9b. The spectrum at this Reynolds number has a slope between $-3$ and $-5/3$.

For each flow rate, the turbulent spectra are evaluated along the path going from the edge to the central subchannel (Figure 4.3b). The peaks found in the spectra reveal periodicities at the associated frequency $f_y$ ascribed to structures occurring in the flow. An average frequency of the structures is calculated based on the values given by the single peaks. The plotted quantities are rendered non dimensional following Equation 4.6. The plots of Figure 4.10 report the average non dimensional frequency of the coherent structures $f_y^*$ in both the edge and the central subchannel against the non dimensional flow velocity $v^*$. An example of the measured velocity signal is shown in Figure 4.10c where the flow oscillations due to coherent structures moving in the flow are visible for $Re=10300$. The peaks detected in the frequency spectra of the velocity signals show a bell-shaped distribution of frequencies so every peak is fitted with a Gaussian bell to obtain the corresponding $\sigma_y$ around the mean. This gives a frequency interval $f_y \pm \sigma_y$ which, in turn, provides a minimum and a maximum wavelength of the structures, as shown in Figure 4.11. The NRMSE resulting from the fitting, and evaluated with Equation 4.16, is lower than 5% for all the considered cases. The wavelength of the coherent structures is evaluated adopting Taylor’s hypothesis (coherent structures in the flow are regarded as frozen entities moving with the stream at velocity $v_m$). The velocity $v_m$ is the stream-wise velocity measured at the inflection point of the profile between the gap and the subchannels [17]. The average wavelength and its minimum and maximum values are
The uncertainty on the wavelength of the structures is estimated through the uncertainty propagation formula as

$$
\delta \lambda = \sqrt{\left( \frac{\partial \lambda}{\partial f_y} df_y \right)^2 + \left( \frac{\partial \lambda}{\partial v_{in}} dv_{in} \right)^2} \approx \left| \frac{\partial \lambda}{\partial f_y} df_y \right| = \frac{v_{in} df_y}{f_y^2},
$$

where the approximation is possible because the error on $v_{in}$ is negligible compared to the uncertainty on $f_y$. The frequency at which the flow structures pass through the measurement region scales almost linearly with the flow velocity. Figure 4.11 shows the wavelength of the structures normalised by the gap region hydraulic diameter suggests that, at high $Re$, the average wavelength is independent of the Reynolds number, confirming previous experimental findings [60, 19, 17, 77]. Recalling the discussion of section 2.5.4 (Chapter 2), if the wavelength of coherent structures can be considered constant within a certain Reynolds interval, the quantity

$$
\frac{Re^3}{\varepsilon_d},
$$

is conserved within such an interval ($\varepsilon_d$ is the turbulent dissipation rate). This means that the turbulent dissipation rate of the flow measured at different Reynolds numbers can be compared.

Considering the lowest and highest Reynolds numbers (based on the total bundle flow area) defining the interval where the wavelength $\lambda$ can be assumed constant,
Figure 4.10: Non dimensional mean frequency of passage of the coherent structures $f^*_{\gamma}$ against the non dimensional velocity $v^*$, measured (a) between the edge subchannel and the gap, and (b) between the gap and the central subchannel. c) Time-dependent velocity signal of the $v$ component, measured at two locations 2mm apart close to the centre of the gap connecting the edge to the central subchannel.

The previous expression becomes

$$\left[ \frac{Re^3}{\varepsilon_d} \right]_{Re=27200} = \left[ \frac{Re^3}{\varepsilon_d} \right]_{Re=6470},$$

from which the turbulent dissipation rate $\varepsilon_d$ of the two cases can be compared:

$$\frac{[\varepsilon_d]_{Re=27200}}{[\varepsilon_d]_{Re=6470}} = \left( \frac{27200}{6470} \right)^3 \approx 74.$$  

This ratio does not change much ($\approx 111$) if the edge or central subchannel’s Reynolds number is considered, instead of the whole bundle’s. The dissipation rate of the flow with a Reynolds of 27200 is approximately a factor 100 higher than with a Reynolds of 6470.

The subject of the next section will be the influence of the geometry of the channel over the structure’s wavelength.
4.5.2 Correlation validation

The correlation derived in section 4.2.3 for predicting the wavelength of coherent structures is now applied to several experimental cases. The predicted (normalised) wavelength $\lambda$, evaluated for different geometries, and the results of the experiments are reported in Figure 4.12a against the hydraulic diameter of the gap region $D_h^*$ (see Figure 4.2 for its definition in the considered geometries) normalised by the hydraulic diameter of the subchannel. The figure suggests that $\lambda/D_h^*$ is constant and approximately equal to

$$\frac{\lambda}{D_h^*} \approx 13,$$

(4.31)

The wavelength of the structures scales linearly with the hydraulic diameter of the gap region $D_h^*$. $\lambda/D_h^*$ constant means that the exponents in Equation 4.14 should be zero, that is $\xi = \zeta = 0$. If one imagines to increase indefinitely the hydraulic diameter of the main subchannel $D_h$ while keeping the gap region unaltered (hence keeping $D_h^*$ and $Re_{gap}$ constant), the wavelength of the structures forming close to the gap should not change much. This means that at some point $\lambda/D_h^*$ will not depend on $(D_h^*/D_h)^{\xi}$ any more, so it is reasonable to assume that

$$\xi = 0 \quad \text{for} \quad \frac{D_h^*}{D_h} \lesssim 1.$$  

(4.32)

We see that the correlation is valid even for $D_h^*/D_h = 1.15$, which is the case of the near wall subchannel of this work’s bundle (Figure 4.2f). The practical meaning of $D_h^*/D_h \lesssim 1$ in a bundle is that the rod are moved farther (Figure 4.12b). In the particular case of a hexagonal bundle, this ratio has a non-zero upper limit that is reached when the rods come into contact with each other ($P/D=1$), being 2.7 and 1.6 for the central and the edge subchannel, respectively. Obviously, this case falls...
out of the scope of this thesis as the contact between the rods would damage the fuel elements of a nuclear reactor. The lower limit of the $D_h^*/D_h$ ratio, however, differs per subchannel. Figure 4.12b shows that the $D_h^*/D_h$ ratio goes to zero if the central subchannel is considered, whereas it goes to the value $1/\pi$ for the edge (wall-bounded) subchannel.

The experiment performed with an eccentric rod inside a circular channel by Choueiri and Tavoularis [13] are included in the plot (∇ in Figure 4.12a) as well. However, this case deserves special care due to geometry very different from the bundle since the borders of the gap region are not clearly identifiable.

It is to be noted that the correlation might have a different expression for low values of the Reynolds number. Experimental results discussed in Chapter 2 of this thesis and reported in [17] show that the length of coherent structures increases at low values of the gap Reynolds number, which cannot be neglected from the correlation. Recalling the general form expressed in Equation 4.14, the exponent of the term $\text{Re}_\text{gap}^\zeta$ could, thus, become lower than zero for low values of the Reynolds.

### 4.5.3 Interaction between the rod and the flow

This section discusses the results of the fluid-structure interaction measurements performed with the high speed camera. Three frequencies are considered for this analysis: the frequency of oscillation of the silicone rod, its natural frequency, and the frequency at which coherent structures move in the axial direction. The average frequency of vibration of the silicone rod’s wall, $f_{\text{sil}}$, the average displacement $\overline{\varepsilon}$, and the $\varepsilon_{\text{rms}}$ are obtained through ten series of measurements repeated on different days for each value of the flow rate. The natural frequency of the silicone rod is estimated depending on the local velocity around the central flexible rod applying the theory explained in section 4.2.2. The stream-wise rate of passage of the coherent structures is measured with LDA in the central subchannel close to the rod (Figure 4.10b). The three frequency series, rendered non dimensional, are plotted in Figure 4.13 against the non dimensional velocity. The frequency of the coherent structures, $f_y^*$ increases linearly with the flow speed as previously discussed. The natural frequency of the silicone rod, $f_{\text{sil},n}$, decreases with the velocity of the surrounding fluid: as the flow rate increases, the frequency of the silicone rod $f_{\text{sil}}^*$ decreases and becomes nearly constant for $v^* \geq 9.4$ ($\text{Re}_e \geq 29000$, as shown in more detail in Figure 4.14a. The frequency of the structures $f_y^*$ approaches twice the natural frequency of the rod $2f_{\text{sil},n}$ when the measured frequency of oscillation of the rod wall $f_{\text{sil}}^*$ matches $f_{\text{sil},n}^*$ for $v^* = 5.7$ ($\text{Re}_e \approx 22700$).

Both trends of the mean displacement of the wall, $\overline{\varepsilon}$, and its root mean square, $\varepsilon_{\text{rms}}$, (Figure 4.15) display a clear peak in the Reynolds number range where $f_y^* = 2f_{\text{sil},n}^*$. The results shown in Figure 4.13 can therefore have the following interpretation. Choueiri and Tavoularis [13] measured that the lateral velocity component of the vortex street oscillated with half the rate of passage of the coherent structures.
in the axial direction $f_y/2$. This was consistent with Meyer and Rehme’s model (sketched in Figure 4.14b) that regarded coherent structures near a gap as counter-rotating vortices which induce a fluctuating velocity field. The components of such a field along the span-wise and stream-wise directions $x$ and $y$ gives a velocity that fluctuates twice as fast along $y$ ($V_y$ in Figure 4.14b). Conversely, the span-wise component ($U_x$ in Figure 4.14b) would oscillate twice as slow around the mean value. The fluctuating lateral velocity component would produce an external force on the rod, oscillating in time with frequency $f_y/2$. When such force oscillates with $f_y/2=f_{sil,n}$ (Figure 4.13), the rod and the vortex street synchronise and the magnitude of the oscillations increases [30], as shown in Figure 4.15.

Alternatively, the results can also be interpreted as a parametric resonance. In parametric oscillations, a parameter can be adjusted in order to change the oscillation frequency of the system. It is known that parametric resonance occurs, in its most energetic mode, when the frequency reaches twice the natural frequency of the system [31, 24]. Therefore, the increase in the amplitude of oscillation observed in our experiments can be regarded as parametric resonance.
4.5. RESULTS AND DISCUSSION

Figure 4.12: a) Normalised wavelength of coherent structures $\lambda/D_h^*$ against $D^*_h/D_h$. The Reynolds numbers in the legend are based on the total flow area of the test section. ○: this work’s experiments; ×: data from [77]; □: data from [17]; ◊: data from [19]; △: data from [88]; ▽: data from [13]; * data from [62].

b) Trend of $D^*_h/D_h$ as a function of $P/D$ for edge and central subchannels of a hexagonal bundle; as the bundle becomes indefinitely large the ratio tends to zero (centre subchannel) and to a finite number (edge subchannel).
Figure 4.13: Plot of $f_y^*/2$ (○), estimated natural frequency of the central silicone rod $f_{sil,n}$ (blue line), and frequency of vibration of the silicone rod wall $f_{sil}^*$ (□) measured with the high-speed camera system.

Figure 4.14: a) Non dimensional response frequency of the silicone rod, depending on the non dimensional velocity $v^*$. b) Gap vortex streets moving with the axial flow along a gap, identified by the dashed borders; originally proposed by Meyer and Rehme in [18].
4.6 Practical application of this chapter’s findings

The findings of this chapter are applied to a couple of real cases, being the Super Phenix and the MYRRHA (Multi-purpose hYbrid Research Reactor for High-tech Applications) reactors, cooled with liquid sodium and lead-bismuth eutectic (LBE) respectively. Both designs have the fuel rods arranged in a hexagonal lattice.

![Graph](image-url)

**Figure 4.15:** Average displacements of the silicone rod border (a) and displacement root mean square (b) depending on the Reynolds. A clear peak in both plots, once compared with the measured response frequency, indicates synchronisation between the rod and the flow structures.

The Super Phenix has a $P/D$ ratio of 1.12, pin diameter of $8.65 \times 10^{-3}$ mm and active core length of 2m [99, 90]. The average sodium temperature through the core is 470°C and the mean flow speed $5\text{ms}^{-1}$. Using the correlation derived in this chapter, one can estimate that the expected coherent structures inside one of the main triangular subchannels of the core of the Super Phenix have a wavelength of 4.9 cm. This corresponds to a stream-wise frequency $f_y$ of 100Hz. Based on the findings discussed in this chapter, a lateral frequency $f_{y2}$ of 50Hz is expected. The theory discussed in section 4.2 to estimate the natural frequency of a rod surrounded by other six arranged in a hexagonal lattice yields a frequency of 5.4Hz, which is well below that of the coherent structures, seemingly not posing issues of fatigue phenomena on the fuel elements.

The second practical application is to the MYRRHA reactor, which has a $P/D$ ratio of 1.28, pin diameter of $6.55 \times 10^{-3}$ mm, wall thickness of 0.51 mm, core length of 1.4 m, and the average LBE temperature through the core 340°C [100]. The estimated coherent structures wavelength is 5.4 cm that, moving with the average flow speed of $1.7\text{ms}^{-1}$, corresponds to a stream-wise frequency of 31Hz, hence the expected lateral frequency $f_{y2}=15.5$Hz. The natural frequency of the fuel rod at the considered flow velocity is 5.4Hz.

Based on these estimations, the frequency of the coherent structures occurring in MYRRHA is closer to the natural frequency of the fuel rod than in the Super Phenix, making that design more prone to synchronization with flow structures.
However, both Super Phenix and MYRRHA employ helicoid wires wrapped around the rods to avoid contact and to prevent damage of the fuel pins, which are not taken into account in these examples.

### 4.7 Summary

The work described in this chapter aimed at studying the structural response of the central rod to large coherent structures occurring in the flow through a hexagonal bundle of rod tightly clustered ($P/D=1.11$). The flow was studied with Laser Doppler Anemometry while the vibrations induced on the flexible rod were recorded with a high-speed camera. Optical access to the measurement region was achieved by means of the refractive index-matching technique.

The measurements of frequency and displacements of the rod wall suggested synchronization between the rod and the coherent structures when these move axially with twice the natural frequency of the rod. This condition is characterised by the increased magnitude of the oscillations and by a response near to the natural frequency of the rod.

A new correlation for estimating the wavelength of the structures is derived based on dimensional analysis and experiments, resulting in a wavelength that scales linearly with the hydraulic diameter of the gap region. Such a correlation is valid for different geometries, involving channels with single rods or more complex rod bundles with $P/D$ (or $W/D$) ranging from 1.02 to 1.20.

The findings presented in this chapter contribute to explain further the physics of the flow-induced vibrations of coherent structures arising in axial rod bundle flows, typical of industrial applications. Furthermore, the proposed correlation may be helpful in designing industrial components that are not prone to resonance phenomena and, thus, mechanical fatigue.
## 4.8 Nomenclature

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1, a_2, a_3, a_4, a_5, a_6 )</td>
<td>Exponents of the dimensional analysis</td>
<td>–</td>
</tr>
<tr>
<td>( C_{fT,c}, C_{fT,e} )</td>
<td>Coefficients for the pressure drop across the hexagonal section</td>
<td>–</td>
</tr>
<tr>
<td>( C_m )</td>
<td>Added mass coefficient</td>
<td>–</td>
</tr>
<tr>
<td>( C_N )</td>
<td>Lateral drag force coefficient</td>
<td>–</td>
</tr>
<tr>
<td>( C_T )</td>
<td>Longitudinal force coefficient</td>
<td>–</td>
</tr>
<tr>
<td>( C_V )</td>
<td>Viscous force coefficient</td>
<td>–</td>
</tr>
<tr>
<td>( C_1, C_2 )</td>
<td>Coefficients</td>
<td>–</td>
</tr>
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<td>( D_h )</td>
<td>Hydraulic diameter of the subchannel</td>
<td>m</td>
</tr>
<tr>
<td>( D_{h,c}, D_{h,e} )</td>
<td>Hydraulic diameter of central and edge subchannels, respectively</td>
<td>m</td>
</tr>
<tr>
<td>( D'_h )</td>
<td>Hydraulic diameter of the gap region</td>
<td>m</td>
</tr>
<tr>
<td>( D_o )</td>
<td>Equivalent diameter of the outer channel</td>
<td>m</td>
</tr>
<tr>
<td>( D_r )</td>
<td>Rod diameter</td>
<td>m</td>
</tr>
<tr>
<td>( D_{sil,i}, D_{sil,o} )</td>
<td>Inner and outer diameter of the silicone rod</td>
<td>m</td>
</tr>
<tr>
<td>( f_c, f_e )</td>
<td>Friction coefficients of central and edge subchannels, respectively</td>
<td>–</td>
</tr>
<tr>
<td>( f_{gap}, f_{ch} )</td>
<td>Friction coefficients of the gap region and main subchannel, respectively</td>
<td>–</td>
</tr>
<tr>
<td>( E )</td>
<td>Young (or elasticity) modulus</td>
<td>Pa</td>
</tr>
<tr>
<td>( f^*<em>{sil,n}, f</em>{sil,n} )</td>
<td>(Non dimensional) natural frequency of the silicone rod, respectively</td>
<td>Hz</td>
</tr>
<tr>
<td>( f^*<em>{sil}, f</em>{sil} )</td>
<td>(Non dimensional) vibration frequency of the silicone rod, respectively</td>
<td>Hz</td>
</tr>
<tr>
<td>( f_{y}, f_y )</td>
<td>(Non dimensional) frequency of coherent structures, respectively</td>
<td>Hz</td>
</tr>
<tr>
<td>( I )</td>
<td>Moment of inertia of the silicone rod</td>
<td>m</td>
</tr>
<tr>
<td>( L_{sil} )</td>
<td>Silicone rod length</td>
<td>m</td>
</tr>
<tr>
<td>( m_a )</td>
<td>Added mass on the silicone rod</td>
<td>kg</td>
</tr>
<tr>
<td>( m_{sil} )</td>
<td>Silicone rod mass</td>
<td>kg</td>
</tr>
<tr>
<td>( N )</td>
<td>Number of recorded frames of the high speed camera</td>
<td>–</td>
</tr>
<tr>
<td>( N_g )</td>
<td>Number of points for fitting the spectral peak</td>
<td>–</td>
</tr>
<tr>
<td>( P )</td>
<td>Pin pitch</td>
<td>m</td>
</tr>
<tr>
<td>( P/D )</td>
<td>Ratio between the pitch and the diameter of the half rods</td>
<td>–</td>
</tr>
<tr>
<td>( [\Delta p]<em>{ch}, [\Delta p]</em>{gap} )</td>
<td>Pressure drop across the main subchannel and the gap region, respectively</td>
<td>Pa</td>
</tr>
<tr>
<td>( [\Delta p]<em>{c}, [\Delta p]</em>{e} )</td>
<td>Pressure drop across central and edge subchannels, respectively</td>
<td>Pa</td>
</tr>
<tr>
<td>( Q )</td>
<td>Volumetric flow rate</td>
<td>( m^3 s^{-1} )</td>
</tr>
<tr>
<td>( x, y, z )</td>
<td>Span-wise (transversal), stream-wise (vertical), and normal directions</td>
<td>m</td>
</tr>
</tbody>
</table>
CHAPTER 4. FLUID-STRUCTURE INTERACTION IN A 7-RODS HEXAGONAL BUNDLE

$S$, $S_{\text{fit}}$  Spectrum and fitted value, respectively
$ar{S}$  Mean value of the spectrum
$W/D$  Nearest wall distance -to-rod diameter ratio

### Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v^*$, $v$</td>
<td>(Non dimensional) stream-wise (vertical) velocity component, respectively</td>
<td>$\text{ms}^{-1}$</td>
</tr>
<tr>
<td>$v_c$, $v_e$</td>
<td>(Non dimensional) stream-wise (vertical) velocity component inside the central and edge subchannels, respectively</td>
<td>$\text{ms}^{-1}$</td>
</tr>
<tr>
<td>$v_{\text{gap}}$</td>
<td>Stream-wise (vertical) velocity component inside the gap region</td>
<td>$\text{ms}^{-1}$</td>
</tr>
<tr>
<td>$v_{\text{in}}$</td>
<td>Value at the inflection point in the profile of the stream-wise (vertical) velocity component</td>
<td>$\text{ms}^{-1}$</td>
</tr>
<tr>
<td>$v'_\text{rms}$</td>
<td>Root mean square of the stream-wise (vertical) velocity component</td>
<td>$\text{ms}^{-1}$</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
<td>s</td>
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### Greek letters

<table>
<thead>
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<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>Gap width</td>
<td>m</td>
</tr>
<tr>
<td>$\varepsilon_d$</td>
<td>Dissipation rate based on Kolmogorov theory</td>
<td>$\text{ms}^{-3}$</td>
</tr>
<tr>
<td>$\varepsilon(t_i), \bar{\varepsilon}$</td>
<td>Instantaneous and mean displacement of the silicone rod</td>
<td>$\mu\text{m}$</td>
</tr>
<tr>
<td>$\varepsilon_{\text{rms}}$</td>
<td>Root mean square of the displacement of the silicone rod</td>
<td>m</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Exponent</td>
<td>–</td>
</tr>
<tr>
<td>$\bar{\lambda}$</td>
<td>Average stream-wise (vertical) wavelength of coherent structures</td>
<td>m</td>
</tr>
<tr>
<td>$\lambda_{\text{Max}}, \lambda_{\text{min}}$</td>
<td>Maximum and minimum estimated stream-wise wavelength of coherent structures</td>
<td>m</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity of water</td>
<td>Pas</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity</td>
<td>$\text{ms}^{-1}$</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Exponent</td>
<td>–</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Fluid density</td>
<td>$\text{kgm}^{-3}$</td>
</tr>
<tr>
<td>$\rho_{\text{ac}}$</td>
<td>Autocorrelation coefficient</td>
<td>–</td>
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<tr>
<td>$\sigma_y$</td>
<td>Width of the Gaussian bell fitting the frequency spectra</td>
<td>Hz</td>
</tr>
<tr>
<td>$\chi_1, \chi_2$</td>
<td>Exponents</td>
<td>–</td>
</tr>
<tr>
<td>$\vec{\omega}$</td>
<td>Vorticity vector</td>
<td>$\text{s}^{-1}$</td>
</tr>
<tr>
<td>Acronyms</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>CMOS</td>
<td>Complementary Metal-Oxide Semiconductor</td>
<td></td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
<td></td>
</tr>
<tr>
<td>FEP</td>
<td>Fluorinated Ethylene Propylene</td>
<td></td>
</tr>
<tr>
<td>FIV</td>
<td>Flow-induced vibration</td>
<td></td>
</tr>
<tr>
<td>FSI</td>
<td>Fluid-structure interaction</td>
<td></td>
</tr>
<tr>
<td>Gen-IV</td>
<td>Generation IV</td>
<td></td>
</tr>
<tr>
<td>LBE</td>
<td>Lead-bismuth eutectic</td>
<td></td>
</tr>
<tr>
<td>LDA</td>
<td>Laser Doppler Anemometry</td>
<td></td>
</tr>
<tr>
<td>LMFBR</td>
<td>Liquid Metal Fast Breeder Reactor</td>
<td></td>
</tr>
<tr>
<td>LWR</td>
<td>Light Water Reactor</td>
<td></td>
</tr>
<tr>
<td>MYRRHA</td>
<td>Multi-purpose hYbrid Research Reactor for High-tech Applications</td>
<td></td>
</tr>
<tr>
<td>NRMSE</td>
<td>Normalised Root Mean Square Error</td>
<td></td>
</tr>
<tr>
<td>RIM</td>
<td>Refractive Index Matching</td>
<td></td>
</tr>
<tr>
<td>SEEDS-1</td>
<td>SEven rods Experiments in Delft for Sesame-1</td>
<td></td>
</tr>
<tr>
<td>TRIGA</td>
<td>Training Research Isotopes General Atomic</td>
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<table>
<thead>
<tr>
<th>Non dimensional groups</th>
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<tbody>
<tr>
<td>Re</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>Re_c, Re_e</td>
<td>Reynolds number of central and edge subchannels</td>
</tr>
<tr>
<td>Re_gap</td>
<td>Reynolds number of the gap region</td>
</tr>
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</table>
5.1 Introduction and motivation

Rod bundles characterise the geometry of many industrial components, such as heat exchangers and the core of light water and liquid metal cooled nuclear reactors. In particular, the core of a Liquid Metal Fast Breeder Reactor (LMFBR) consists of a hexagonal bundle of rods, each wrapped by a wire wound helicoidally. These wires prevent damage by fretting of the fuel rod’s cladding, and guide the liquid metal through the gaps between the pins enhancing radial heat transfer and improving the safe operation of the reactor.

Depending on the position of the wire, the flow was found to unexpectedly bend against the direction of the wire. This was called “migratory flow” by Ohtake et al. and ascribed to the increased hydraulic resistance caused by the wire inside the considered subchannel [45]. A physical model that could predict such a flow bending, however, was not provided, nor it is available nowadays.

This chapter describes the work carried out to measure migratory flow in a 7-rod, wire-wrapped hexagonal bundle to develop a model that explains and quantifies the bending of the flow due to the action of the wire. The facility, SEEDS-2, replaces SEEDS-1 in the experimental loop. Planar Particle Image Velocimetry (PIV) is performed to measure the flow near the wire wrapped around the central rod of the bundle. The optical access needed for the measurements is provided by making the outer casing of transparent Perspex, and by replacing part of the steel of the rods with Fluorinated Ethylene Propylene (FEP), which is a refractive index-matching material.

The content of this chapter has been submitted *Int. J. Heat Fluid Fl.* (2019)
The approach is structured as follows:

1. The flow field at the front of the central wrapping wire is studied with PIV for six equally spaced Reynolds numbers, at room temperature, and with water as working fluid.

2. The pressure gradient field close to the wire is retrieved from the measured velocity field and the discretised, two-dimensional Navier-Stokes equations.

3. A model, derived from the Euler equations, is derived for estimating both the pressure gradient normal to the wire and the bending angle of the flow streamlines.

4. The pressure gradient field obtained from the model is compared with the experimentally obtained field.

5. Finally, the theoretically predicted bending angle of the flow is compared with the experimental values to confirm the validity of the proposed model.

5.2 Experimental apparatus

The measurement system is a planar PIV system consisting of a laser source and a high speed camera. The light source illuminates a region of fluid close to the wire wrapped around the central rod of the hexagonal bundle. The images needed for computing the flow velocity are recorded by the camera from the front side of the Perspex hexagonal casing.

5.2.1 PIV and uncertainty quantification

The light source of the PIV system is a class-IV, diode pumped 5W laser with a wavelength of 532nm (LaVision, Germany). The DaVis software allows for adjusting the separation time between the two laser pulses. The combination of a spherical and a cylindrical lens creates the laser sheet, which has a beam waist (the thinnest region of the sheet) thickness of 1mm. The flow is seeded with borosilicate glass hollow spheres (LaVision, Germany) with an average density of $1.1\text{ g m}^{-3}$ and average diameter of $9 - 13\mu\text{m}$. The camera used for PIV measurements is a Complementary Metal-Oxide Semiconductor (CMOS) Imager MX-4M (LaVision, Germany) with 4 MP of resolution (pixel size of $5.5\mu\text{m}$). The camera mounts an AF-S 50mm F/1.4 (Nikon) lens with a magnification factor $M_0=0.13$. The depth of the laser sheet (out-of-plane direction) $\delta_z$ is given by [101] as

$$\delta_z=4\left(1 + \frac{1}{M_0}\right)^2 f_{\#}^2 \lambda_l=0.6\text{mm},$$

(5.1)

where $f_{\#}$ is the lens aperture, and $\lambda_l$ is the laser wavelength.

Pairs of images are recorded by the camera with a frequency 18Hz where the particles appear as bright spots as they reflect light. The time separation between the images, in each pair, is adjusted by the laser: this can be as low as 100µs.
Cross-correlating each pair of images allows for detecting the displacement of each particle during the time interval between the two images. Based on a calibration factor, the pixel displacement is finally converted into a velocity value. An intensity correction algorithm consisting of a background subtraction is used to diminish the reflection from the internal components of the bundle. The recorded images are pre-processed by subtracting the local intensity of each image (sliding background) to keep intensity fluctuations at a minimum. This method averages the local intensity of light around each particle. The mean value is then subtracted from the intensity value of each pixel to improve the local contrast. The velocity vectors are calculated with a multi-pass option where the initial size of the interrogation window is $64 \times 64$ pixels and two passes are used: the displacement of the particles obtained in the first pass is used as information to shift the interrogation window in the next pass. The initial size of the interrogation window is subsequently decreased to $48 \times 48$ pixels. A reference vector field computed on 500 images is used in the first pass to introduce an initial deformation of the interrogation window. This enhances the signal-to-noise ratio of the correlation peaks. The post-processing consists of a median filter to remove outliers [102, 36]. The resulting blank points are filled with a vector interpolation algorithm.

The uncertainty on the PIV measurements consists of a statistical error and a systematic error. The statistical error $\sigma_{\text{stat}}$ is evaluated following [101] as

$$\sigma_{\text{stat}} = \frac{\sigma}{\sqrt{N}} = \sqrt{\frac{1}{N(N-1)} \sum_{i=1}^{N} (v_i - \bar{v})^2}, \quad (5.2)$$

where $\sigma$ is the standard deviation of the measured velocity samples, $N$ is the number of samples, and $\bar{v}$ is the mean value.

The systematic error $\sigma_{\text{syst}}$ is due to the calibration of the PIV system: the conversion factor $F$ from pixel to mm is based on the image of the wire, whose diameter in mm is known:

$$F = \frac{D_w}{D_{w,p}}, \quad (5.3)$$

being $D_w$ and $D_{w,p}$ the diameter of the wire in mm and pixel, respectively.

Since there is a dimensional tolerance $\delta D_w$ on the wire diameter (reported in table 3.2), $F$ is subject to an uncertainty as well. This is estimated considering that a generic velocity vector is evaluated from the measurements as

$$v_i = \frac{\Delta_p F}{dt} = \frac{\Delta_p D_w}{dt D_{w,p}}, \quad (5.4)$$

where $\Delta_p$ is the particle displacement in pixel between two images and $dt$ is the time between the frames. An uncertainty on the diameter of the wire expressed in pixel $\delta D_{w,p} = 1$ is also taken into account in the systematic error.

The systematic error $\sigma_{\text{syst}}$ is evaluated through the uncertainty propagation formula as

$$\sigma_{\text{syst}} = \sqrt{\left(\frac{\partial v_i}{\partial D_w} \delta D_w\right)^2 + \left(\frac{\partial v_i}{\partial D_{w,p}} \delta D_{w,p}\right)^2}, \quad (5.5)$$
where the two derivative terms are obtained by deriving Equation 5.4 with respect to $D_w$ and $D_{w,p}$:

$$
\frac{\partial v_i}{\partial D_{w}} = \frac{\Delta_p}{dt D_{w,p}}, \quad \frac{\partial v_i}{\partial D_{w,p}} = \frac{\Delta_p}{dt D_{w,p}^2}.
$$

(5.6)

The total uncertainty $\sigma_\tau$ is estimated adding in quadrature $\sigma_{\text{stat}}$ and $\sigma_{\text{syst}}$:

$$
\sigma_\tau = \sqrt{\sigma_{\text{stat}}^2 + \sigma_{\text{syst}}^2}.
$$

(5.7)

The uncertainty thus calculated is included in the line plots of the PIV measurements shown from here on.

## 5.3 Measurement campaign

The measurement campaign consists of six flow rate values for which the corresponding Reynolds numbers are calculated based on the bundle’s hydraulic diameter $D_h = 4A/P_w$, where $A$ is the bundle flow area, and $P_w$ is the corresponding wetted perimeter, according to [90]. Table 5.1 reports the water temperature of the experiments and the Reynolds number for the investigated flow rates. Each

<table>
<thead>
<tr>
<th>$Q$ [$\text{m}^3\text{s}^{-1}$]</th>
<th>$T$ [$\text{o}^\circ\text{C}$]</th>
<th>$Re$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.95 \times 10^{-3}$</td>
<td>32.3</td>
<td>16330</td>
</tr>
<tr>
<td>$2.70 \times 10^{-3}$</td>
<td>31.7</td>
<td>14760</td>
</tr>
<tr>
<td>$2.30 \times 10^{-3}$</td>
<td>32.0</td>
<td>12650</td>
</tr>
<tr>
<td>$1.80 \times 10^{-3}$</td>
<td>32.2</td>
<td>9940</td>
</tr>
<tr>
<td>$1.370 \times 10^{-3}$</td>
<td>32.3</td>
<td>7580</td>
</tr>
<tr>
<td>$9.00 \times 10^{-4}$</td>
<td>32.4</td>
<td>4990</td>
</tr>
</tbody>
</table>

Table 5.1: $Q$: mass flow rate; $T$: temperature of the water; $Re$: Reynolds number.

PIV measurements consists of 10000 images collected at a rate of 18 frames per second. The location of the measurement region is shown in Figure 5.1. The laser sheet enters the outer hexagonal Perspex casing (Figure 5.1a) and it goes through the FEP, reaching the central wire spacer where the measurement region is located (red rectangular area in Figure 5.1b). For a preliminary measurement, the laser sheet is moved closer to the rod (Figure 5.1b, dashed profile in section B-B) in order to study the flow area downstream the wire. The second, and main, measurement campaign is conducted with the laser sheet closer to the bulk region of the subchannel, being tangent to the central rod’s wire spacer. The flow area in front of the helicoid spacer is thus investigated.
5.4 Symmetry of the flow

Due to the closeness of the elements of the bundle, the correct placement of the rods is very important to avoid flow blockages and asymmetric flow during the experiments. Owing to the small size of the bundle, an error in the orientation of the rods would affect the whole flow field. For this reason, a preliminary series of measurements is carried out at $Re=23450$ focusing on the flow around the central rod.

Given the symmetry of the setup, the flow fields at different locations along the central wire should be similar. Therefore, two PIV measurements are performed that study the flow near the centre. One PIV measurement is carried out through the front of the hexagonal Perspex casing, measuring the flow area shown in Figure 5.1; the main measurement campaign will be carried out at this location.

Later, the bundle is turned by 60° clock-wise allowing for a second, additional measurement through the side (Figure 5.2): the measured area is shifted upward in order to have the wire in the same position of the first measurement, relative to the gap. A similar measurement could not be performed through the left side as the measured area could not be reached by the moving system.

The two time-averaged flow fields thus obtained are shown in Figure 5.2b. The flow fields are qualitatively similar, indicating that the rods are correctly oriented inside the hexagonal case.

A quantitative comparison between the two measurements is done by evaluating
Figure 5.2: a) Flow areas through the front (blue) and through the side (red) studied to check the flow symmetry; the red arrow indicates the second side through which the additional measurement is performed. b) The wire in the additional measurement region is in the same position, relative to the gap, of the measurement through the front (detail A).

the velocity magnitude along three arbitrary vertical lines of the flow fields. The velocity magnitude is shown in Figure 5.4, where each plot refers to one of the three lines. The plots show that the magnitude of the velocity measured through the two faces is similar within 6.4% (x=4mm), 3.5% (x=6mm), and 4.1% (x=8mm), suggesting that the flow inside the subchannels is not obstructed and that the rods are installed correctly inside the bundle.
Figure 5.3: Flow field measured through the front side (a) and through the side to the right (b). The vertical lines highlight the paths followed for comparing the two flow fields.
Figure 5.4: Normalised velocity magnitude evaluated along the vertical line (a) \( x=4\,\text{mm} \), (b) \( x=6\,\text{mm} \) and (c) \( x=8\,\text{mm} \) for the two measurements.
5.5 Time-averaged velocity fields

The initial measurement campaign studies the fluid region downstream the wire (Figure 5.1b, view B-B), showing that the flow follows the helical path of the wire, as expected, contributing to the mixing. This is evident from Figure 5.5a where the blue colour indicates a negative direction of the $u$ component (towards the left). The flow field of the lateral component measured downstream the wire for the six Reynolds numbers is shown in Figure 5.6 where the superimposed vectors follow the wire direction (from top right to bottom left in the plots).

![Figure 5.5: a) Contour plot of the lateral velocity component $u$ measured beneath the wire; the dashed line indicates the wire’s position, whereas the solid line bounds the area affected by reflection, which is masked out. b) Contour plot of the $u$ component measured at the front of the wire, closer to the bulk of the subchannel; the dashed line marks the borders of the wire.](image)

Figure 5.5b shows that if the measurement region is moved at the front of the wire, closer to the bulk of the subchannel, the fluid changes direction moving against the wire’s path. This is clear from the red colour that shows a positive $u$, hence a flow in the positive $x$ direction. This flow behaviour, called “migratory flow” by Ohtake et al. [45], was ascribed to the increased hydraulic resistance that the wire causes in the subchannel. A similar behaviour is found in Song et al. [46].

The velocity fields shown hereafter focus on the flow in front of the central wire where the migratory flow is observed. The measured quantities are the stream-wise and span-wise velocity components $v$ and $u$, respectively.

Figure 5.7 shows the contour plot of the time-averaged axial velocity component $v$, for the considered values of Reynolds numbers (see table 5.1). A low-speed region appears at the downstream side of the wire, which is also found by [34].
This region is more spread-out at the lower Reynolds numbers and it becomes narrower as the Reynolds increases. This is quantitatively more clear in Figure 5.8a where $v$ is evaluated along the vertical line $x=5.6\text{mm}$ for $Re=4990$, $Re=12650$ and $Re=16330$ (the flow direction in the line plot is from left to right). At $Re=4990$ the low-speed region downstream of the wire is more pronounced and it affects a larger area of the wake.

Figure 5.9 shows the contour plot of the normalised time-averaged span-wise velocity component $u$ for the six studied Reynolds numbers. The positive sign of the lateral velocity component means that the flow coming from the top bends towards the right as it approaches the wire. The figure shows that the relative magnitude of $u$ is slightly higher at lower Reynolds numbers, meaning that the flow tends to bend more markedly at lower flow rates. The lateral component $u$ is evaluated along the same vertical line and the values are plotted in Figure 5.8b. The lateral component becomes increasingly higher as the wire is approached; the bending towards the wire is relatively stronger as the Reynolds is decreased, reaching a maximum for $Re=4990$. 

**Figure 5.6:** Span-wise time-averaged velocity component $u$ normalised to the bulk velocity $V_b$, measured downstream the wire, shown as a dashed line. a) $Re=4990$; b) $Re=7580$; c) $Re=9940$; d) $Re=12650$; e) $Re=14760$; f) $Re=16330$. 
5.5.1 Determining the pressure gradient field from the 2D Navier-Stokes equations

The pressure gradient normal to the wire, later on needed to describe the migratory flow, is evaluated by means of the Navier-Stokes equations applied to the measured velocity fields. Due to the action of the wire that pulls the fluid transversally throughout the gaps, the fluid measured close to the rod is assumed to have two major velocity components: the lateral flow direction induced by the wire, and the main axial direction. Based on the experiments described in [103], the out-of-plane velocity component is estimated to be equal at most to $10\% V_b$ for a Reynolds of 6300. Moreover, a recent numerical work [46] showed that the local flow behaviour near the wire is independent of the Reynolds number. It is thus reasonable to assume that the relative magnitude of the out-of-plane velocity component near the wire does not strongly depend on the flow rate. Hence, the flow is considered two-dimensional. The Navier-Stokes equations are discretised in space and time.
5.5. TIME-AVERAGED VELOCITY FIELDS

Figure 5.8: a) Stream-wise (axial) and b) span-wise (lateral) velocity components evaluated along the vertical line \( x = 5.6 \text{mm} \) for three Reynolds numbers. The flow direction is from left to right. Due to light reflection from the wire, some values measured at the wire’s position are not included.

following the approach of [104, 105]. The pressure gradient is expressed as follows:

\[
\nabla p = -\rho \left[ \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} - \nu \nabla^2 \vec{v} \right],
\]

(5.8)

where \( \nu \) is the kinematic viscosity. Assuming a two-dimensional flow, Equation 5.8 is decomposed along \( x \) (and similarly along \( y \)) as

\[
\frac{\partial p(x,y,t)}{\partial x} = -\rho \left[ \frac{\partial u(x,y,t)}{\partial t} + u(x,y,t) \frac{\partial u(x,y,t)}{\partial x} + v(x,y,t) \frac{\partial u(x,y,t)}{\partial y} + \right. \\
\left. -\nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \right].
\]

(5.9)

The time derivative term is discretized with a central finite difference scheme following [105], and considering a separation time between two consecutive images of 0.055s. The same discretization scheme is used for the spatial derivatives of \( u \) and \( v \). This leads, for the \( x \) direction (and similarly for the \( y \) direction), to

\[
\frac{\partial p(i,j,\tau)}{\partial x} = -\rho \left[ \frac{u(i,j,\tau+1) - u(i,j,\tau-1)}{2\Delta t} + u(i,j,\tau) \frac{u(i+1,j,\tau) - u(i-1,j,\tau)}{2\Delta x} + \\
+ v(i,j,\tau) \frac{u(i,j,\tau+1) - u(i,j,\tau-1)}{2\Delta y} + \\
-\nu \left( \frac{u(i+1,j,\tau) - 2u(i,j,\tau) + u(i-1,j,\tau)}{(\Delta x)^2} + \right. \\
\left. + \frac{u(i,j+1,\tau) - 2u(i,j,\tau) + u(i,j-1,\tau)}{(\Delta y)^2} \right) \right],
\]

(5.10)
Figure 5.9: Span-wise velocity component $u$ normalised to the bulk velocity $V_b$, overlapped to the vectorial field. a) $Re=4990$; b) $Re=7580$; c) $Re=9940$; d) $Re=12650$; e) $Re=14760$; f) $Re=16330$. The main flow is from top to bottom.

where $\Delta x$ and $\Delta y$ are the spatial pitches between consecutive vectors of the matrix along $x$ and $y$, respectively. Equation 5.10, and the corresponding one for the $y$ direction, are averaged over the total recorded images to obtain the time-averaged pressure gradient components along $x$ and $y$:

$$\overline{\frac{\partial p(i,j)}{\partial x}} = \frac{1}{N} \sum_{\tau=1}^{N} \frac{\partial p(i,j,\tau)}{\partial x},$$

and

$$\overline{\frac{\partial p(i,j)}{\partial y}} = \frac{1}{N} \sum_{\tau=1}^{N} \frac{\partial p(i,j,\tau)}{\partial y},$$

where $N$ is the number of recorded PIV images. Finally, the time-averaged pressure gradient normal to the wire is estimated as

$$\frac{\partial p(i,j)}{\partial \tilde{x}} = \frac{\partial p(i,j)}{\partial x} \sin \theta + \frac{\partial p(i,j)}{\partial y} \cos \theta,$$

where $\theta=77^\circ$ is the angle that the wire forms with the horizontal (later sketched in Figure 5.11a).
5.6 Modelling of migratory flow

The time-averaged velocity fields shown in the previous section show that the flow near the wire bends against the wrapping direction of the helicoid spacer (Figure 5.6), if the flow is measured at the front of the wire. This section discusses the theory through which this behaviour is modelled and explained.

5.6.1 Euler equations in the streamline’s coordinate system

\[
\sum \vec{F}_{\text{ext}} = \frac{D(M\vec{V})}{Dt} = \frac{\partial (M\vec{V})}{\partial t} + (\vec{V} \cdot \nabla)M\vec{V}, \tag{5.14}
\]

where \( \vec{F}_{\text{ext}} \) is the sum of external forces acting on the control volume element, \( \vec{V} \) is the velocity vector, \( \frac{D}{Dt} \) is the substantial (or Lagrangian) derivative, and \( M = \rho dV \) is the mass of the control volume, with \( \rho \) the fluid density. If \( M \) is constant, the previous equation becomes

**Figure 5.10:** Force balance in a Lagrangian frame of reference following a fluid element of volume \( dV = dn ds \) per unit of depth along a streamline; \( \beta \) is the angle that the streamline forms with the \( y \) axis.
\[ \sum \vec{F}_{\text{ext}} = M \left( \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right). \] (5.15)

The definition of streamline implies that the velocity vector is always tangent to the streamline, so it has no component along the normal direction \( n \) [101], therefore \( \vec{v} \) can be re-written as

\[ \vec{v} = U_s \hat{e}_s, \]

being \( U_s \) the modulus of the vector and \( \hat{e}_s \) the unit vector along the tangent direction \( s \).

By separating the external forces along the \( s \) and \( n \) directions, Equation 5.15 becomes

\[ \sum F_{\text{ext},s} \hat{e}_s + \sum F_{\text{ext},n} \hat{e}_n = \rho \frac{\partial (U_s \hat{e}_s)}{\partial t} + (\vec{v} \cdot \nabla) (U_s \hat{e}_s) \]

\[ = \rho \left[ \frac{\partial (U_s \hat{e}_s)}{\partial t} + \frac{\partial U_s}{\partial s} (U_s \hat{e}_s) \right] \] dnds

The sum of the external forces acting along the \( s \) direction is

\[ \sum F_{\text{ext},s} = \rho \left[ \frac{\partial U_s}{\partial t} + \frac{\partial U_s}{\partial s} U_s \right] \] dnds. (5.17)

The left-hand side term is obtained from a balance of the forces acting on the control volume along \( s \):

\[ \sum F_{\text{ext},s} = \rho dn - \left( p + \frac{\partial p}{\partial s} ds \right) dn = -\frac{\partial p}{\partial s} ds dn. \] (5.18)

Substituting Equations 5.17 in 5.18 leads to

\[ -\frac{\partial p}{\partial s} ds dn = \rho \left[ \frac{\partial U_s}{\partial t} + \frac{\partial U_s}{\partial s} U_s \right] dsdn, \] (5.19)

which is re-written as

\[ -\frac{1}{\rho} \frac{\partial p}{\partial s} = \frac{\partial U_s}{\partial t} + \frac{\partial U_s}{\partial s} U_s. \] (5.20)

This equation describes the acceleration of an element of fluid due to the pressure gradient \( \frac{\partial U_s}{\partial s} \) along the followed streamline. The control volume \( dV \) moves along the streamline (i.e. \( s \) direction) with no displacement along the \( n \) direction. This means that there is an equilibrium condition along \( n \). The right-hand side terms of Equation 5.16 are entirely acting along the \( s \) direction, meaning that along \( n \)

\[ \sum F_{\text{ext},n} = 0. \] (5.21)

Analogously to the balance of external forces along the \( s \) direction (Equation 5.18), a balance can be written for the \( n \) direction:
\[ p ds - \left( p + \frac{\partial p}{\partial n} dn \right) ds + M \frac{U_s^2}{R} \frac{\partial p}{\partial n} ds + \rho \frac{U_s^2}{R} ds = 0, \quad (5.22) \]

where \( R \) is the radius of curvature of the streamline defined as in [106]

\[ R = \left[ 1 + \left( \frac{dx}{dy} \right)^2 \right]^{3/2} \left( \frac{d^2 x}{dy^2} \right)^{-1}, \quad (5.23) \]

where \( \frac{dx}{dy} \) is the derivative of the streamline along the stream-wise direction \( y \).

Finally, Equation 5.22 becomes

\[ \frac{\partial p}{\partial n} = \rho \frac{U_s^2}{R}. \quad (5.24) \]

This equation means that a streamline bends under the action of the transversal pressure gradient \( \frac{\partial p}{\partial n} \), with the pressure decreasing towards the centre of curvature of the streamline [107]. Equations 5.20 and 5.24 constitute the Euler equations in the streamline reference frame [108]:

\[ \frac{\partial p}{\partial n} = \rho \frac{U_s^2}{R}, \quad \frac{\partial p}{\partial s} = \rho \left( \frac{\partial U_s}{\partial t} + \frac{\partial U_s}{\partial s} U_s \right). \quad (5.25) \]

The reader is directed to Appendix C for a derivation of the previous set of equations by following an Eulerian approach, instead of Lagrangian.

Figure 5.11a sketches a streamline that bends as the fluid approaches the wire spacer. The main flow is along the \( y \) direction and from top to bottom in the figure. A sample of the real time-averaged streamline distribution is shown in Figure 5.11b. The streamlines are superimposed to the field of the lateral (span-wise) velocity component.

The derivative of the velocity along a streamline, \( \frac{\partial U_s}{\partial s} \), is computed as

\[ \frac{\partial U_s}{\partial s} = \frac{U_s(i + 1) - U_s(i - 1)}{s(i + 1) - s(i - 1)}, \quad (5.26) \]

with the indices \( i + 1 \) and \( i - 1 \) referring to the points along the streamline. This is substituted in Equation 5.25 and the pressure gradient normal to the wire (\( \tilde{x} \) direction) is finally evaluated as

\[ \frac{\partial p}{\partial \tilde{x}} = \frac{\partial p}{\partial n} \sin(\theta - \beta) + \frac{\partial p}{\partial s} \cos(\theta - \beta), \quad (5.27) \]

where \( \theta = 77^\circ \) is the wire’s pitch angle with the horizontal axis (see Figure 5.11a for reference) and \( \beta \) is the angle that the streamline forms with the vertical (see Figure 5.11a for reference). This equation can be used to estimate the pressure gradient normal to the wire as a tool to explain the experimental results regarding the flow direction.
This section aims at deriving an equation that describes and quantifies the bending of the flow streamline due to the proximity of the wire spacer. By doing so, our understanding of the physics governing the migratory flow improves. We start from the Euler equation across a streamline (Equation 5.24) that, without gravitational effects, is

\[
- \frac{U_s^2(\vec{r})}{R(\vec{r})} \hat{e}_n = - \frac{1}{\rho} \frac{\partial p(\vec{r})}{\partial n} \hat{e}_n,
\]

(5.28)

where \(\vec{r} = (x, y)\) is the position in the two-dimensional flow field.

A streamline is defined as the locus of points always tangent to the local velocity vector [101]:

\[
\frac{dx}{dy} = \frac{u(\vec{r})}{v(\vec{r})}
\]

(5.29)

where \(u(\vec{r})\) and \(v(\vec{r})\) are the velocity components along a streamline. For any
The second order derivative is rearranged as

\[
\frac{d^2 \psi(y)}{dy^2} = \frac{u(\bar{r})}{v(\bar{r})} = \frac{U_s \sin \beta(\bar{r})}{U_s \cos \beta(\bar{r})} = \tan \beta(\bar{r}), \tag{5.30}
\]

which is expressed in terms of the angle \( \beta \) formed by the streamline with the vertical (see Figure 5.11a). The velocity magnitude \( U_s \) is then expressed as

\[
U_s^2(\bar{r}) = u^2(\bar{r}) + v^2(\bar{r}) = \begin{bmatrix} 1 + \frac{u^2(\bar{r})}{v^2(\bar{r})} \end{bmatrix} = v^2(\bar{r}) \left[ 1 + \tan^2 \beta(\bar{r}) \right]. \tag{5.31}
\]

Upon substitution of Equation 5.31 in Equation 5.28, this becomes

\[
\rho \frac{v^2(\bar{r})}{R(\bar{r})} \left[ 1 + \tan^2 \beta(\bar{r}) \right] \frac{\rho}{\partial n} \hat{e}_n = \frac{\partial p(\bar{r})}{\partial n} \hat{e}_n. \tag{5.32}
\]

Considering a single streamline \((\psi(y),y)\) and recalling the definition of \(R\) (Equation 5.23), the previous equation becomes

\[
\rho v^2(\bar{r}) \left[ 1 + \tan^2 \beta(\bar{r}) \right] \frac{d^2 \psi(y)}{dy^2} \left[ 1 + \left( \frac{d\psi(y)}{dy} \right)^2 \right]^{3/2} \hat{e}_n = \frac{\partial p(\bar{r})}{\partial n} \hat{e}_n. \tag{5.33}
\]

Following Equation 5.30 \( \tan \beta(\bar{r}) \) replaces the \( d\psi(y)/dy \) term, hence

\[
\rho v^2(\bar{r}) \frac{1 + \tan^2 \beta(\bar{r})}{[1 + \tan^2 \beta(\bar{r})]^{3/2}} \frac{d^2 \psi(y)}{dy^2} \hat{e}_n = \frac{\partial p(\bar{r})}{\partial n} \hat{e}_n. \tag{5.34}
\]

The second order derivative is rearranged as

\[
\frac{d^2 \psi(y)}{dy^2} = \frac{d}{dy} \left( \frac{d\psi(y)}{dy} \right) = \frac{d}{dy} \tan \beta(\bar{r}) = \left[ 1 + \tan^2 \beta(\bar{r}) \right] \frac{d\beta(\bar{r})}{dy}, \tag{5.35}
\]

and substituted into Equation 5.34, to obtain an expression linking the pressure gradient normal to the streamline with the local angle of inflection \( \beta(\bar{r}) \):

\[
\frac{1}{\rho v^2(\bar{r})} \frac{\partial p(\bar{r})}{\partial n} = \left[ 1 + \tan^2 \beta(\bar{r}) \right]^{1/2} \frac{d\beta(\bar{r})}{dy}. \tag{5.36}
\]

In order to obtain \( \beta(\bar{r}) \), Equation 5.36 is integrated in \( dy \) on both sides along the considered streamline \( s=(\psi(y),y) \). The integral is estimated between \( y=\pm \infty \), far upstream, where the influence of the wire is negligible and the streamline is straight \((\beta=0)\), and the arbitrary coordinate \( y=\sigma \) along the streamline. By moving all the terms that do not contain \( \beta(\bar{r}) \) to the right-hand side of the equation, we get

\[
\int_{\infty}^{\sigma} [1 + \tan^2 \beta(\bar{r})]^{1/2} \frac{d\beta(\bar{r})}{dy} \ dy = \int_{\infty}^{\sigma} \frac{1}{\rho v^2(\bar{r})} \frac{\partial p(\bar{r})}{\partial n} \ dy.
\]

The solution of this equation is
\[ \beta(y=\sigma) = \int_{-\infty}^{\sigma} \frac{1}{\rho v^2(\vec{r})} \frac{\partial p(\vec{r})}{\partial n} dy. \] (5.37)

For the complete derivation of the previous equation, the reader is directed to Appendix D.

Considering that \( \frac{\partial y}{\partial s} = \cos \beta \) (see Figure 5.10), the previous equation can be rewritten as

\[ \beta(y=\sigma) = \int_{-\infty}^{s} \frac{1}{\rho v^2(\vec{r})} \frac{\partial p(\vec{r})}{\partial n} \cos \beta(\vec{r}) ds. \] (5.38)

For small \( \beta \), the cosine can be neglected:

\[ \beta(y=\sigma) = \int_{-\infty}^{s} \frac{1}{\rho v^2(\vec{r})} \frac{\partial p(\vec{r})}{\partial n} ds. \] (5.39)

This final equation gives the value of \( \beta \) as the result of a path integral over a streamline. The bending of the flow is the result of the interaction between the transverse pressure gradient and inertia.

In an LMFBR, the inertial term \( \rho v^2(\vec{r}) \) may differ from our experiments due to the different fluid properties and flow velocities through the subchannels of the core. Two cases are taken as reference to estimate such a difference: the sodium-cooled Super Phenix and the LBE-cooled MYRRHA designs. The density for sodium and LBE at operation conditions is, respectively, \( \rho_{\text{Na}} = 840 \text{kgm}^{-3} \) at \( 470^\circ \text{C} \) and \( \rho_{\text{LBE}} = 10360 \text{kgm}^{-3} \) at \( 340^\circ \text{C} \). As for the flow speed, typical values are \( 5 \text{ms}^{-1} \) for a sodium reactor and \( 2 \text{ms}^{-1} \) for LBE. The inertial term \( \rho v^2(\vec{r}) \) results 5 and 10 times higher for sodium and LBE, respectively, meaning a smaller bending angle and a more straight flow. However, in a LMFBR the angle of the wire’s helix might be different, affecting the \( \frac{\partial p(\vec{r})}{\partial n} \) term; this is not taken into account in the comparison.

Furthermore, it is interesting to observe that the value of the bending angle at a certain coordinate \( s \) along a streamline depends on the upstream path, as indicated by the need of solving the integral. The small \( \beta \) approximation is accurate within an error of 5% for \( \beta \in [0, \pi/3] \), and within 12% for \( \beta \in [0, \pi/4] \).

5.7 Results and discussion

In the first part of this section, the results of the model based on the Euler equations are shown; they consist of the pressure gradient fields evaluated normal to the wire. These are then compared to the experimental results, being the pressure gradient field derived from the 2D Navier-Stokes equations. Finally, Equation 5.39, which predicts the bending angle \( \beta \), is also compared to the experiments.
5.7.1 Pressure gradient normal to the wire predicted by the Euler equations

This section presents the reconstructed pressure gradient modelled through the Euler equation, following the approach discussed in section 5.6. Equation 5.27 is applied to the streamlines to estimate the local, non-dimensional pressure gradient normal to the wire, \( \frac{2D_w}{\rho V_b^2} \frac{\partial p}{\partial \tilde{x}} \), where \( 1/2\rho V_b^2 \) is the dynamic pressure, and \( D_w \) the wire diameter. Figure 5.12 shows the pressure gradient field normal to the wire, along the \( \tilde{x} \) direction (see Figure 5.11a for the reference system). Figure 5.12 shows that the negative pressure gradient bends the streamlines towards the wire, leading to the lateral velocity field previously shown in Figure 5.9. Thereafter, the flow enters the downstream region, where the pressure increases with \( \tilde{x} \) (positive \( \partial p/\partial \tilde{x} \)) throughout the recirculation region, similarly to the experiments performed by Biswas et al. [109] with a backward-facing step. Here the positive pressure

![Image](image_url)

**Figure 5.12:** Mean non-dimensional pressure gradient normal to the wire spacer modelled through the Euler equations. The flow is from top to bottom; a) \( Re=4990 \); b) \( Re=7580 \); c) \( Re=9940 \); d) \( Re=12650 \); e) \( Re=14760 \); f) \( Re=16330 \); an exemplifying streamline is also reported. The region over the wire is not shown due to reflection.
gradient $\partial p/\partial \tilde{x}$ straightens the streamline, decreasing thus the bending angle $\beta$. The region over the wire is not shown as it is affected by reflection coming from the metal surface.

### 5.7.2 Comparison with the pressure gradient predicted by the Navier-Stokes equation

The pressure gradient normal to the wire estimated by the model based on the Euler equations is compared with the solution given by the Navier-Stokes equation (Equation 5.13). Figure 5.12 shows the pressure gradient field predicted by the Euler equations, whereas the pressure gradient obtained from solving the two-dimensional Navier-Stokes equations is shown in Figure 5.13. The pressure gradient normal to the wire based on the Euler equations is confirmed by the results of the Navier-Stokes equations, where the viscous effects are taken into account and

**Figure 5.13:** Time-averaged non-dimensional pressure gradient normal to the wire, computed from the two-dimensional Navier-Stokes equations. The flow is from top to bottom. a) $Re=4990$; b) $Re=7580$; c) $Re=9940$; d) $Re=12650$; e) $Re=14760$; f) $Re=16330$; the black line is followed for extracting the numerical values for comparison. The region over the wire is not shown due to reflection.
the only assumption is that the flow is two-dimensional.

A quantitative comparison is performed following a sampling line normal to the wire, drawn in Figure 5.13f, for the six investigated Reynolds numbers. Figure 5.14 shows the plot of $\frac{\partial p}{\partial \tilde{x}}$ modelled through the Euler equation (○) and calculated from the Navier-Stokes equations (□). The abscissa $\tilde{x}$ is the coordinate over the line normal to the wire selected for the comparison. The position of the wire is also indicated for reference. The error bars associated with the model (○) are based on the error resulting from fitting the velocity field with the streamlines. The

![Graphs showing non-dimensional pressure gradient normal to the wire predicted by the Euler equations (○, Equation 5.27) compared to the results obtained with the Navier-Stokes equations (□, Equation 5.13).](image)

Figure 5.14: Non-dimensional pressure gradient normal to the wire predicted by the Euler equations (○, Equation 5.27) compared to the results obtained with the Navier-Stokes equations (□, Equation 5.13). a) $Re=4990$; b) $Re=7580$; c) $Re=9940$; d) $Re=12650$; e) $Re=14760$; f) $Re=16330$.

error bars associated with the results of the Navier-Stokes equations applied to the measured velocity fields (□) are based on statistical deviation around the mean value. The figure shows good agreement between the two data series indicating, thus, that the model is capable of predicting the pressure gradient caused by the wire. Moreover, the agreement between the model (where the viscous effects are neglected) and the Navier-Stokes equations (where they are included) suggests
that the viscous term does not play a significant role in such a flow.

5.7.3 Validating the predicted bending angle against the experiments

In this section the bending angle $\beta$, predicted by Equation 5.39, is compared with the angle obtained from the time-averaged velocity fields measured with PIV. Based on the measurements, $\beta$ is evaluated along the streamline following Equation 5.30:

$$\beta (\vec{r}) = \arctan \left( \frac{u(\vec{r})}{v(\vec{r})} \right).$$

(5.40)

The results are plotted in Figure 5.15a against the normalised coordinate $s$ along the considered streamline (drawn in Figure 5.12).

![Figure 5.15: a) Bending angle $\beta$ along a streamline evaluated with Equation 5.39 (○) compared to the angle obtained from the PIV measurements (□). The abscissa is normalised by the streamline’s total length. $Re=16330$. b) The path $\Gamma_w$ of the streamline over the wire (red) is considered straight to simplify Equation 5.39; $\theta$ is the pitch angle of the wire.

The accuracy on the prediction of $\beta$ is expressed as the normalised root mean square error (NRMSE):

$$\text{NRMSE} = \sqrt{\frac{1}{N_w} \sum_{i=1}^{N_w} \left( \beta(i)_{\text{Eq.5.39}} - \beta(i)_{\text{PIV}} \right)^2} / \left( \beta_{\text{max}}_{\text{PIV}} - \beta_{\text{min}}_{\text{PIV}} \right),$$

(5.41)

where $N_w$ is the number of points along the streamline where the pressure gradient is evaluated, and the subscript PIV refers to the angle retrieved from the experiments. The error thus evaluated is 8%: most of the contribution to the error is located in the measured region closest to the wire, being $0.47 \leq s(y)/L \leq 0.55$, where
reflections of light coming from the metal may affect the measurement. Based on
the agreement between the angle measured from the experiments and the predic-
tion, the new expression derived for estimating $\beta$ (Equation 5.39) shows that it is
a valid alternative to the Euler equations for linking the pressure gradient to the
corresponding bending angle $\beta$, being more easily measurable than the curvature
radius.

Equation 5.39 is derived under the assumption that the viscous effects are negligi-
ble. The comparison between the model and the Navier-Stokes equations proves
that this assumption is valid for these experiments, where water is the working
fluid. However, this simplification may not be valid in a LMFBR.

In order to assess its validity, two reference designs are considered being Su-
per Phenix and MYRRHA reactors, whose average core temperatures are 470°C
and 340°C respectively. The kinematic viscosity of sodium and LBE at operation
conditions is $\nu_{Na}=3 \times 10^{-7} \text{m}^2\text{s}^{-1}$ and $\nu_{LBE}=1.6 \times 10^{-7} \text{m}^2\text{s}^{-1}$. These are of
the same order of magnitude as the water’s viscosity in the experiments, being
$8 \times 10^{-7} \text{m}^2\text{s}^{-1}$. For both cases, the inertial forces are more important than those
of the experiments, being the Reynolds number equal to 160000 and 120000 for
Super Phenix and MYRRHA respectively. Therefore viscous forces are even less
important when the model is applied to a liquid metal-cooled reactor.

5.8 Approximating the bending in the wire region

The bending angle of the flow can be predicted through Equation 5.39 following
the path of a streamline. Limiting our attention to the wire region (red path
in Figure 5.15b), $\beta$ can be estimated relying on a number of assumptions that
simplify the integral in Equation 5.39. Recalling Equation 5.27 for the pressure
gradient normal to the wire

$$\frac{\partial p}{\partial \hat{x}} = \frac{\partial p}{\partial n} \sin(\theta - \beta) + \frac{\partial p}{\partial s} \cos(\theta - \beta),$$

(5.42)

the contribution of the $\partial p/\partial s$ term is negligible compared to the $\partial p/\partial n$ term:
from a comparison it appears that this term is three orders of magnitude smaller
than $\partial p/\partial n$ throughout the entire path of the streamline. This means that the
accelerating force along the streamline is negligible compared to the centripetal one
responsible for the bending of the streamline. Hence the following approximation
is done:

$$\frac{\partial p}{\partial \hat{x}} \approx \frac{\partial p}{\partial n} \sin(\theta - \beta).$$

It follows that the pressure gradient normal to the wire is

$$\frac{\partial p}{\partial n} \approx \frac{\partial p}{\partial \hat{x}} \frac{1}{\sin(\theta - \beta)},$$

which can be substituted in Equation 5.39 derived in the previous section, leading to
\[ \beta(y=\sigma) = \int_{-\infty}^{s} \frac{1}{\rho v^2(\vec{r}) \sin[\theta - \beta(\vec{r})]} \frac{\partial p(\vec{r})}{\partial \vec{x}} ds. \]

The previous integral is split into two parts, being

\[ \beta(y=\sigma) = \int_{-\infty}^{\text{lead.}} \ldots ds + \int_{-\infty}^{\text{trail.}} \ldots ds, \]

where the two extremes of integration refer to the position of the leading (lead.) and trailing (trail.) edge of the wire along a streamline.

The second approximation consists of linearizing the pressure gradient in the second integral across the wire of diameter \( D_w \):

\[ \left( \frac{\partial p}{\partial \vec{x}} \right)_w \approx \frac{(\Delta p)_w}{D_w}, \]

where \( (\Delta p)_w \) is the pressure drop across the wire in the perpendicular direction. This can be expressed as

\[ (\Delta p)_w = \frac{1}{2} \rho C_p U_\bot^2, \]

being \( C_p \) a local drag coefficient and \( U_\bot \) the velocity perpendicular to the wire. This linearization leads to

\[ \beta(y=\sigma) = \int_{-\infty}^{\text{lead.}} \ldots ds + \int_{-\infty}^{\text{trail.}} \frac{C_p U_\bot^2}{2D_w} \int_{\text{lead.}}^{\text{trail.}} \frac{1}{v^2(\vec{r}) \sin[\theta - \beta(\vec{r})]} ds, \]

which, with \( \theta \gg \beta \) (see Figure 5.11 as an example), simplifies into

\[ \beta(y=\sigma) = \int_{-\infty}^{\text{lead.}} \ldots ds + \frac{C_p U_\bot^2}{2D_w \sin \theta} \int_{\text{lead.}}^{\text{trail.}} \frac{1}{v^2(\vec{r})} ds. \]

Figure 5.15b sketches the path followed by a streamline across the wire. For a cylinder lying on a flat plate in the presence of a flow in the perpendicular direction, Bearman et al. [110] measured a value of the pressure coefficient \( C_p \) of 0.5, which accounts for the pressure drop in the wake of the cylinder. Since the wire is small compared to the rod, it can be locally approximated to a cylinder on a flat plate, and \( C_p \) can be used for the estimation of \( \beta \). The velocity perpendicular to the wire is calculated as

\[ U_\bot = U_\infty \cos \theta, \]

being \( U_\infty \) the free stream velocity evaluated far upstream of the wire. The \( \beta \) angle at the trailing edge of the wire is finally evaluated as

\[ \beta(y=\sigma) = \int_{-\infty}^{\text{lead.}} \frac{1}{\rho v^2(\vec{r}) \sin \theta} \frac{\partial p(\vec{r})}{\partial \vec{x}} ds + \frac{C_p (U_\infty \cos \theta)^2}{2D_w \sin \theta} \int_{\text{lead.}}^{\text{trail.}} \frac{1}{v^2(\vec{r})} ds. \]
which yields $\beta = 6.5^\circ$, whereas the measured angle is $7.4^\circ$, corresponding to an error of 14%. The difference between the values can be ascribed to the geometry where $C_p$ is measured, which differs from the wire-wrapped rod case, being a cylinder lying on a flat plate. Moreover, the approximation $\theta \gg \beta$ could also have an effect on the final result. Nevertheless, Equation 5.48 could be a first step towards a correlation that estimates the bending of the flow near the wire based on macroscopic variables. In fact, this equation could also be used to estimate the pressure drop across a wrapping wire based on the value of the $\beta$ angle, which can be obtained from the streamline field.

![Figure 5.16](image)

**Figure 5.16:** a) Bending angle measured upstream ($\beta_1$), downstream ($\beta_2$) of the wire, and mean value ($\beta_w$). The wire’s pitch angle with the vertical axis $\gamma = 13^\circ$ is determined from $\tan \gamma = \pi \phi_{pin}/h$. b) Triangular subchannel: the hydraulic diameter $\phi_h$ is based on the main subchannel area (hatched).

PIV measurements of the flow in front of the wire are disturbed by the reflection of the light by the metal, thus $\beta$ is measured at two points located respectively at the leading edge ($\beta_1$) and at the trailing edge ($\beta_2$) of the wire, 12 mm apart. $\beta$ is measured at these two location as the angle formed by the streamline with the vertical axis. The measurement is repeated for all the investigated Reynolds numbers, and the results are reported in Figure 5.16. The plot shows that $\beta$ does not strongly depend on the flow rate, for $Re \geq 7580$ (note that this condition could be already reached for $4990 < Re \leq 7580$). The near-constant trend of $\beta$ shown in the figure means that the ratio between the velocity components $u$ and $v$ remains constant as well ($\tan \beta = u/v$), although the single components change with the Reynolds number. Thus, the flow rate does not affect the direction of the flow, as
concluded also by Song et al. [46].

5.9 Estimating the bending with dimensional analysis

The expression derived for estimating the bending angle of the flow (Equation 5.39) depends on the characteristics of the flow field (velocity components and flow streamlines). It follows directly from theory and it has its roots in the physics of the Euler equations. However, it may be useful to develop a correlation to predict such an angle based on macroscopic variables such as the design parameters of the rod bundle (rod diameter, wire diameter, wire pitch, fluid properties).

The bending angle across the wire, $\beta_w$, can be derived from dimensional analysis. The considered geometry is a triangular subchannel of an arbitrary wire-wrapped, hexagonal rod bundle (Figure 5.16b). The velocity components $u$ and $v$ that determine the bending angle are affected by the geometry of the channel, accounted for in the hydraulic diameter term $\phi_h$, the fluid properties $\rho$ and $\mu$, the bulk velocity $V_b$, the angle of the helicoid wire $\Upsilon$ and the wire diameter $\phi_w$. The angle can, thus, be written as a function of these variables as

$$\tan \beta_w = \frac{u}{v} = \left(\frac{h}{\phi_{pin}}\right)^{b_1} \cdot \rho^{b_2} \cdot \mu^{b_3} \cdot \phi_h^{b_4} \cdot V_b^{b_5} \cdot \phi_w^{b_6},$$

(5.49)

where $h/\phi_{pin}$ is the ratio between the wire wrapped pitch and the rod diameter, commonly used in wire-wrapped rod bundles, and related to the helicoid angle $\Upsilon$ through

$$\tan \Upsilon = \frac{\phi_{pin}}{h},$$

being $\Upsilon$ defined in Figure 5.16b. For a given $\phi_{pin}$ and $\phi_w$, the $P/D$ ratio determines the hydraulic diameter $\phi_h$ of the subchannel containing the wire [90]:

$$\phi_h = \frac{2\sqrt{3}\phi_{pin}^2 (P/D)^2 - \pi \phi_{pin}^2 - \phi_w^2}{\pi \phi_{pin} + \phi_w},$$

(5.50)

The dimensional analysis of Equation 5.49 yields the following expression:

$$[-] = [-]^{b_1} \cdot [\text{kg m}^{-3}]^{b_2} \cdot [\text{kg m}^{-1} \text{s}^{-1}]^{b_3} \cdot [\text{m}]^{b_4} \cdot [\text{ms}^{-1}]^{b_5} \cdot [\text{m}]^{b_6},$$

(5.51)

from which the following system of equations can be written:

$$\begin{align*}
b_1 &= 1 \\
b_2 + b_3 &= 0 \\
-3b_2 - b_3 + b_4 + b_5 + b_6 &= 0 \\
-b_3 - b_5 &= 0
\end{align*}$$

(5.52)

The solution of the system gives the following correlation:
\[ \tan \beta_w = \frac{h}{\phi_{pin}} R e^{b_3} \left( \frac{\phi_w}{\phi_h} \right)^{b_6} . \] (5.53)

Experimental evidence from this work shows that \( \beta_w \) is not a function of the Reynolds number (Figure 5.16a) meaning that \( b_3 \approx 0 \), simplifying the previous equation into

\[ \tan \beta_w = \frac{h}{\phi_{pin}} \left( \frac{\phi_w}{\phi_h} \right)^{b} . \] (5.54)

The value of the exponent \( b \) is determined as

\[ b = \frac{\ln \left( \frac{\phi_{pin} \tan \beta_w}{h} \right)}{\ln \left( \frac{\phi_w}{\phi_h} \right)} . \] (5.55)

Substituting the dimensions of SEEDS-2 experimental setup (table 3.2) in the previous formula yields

\[ b = \frac{\ln \left( \frac{D_r}{h} \tan \beta_w \right)}{\ln \left( \frac{D_w}{D_h} \right)} \approx 4.6 , \] (5.56)

where \( D_r, h, D_w \) and \( D_h \) are the dimensions of this work’s bundle, and \( \beta_w \) is the mean value of the bending angle measured from the experiments.

The narrowness of the subchannel is taken into account through the ratio \( \phi_w/\phi_h \). If \( P/D \) is given, the ratio \( \phi_w/\phi_{pin} \) is also fixed. For example, a hexagonal rod bundle with \( P/D \) of 1.11, such as the one of this research, yields \( \phi_w/\phi_h = 0.35 \), independently from the diameter of the wire and rod.

The pitch angle \( \gamma \) is defined as \( \tan \gamma = \pi \phi_{pin}/h \). It follows that hexagonal bundles with the same \( P/D \) ratio and \( \gamma \) (hence the same \( h/\phi_{pin} \) ratio ) should bend by the same amount close to the wire spacer. Note that these findings are valid for a geometry of the triangular subchannel where the wire is in contact with the adjacent rod (see Figure 5.16b), as in the case of the experimental setup of this work.

To the author’s knowledge, there is no other study from literature that focuses on measuring the bending angle induced by the wire spacer. The correlation presented in this section is the first step towards a more general form. Validating this correlation would thus require to modify the geometry of the setup and repeat the experiments to assess the influence of the individual parameters such as \( P/D \), wire pitch, and wire diameter. However, this was not possible for this research. Nevertheless, an attempt at quantifying the uncertainties on this correlation shows that the error on the predicted \( \beta_w \) is 5\% (for the estimation of the error, the reader is directed to Appendix E).

5.10 Summary

This chapter described the work that aimed at modelling and explaining the migratory flow close to the helicoid wire spacer inside a 7-rods, wire-wrapped hexagonal
bundle by reconstructing the pressure gradient in the direction normal to the wire. Experiments have been performed inside the rod bundle, where the axial flow has been investigated with PIV, making use of a refractive index-matching technique to reduce refraction of light inside the test section. Six Reynolds numbers were considered while measuring the time-averaged velocity fields.

The results show that the flow downstream the wire followed the helicoid path, as expected. However, if the investigated region was moved to the front of the wire, closer to the bulk of the subchannel, the flow changed the direction by moving against the wire’s helicoid path. The pressure gradient normal to the wire has been modelled through the Euler equations applied to the streamlines, thereby showing that the bending of the flow is caused by the pressure gradient imposed by the wire.

The results modelled through the Euler equations were compared with the solution of the two-dimensional Navier-Stokes equations leading to a good qualitative and quantitative agreement. Furthermore, an expression to predict the bending angle of the flow has been derived and successfully validated against the experimental results.

The findings presented in this chapter give more insight into the physics governing the bending of the flow close to the wire spacer, highlighting how the flow bending is the result of the interaction between the transverse pressure gradient and inertia of the flow. The model hereby presented leads the way to future possibilities for evaluating the bending angle through macroscopic, known variables such as bundle dimensions and flow rate. In this regard, a correlation based on dimensional analysis, and supported by experimental findings, has been proposed to estimate the bending angle from the dimensions of the bundle (diameter of wire and rod, $P/D$ ratio, and helicoid wire pitch).
### 5.11 Nomenclature

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Flow area</td>
<td>m</td>
</tr>
<tr>
<td>$b_1, b_2, b_3, b_4, b_5, b_6$</td>
<td>Exponents of the dimensional analysis</td>
<td>–</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Local drag coefficient</td>
<td>–</td>
</tr>
<tr>
<td>$D_h$</td>
<td>Hydraulic diameter</td>
<td>m</td>
</tr>
<tr>
<td>$D_w, D_{w,p}$</td>
<td>Wire diameter, and in pixel respectively</td>
<td>mm</td>
</tr>
<tr>
<td>$\hat{e}_n, \hat{e}_s$</td>
<td>Normal and tangential unit vectors in the streamline’s frame of reference</td>
<td>–</td>
</tr>
<tr>
<td>$F$</td>
<td>mm-to-pixel conversion factor</td>
<td>mmpx$^{-1}$</td>
</tr>
<tr>
<td>$\vec{F}_{ext}$</td>
<td>External forces acting on a volume of fluid</td>
<td>N</td>
</tr>
<tr>
<td>$f_#$</td>
<td>Lens aperture</td>
<td>–</td>
</tr>
<tr>
<td>$h$</td>
<td>Helicoidal wire pitch</td>
<td>mm</td>
</tr>
<tr>
<td>$i, j, \tau$</td>
<td>Indices</td>
<td>–</td>
</tr>
<tr>
<td>$L$</td>
<td>Streamline’s length</td>
<td>mm</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of recorded frames</td>
<td>–</td>
</tr>
<tr>
<td>$N_s$</td>
<td>Number of points along a streamline</td>
<td>–</td>
</tr>
<tr>
<td>$n, s$</td>
<td>Indices pertaining to the normal and tangential coordinate along a streamline</td>
<td>–</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass of fluid</td>
<td>kg</td>
</tr>
<tr>
<td>$M_0$</td>
<td>Magnification factor</td>
<td>–</td>
</tr>
<tr>
<td>$P/D$</td>
<td>Ratio between the pitch and the diameter of the half rods</td>
<td>–</td>
</tr>
<tr>
<td>$P_w$</td>
<td>Wetted perimeter</td>
<td>m</td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$(\Delta p)_w$</td>
<td>Pressure drop across the wire</td>
<td>Pa</td>
</tr>
<tr>
<td>$Q$</td>
<td>Volumetric flow rate</td>
<td>m$^3$s$^{-1}$</td>
</tr>
<tr>
<td>$R$</td>
<td>Radius of curvature</td>
<td>m</td>
</tr>
<tr>
<td>$\vec{r}$</td>
<td>Vector of position</td>
<td>m</td>
</tr>
<tr>
<td>$v_i, \bar{v}$</td>
<td>Instantaneous and mean velocity</td>
<td>ms$^{-1}$</td>
</tr>
<tr>
<td>$\vec{v}$</td>
<td>Velocity vector</td>
<td>ms$^{-1}$</td>
</tr>
<tr>
<td>$V_b$</td>
<td>Bulk velocity</td>
<td>ms$^{-1}$</td>
</tr>
<tr>
<td>$dV$</td>
<td>Control volume</td>
<td>–</td>
</tr>
<tr>
<td>$\delta D_w$</td>
<td>Uncertainty on the wire diameter</td>
<td>mm</td>
</tr>
<tr>
<td>$\delta D_{w,p}$</td>
<td>Uncertainty on the wire diameter in pixels</td>
<td>px</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
<td>°C</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
<td>s</td>
</tr>
<tr>
<td>$dt, \Delta t$</td>
<td>Time interval between two consecutive frames, and two consecutive images</td>
<td>s</td>
</tr>
<tr>
<td>$U_s$</td>
<td>Velocity tangent to the streamline</td>
<td>ms$^{-1}$</td>
</tr>
<tr>
<td>$U_L$</td>
<td>Velocity component perpendicular to the wire</td>
<td>ms$^{-1}$</td>
</tr>
<tr>
<td>$U_\infty$</td>
<td>Free stream velocity, far upstream of the wire</td>
<td>ms$^{-1}$</td>
</tr>
<tr>
<td>$u$</td>
<td>Span-wise (transversal) velocity component</td>
<td>ms$^{-1}$</td>
</tr>
</tbody>
</table>
\( x, y, z \)  
Span-wise (transversal), stream-wise (vertical), and normal directions  
\( \Delta x, \Delta y \)  
Transversal and vertical separation interval between velocity vectors  
\( \mathring{x} \)  
Coordinate normal to the wire  
\( \delta_z \)  
Depth of focus

**Greek letters**  

<p>| ( \beta ) | Bending angle of the streamline | ° |
| ( \beta_1, \beta_2 ) | Bending angle of the streamline measured upstream and downstream of the wire | ° |
| ( \beta_{\text{Max}}, \beta_{\text{min}} ) | Maximum and minimum angle of bending along the path of a streamline | ° |
| ( \beta_w ) | Mean bending angle of the streamline across the wire | ° |
| ( \Gamma_w ) | Path followed by a streamline over the wire | m |
| ( \gamma ) | Helicoid wire angle of SEEDS-2 facility | ° |
| ( \Delta_p ) | Particle displacement between two consecutive images | pixel |
| ( \theta ) | Angle formed by the wire with the horizontal | ° |
| ( \lambda_l ) | Wavelength of the laser | nm |
| ( v, v_{\text{LBE}}, v_{\text{Na}} ) | Kinematic viscosity of water, lead-bismuth eutectic, and sodium | ms(^{-1}) |
| ( \rho, \rho_{\text{LBE}}, \rho_{\text{Na}} ) | Density of water, lead-bismuth eutectic, and sodium | kg m(^{-3}) |
| ( \sigma_{\text{stat}}, \sigma_{\text{syst}} ) | Statistical and systematic errors on the measurements | ms(^{-1}) |
| ( \sigma_v ) | Mean error on the measurements | ms(^{-1}) |
| ( \Upsilon ) | Helicoid wire angle of a generic hexagonal bundle, wire-wrapped facility | ° |
| ( \phi_{\text{pin}}, \phi_w, \phi_h ) | Pin diameter, wire diameter, and hydraulic diameter of the subchannel of a generic hexagonal bundle, wire-wrapped facility | m |</p>
<table>
<thead>
<tr>
<th>Acronyms</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AOI</td>
<td>Area of interest</td>
</tr>
<tr>
<td>CMOS</td>
<td>Complementary Metal-Oxide Semiconductor</td>
</tr>
<tr>
<td>FEP</td>
<td>Fluorinated Ethylene Propylene</td>
</tr>
<tr>
<td>LBE</td>
<td>Lead-bismuth eutectic</td>
</tr>
<tr>
<td>LMFBR</td>
<td>Liquid Metal Fast Breeder Reactor</td>
</tr>
<tr>
<td>MYRRHA</td>
<td>Multi-purpose hYbrid Research Reactor for High-tech Applications</td>
</tr>
<tr>
<td>NRMSE</td>
<td>Normalised Root Mean Square Error</td>
</tr>
<tr>
<td>PIV</td>
<td>Particle Image Velocimetry</td>
</tr>
<tr>
<td>SEEDS-1</td>
<td>SEven rods Experiments in Delft for Sesame-1</td>
</tr>
<tr>
<td>SEEDS-2</td>
<td>SEven rods Experiments in Delft for Sesame-2</td>
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</table>

<table>
<thead>
<tr>
<th>Non dimensional groups</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Re)</td>
<td>Reynolds number</td>
</tr>
</tbody>
</table>
Chapter 6

Conclusions and recommendations

When this research started but one correlation was available for predicting the length of coherent structures in bundle geometries; however this had been developed for a rather specific shape of the channel, lacking thus general validity. This work proposes, thus, a correlation applicable to different geometries, including full rod bundles.

Literature is rich in experimental works on Fluid-Structure Interaction (FSI) with rod bundle geometries. However, an experimental study that specifically addressed the role of coherent structures in inducing vibrations inside rod bundles was still missing. For that reason, this research aimed at providing, and provides, novel experimental evidence regarding vibrations induced by coherent structures on the rods of a bundle.

Migratory flow was first observed in the seventies in rod bundles with helicoid wire spacers. A rather general explanation was proposed, which ascribed the phenomenon to the hydraulic resistance caused by the wire inside the subchannel. Only recently, was a numerical study conducted that partially focussed again on the effect of migratory flow. However literature was still missing any attempt to model this phenomenon, which therefore became one of the goal of this research.

This experimental research focusses on large coherent structures in rod bundles, their contribution to flow-induced vibrations (FIV), and the so-called migratory flow observed in hexagonal rod bundles with wire spacers wrapped around the rods. For these purposes, three experimental facilities have been designed and built. A rectangular channel hosting two half-rods, and two 7-rod hexagonal bundles: one hosting a flexible rod for FSI experiments, and the other featuring helicoid wires wrapped around the rods to investigate migratory flow. Laser Doppler Anemometry (LDA), Particle Image Velocimetry (PIV), and a high speed camera have been used in combination with the refractive index-matching technique (RIM) to achieve optical access near the measured area. The experiments have been conducted with water as working fluid, in isothermal conditions and ambient pressure.
Large coherent structures have been detected in the gap between the rods, suggesting a lateral movement of these structures across the subchannels (cross-flow). The measurements have shown that this phenomenon has a threshold in the Reynolds number, above which cross-flow through the gap region has not been detected. This is also confirmed by previous numerical works performed at similar values of Reynolds for an annular gap. A similar threshold effect of the Reynolds number has been observed in the velocity difference (% bulk velocity) between bulk and gap region. This difference increases below the same value of Reynolds, and it becomes constant at higher Reynolds numbers.

Coherent structures carried by the flow in the axial direction of the bundle synchronise with the flow-induced oscillations of the flexible rod, leading to an increased amplitude of the rod’s oscillation.

Based on experiments and on physical reasoning, our understanding of what determines the size of coherent structures has improved. This research shows that the wavelength is a function of the hydraulic diameter of the main channel where the bulk flow is found, and of the local hydraulic diameter, being this defined by the gap flow area. In fact, the wavelength appears to be function of their ratio, thereby depending on the velocity difference between the two regions. This study has eventually led to a new correlation for estimating the size of coherent structures, which has been tested against the experiments conducted in several geometries.

Migratory flow has been explained as the result of the pressure gradient induced by the wire in the fluid that bends the flow. Moreover, deriving a theoretical model to explain such a phenomenon has increased our understanding of the physics at play. The model shows that the bending is the result of the interaction between the pressure gradient and the inertial forces applied to the fluid.

The remainder of this chapter describes in more detail the conclusions of this research, and ultimately provides recommendations for future works to address some still unanswered questions.

6.1 Coherent structures in a compound channel

The flow of coolant through the core of a nuclear reactor can generate large coherent structures that can be beneficial to the safety of the core as they prevent hot spots on the cladding of the fuel elements. Designing safer and more efficient nuclear reactors requires that some questions regarding these coherent structures be addressed, such as

*How do Reynolds number and gap width affect the behaviour of coherent structures in a gap geometry?*

To answer these questions, a facility has been assembled consisting of a rectangular channel hosting two half-rods facing each other; the idea of such a setup originated
from the work of Mahmood [17]. Owing to the relative easiness of access to the measured region, this setup allowed for detailed measurements of the flow in the gap between the rods. The gap width between the half-rods could be adjusted to 1, 2, or 3 millimeter, corresponding to a pitch-to-rod diameter ratio of 1.07, 1.13, or 1.20. The flow between the rods has been measured with LDA, a non-intrusive optical measurement technique, within the Reynolds number range $600 - 29000$. Optical access to the gap region has been achieved by making one of the rods of Fluorinated Ethylene Propylene (FEP), a refractive index-matching material with nearly the same refractive index of water.

Large coherent structures carried by the flow near the gap, also known as gap vortex streets, have been measured from the analysis of the time-dependent signal of the velocity components. Their axial length is estimated through the Taylor hypothesis, which regards these structures as “frozen” entities carried by the flow with a convection speed lower than the bulk velocity. The measurements have shown that the length of the structures is not a function of Reynolds above a certain value of this, and that their length tends to an asymptotic value. The velocity difference between the bulk flow and the centre of the gap (% bulk velocity), which is the driving force of coherent structures, becomes also independent of the Reynolds number above a certain threshold. This might explain why the wavelength is not affected by the Reynolds number.

However, for lower values of Reynolds, coherent structures become longer, extending farther into the bulk of the main subchannels, suggesting a possible growth also in the lateral direction due to the extending of the region where the average velocity changes. This could be due to the widening shear layer region between the gap and the bulk, similarly to wall-bounded flows with decreasing Reynolds number. Measurements with different $P/D$ ratio have shown that coherent structures become longer with increasing gap width.

Cross-flow of coherent structures across the gap has been revealed by periodicities in the span-wise (lateral) velocity component measured near the centre of the gap, showing that the frequency of cross-flow decreases with larger gaps. The plot of the stream-wise velocity root mean square has shown the presence of an additional peak in the middle of the gap, which occurs below a certain value of the Reynolds number. This threshold effect can be related to the velocity difference between the main channel and the gap, which is the driving force of coherent structures, and which becomes larger below the same Reynolds number. The peak in the velocity root mean square can indicate cross-flow of structures across the gap, thereby increasing the turbulence level in this region.

6.2 Size of large coherent structures in rod bundle geometries

The size (i.e. the wavelength) of large coherent structures occurring in a rod bundle has a direct impact on the design of the core of a nuclear reactor. Their size affects the lateral mixing across the subchannels, as shown by Mahmood [17], and estimating the length of these structures is important for flow-induced vibration
6.3 Vibrations induced by gap vortex streets

Coherent structures forming in the flow may enhance the heat exchange between the fuel pins and the coolant of a nuclear reactor. However, these structures might
trigger vibrations of the fuel rods due to the oscillation that they cause in the
flow. This could cause structural damage to the fuel element’s cladding through
fretting and fatigue. In rod bundles, vibrations induced by periodical vortices in a
transversal flow have been thoroughly studied. In axial flows, however, their role
was still unclear. Hence the following question was formulated:

What is the role of coherent structures in inducing vibrations in a rod bundle with
axial flow?

To answer this question, optical measurements have been conducted in a 7-rod
hexagonal bundle with $P/D = 1.11$. The flow has been measured with LDA to detect
coherent structures and calculate their frequency. Displacements and frequency
of oscillation of the central rod, made of silicone, have been measured with a
high speed camera. Optical access has been achieved by using FEP as refractive
index-matching material for the outer rods of the bundle.
The experiments show that the frequency of coherent structures moving axially
along the bundle increases linearly with the Reynolds number, meaning that the
structures move faster as the flow rate is increased.
If coherent structures move in the axial direction with a frequency that is twice
the first natural frequency of the rod (i.e. $f_y = 2f_{s\text{il,n}}$), the rod oscillates with a
frequency equal to its natural frequency. As coherent structures are entities carried
by the flow, their stream-wise frequency $f_y$ increases with the bulk flow velocity.
When $f_y$ reaches the value of $f_y/2 = f_{s\text{il,n}}$, there is synchronisation between the
rod and coherent structures. The increased magnitude of the rod displacements
confirms this finding. This condition can also be regarded as parametric resonance,
where the most energetic mode of vibration occurs whenever the frequency reaches
the twice the natural frequency of the system, as in the case of these experiments.

6.4 Migratory flow in a rod bundle with wire spacers

The core of a liquid metal-cooled reactor typically employs wire spacers as a solu-
tion to prevent fuel elements from touching each other. These wires are wrapped
helicoidally around the rods to enhance the mixing of coolant (hence heat removal)
and to avoid hot spots on the fuel cladding. Due to the presence of this additional
component, the flow inside a rod bundle is very complex to understand and to
model. In 1976, Ohtake et al. measured the flow inside a 37-rod, wrapped-wire
hexagonal bundle by means of hot wire anemometry. They observed that the flow
near the wire bent transversally against the wrapping direction of the wire. The
phenomenon was called migratory flow and ascribed to the hydraulic resistance
caused by the wire inside the considered subchannel. The numerical work of Song
et al. suggested that this was caused by a pair of counter-rotating vortices forming
downstream of the wire. However, a physical model and a general understanding
of migratory flow was still missing.
Can migratory flow be explained and modelled to predict the associated bending angle of the flow close to the wire spacer?

To answer this question, a model has been derived following a theoretical approach to predict the pressure gradient and the bending of the flow. Then the predicted results have been compared with the experiments performed in the 7rod, wire-wrapped hexagonal bundle by means of planar PIV, for several values of the Reynolds number.

The model for the pressure gradient has been derived from the steady state, two-dimensional, inviscid Navier-Stokes equations applied to the flow streamlines, and successfully validated against the results based on the experiments.

An equation for estimating the bending angle of the flow at an arbitrary point along a streamline has been derived and compared with the measurements. The bending angle is expressed as an integral along the streamline, upstream of the considered point $s$:

$$\beta(\vec{r}) = \int_{-\infty}^{s} \frac{1}{\rho v^2(\vec{r})} \frac{\partial p(\vec{r})}{\partial n} ds.$$ 

The underlying meaning of this equation is that the bending of the flow is the result of the interaction between the pressure gradient normal to the streamline $\partial p(\vec{r})/\partial n$ and the inertial forces acting on the fluid $\rho v^2(\vec{r})$, which tends to straighten the flow back.

The measurements have shown that the flow close to the wire follows the helicoid path in the downstream region. If the measurement region is shifted closer to the main subchannel, however, the flow bends towards the wire, against the helicoid path, suggesting the presence of migratory flow.

The bending angle measured on the flow streamlines does not depend on the Reynolds number, above a certain threshold. It follows that the ratio between the lateral and axial velocity components $u/v$ is also independent of the Reynolds number, since this ratio is linked to the bending angle by the definition of streamline.

Furthermore, a correlation for estimating the angle by which the flow bends close to the wire has been derived based on experiments and through dimensional analysis.

$$\tan \beta_w = \frac{h}{\phi_{pin}} \left( \frac{\phi_w}{\phi_h} \right)^b.$$ 

This correlation is different from the integral shown above in that it depends on macroscopic variables such as the dimensions of the bundle (wire and rod diameter), the wire pitch, and the geometry of the subchannel through the hydraulic diameter. The correlation predicts that the flow through bundles characterised by the same $P/D$ ratio and helicoid pitch angle should bend by the same amount close to the wire spacer. This represents a first step towards a more general expression that links the bending of the flow to the parameters of the hexagonal lattice and to the flow rate.
6.5 Perspectives

One of the main limitations of this research, concerning with FSI, is that the flow and the oscillations of the rod could not be measured at the same time. The LDA system requires a dark background to enhance the detection of the seeding particles (hence the local velocity) added to the fluid. On the other hand, measuring the oscillations induced on the rod require a powerful flashlight to improve the contrast between the white silicone rod and the darker surrounding fluid. Measuring simultaneously the flow field and the oscillations of the rod would give more insight into the mutual interplay between the vibrating rod and the coherent structures occurring in the flow. Future research should then address this issue, for example by installing accelerometers on the rod surface that do not interfere with the requirements for measuring the flow.

The effect that the oscillating rod has on the surrounding fluid would lead to a better understanding of the coupling between the rod and coherent structures that may occur in the flow. In this research, flow measurements have been conducted only in the presence of the vibrating rod. For future works, it is thus encouraged that flow measurements be performed first without any moving body, and then in the presence of the oscillating rod. The effect that vibrations may have on coherent structures could thus be evaluated.

The research described in Chapter 4 focused on measuring the displacements of only one edge of the silicone rod due to the limited resolution of the high speed camera. Measuring both edges at the same time can be helpful to know more about the mode of vibration of the rod. For example, the instantaneous displacement of the two edges could be compared by means of cross-correlation to understand if the rod oscillates in a beam mode or in a shell mode (for example with the edges moving inward and outward at the same time). Adding one camera would also make possible to measure the out-of-plane displacement of the rod, which has not been possible in this work.

Temperature fluctuations of the coolant through the core of a fast nuclear reactor might lead to deformation or bending of the fuel elements due to different thermal expansion of the cladding, as discussed by Hartig [111]. The bent fuel elements would, in turn, confine the flow even more inside some of the surrounding subchannels, which would lead to a further deterioration of the heat transfer. This would trigger a self-sustaining, oscillatory mechanism that, on the long term, could lead to mechanical fatigue on the rods. This phenomenon was observed in the sodium-cooled KNK-II pilot plant in Germany, as reported by Heinecke [112]. For this reason, it would be interesting to measure coherent structures and flow-induced vibrations inside a bundle hosting a central off-centred rod (i.e. closer to some of the surrounding rods) to deformed rod case. The correlation derived in this research predicts that the eccentric rod could cause coherent structure to shorten due to the decreased hydraulic diameter of the gap region. This would eventually increase the frequency of coherent structures in the fluid, thereby affecting the interaction with the rods.

The experimental results shown in Chapter 5 suggest that migratory flow transfers
fluid from the near-gap region towards the main subchannel. At the same time, measurements of the flow closer to the rod have shown that the fluid follows the wire, moving from the main subchannel towards the gap. For a nuclear reactor, this combined effect is important because it transfers hotter fluid from the low-speed gap region to the bulk of the subchannel. At the same time, the gap region is replenished with colder fluid from the main subchannel, thereby enhancing heat transfer and avoiding hot spots on the fuel cladding. Future research could, therefore, aim at quantifying the lateral mass transfer between the gap region and the subchannel. For example, several “slices” of fluid could be measured with PIV as the laser is moved farther away from the rod, towards the bulk of the subchannel. A mass balance could thus be applied to the resulting measured volume of total lateral surface $S$:

$$\frac{dm}{dt} = \int_S \rho \vec{v} \cdot n \, dS.$$ 

This approach could provide information on the net transfer of fluid between the gap and the subchannel due to the presence of the wire.

An attempt at providing a correlation to predict the bending angle of the flow near the wire has been done in the latter part of Chapter 5. The correlation involves several macroscopic variables such as the dimensions of the bundle (wire and rod diameters, helicoid wire’s pitch). Although the influence of the dimensional tolerances of these parameters has been estimated, the correlation should be validated on different wire-wrapped rod bundle geometries, for example by changing the wire-to-rod diameter ratio or the pitch of the helicoid path. This could not be done in this research and it is therefore recommended as future work.

Optical measurements in a 7-rod bundle offer easier optical access and may be cheaper due to lower material costs, compared to larger bundles with 19, 37 or 61 pins. However, due to the reduced size of a 7-rod bundle, material tolerances of the components (i.e. rod and wire diameter, thickness of the hexagonal casing) might have a larger relative influence on the flow area. For example, a deviation from the nominal value of the thickness of the outer hexagonal wall could reduce the flow area in the cross section, affecting the flow and the estimation of the bulk velocity. This aspect is thus to be taken in careful consideration when designing a rod bundle facility.
6.6 Nomenclature

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Description</th>
<th>Units</th>
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</thead>
<tbody>
<tr>
<td>$D_h^*$</td>
<td>Hydraulic diameter of the gap region</td>
<td>m</td>
</tr>
<tr>
<td>$f_{sil,n}$</td>
<td>Natural frequency of the oscillating silicone rod</td>
<td>Hz</td>
</tr>
<tr>
<td>$f_y$</td>
<td>Frequency of coherent structures</td>
<td>Hz</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass of fluid</td>
<td>kg</td>
</tr>
<tr>
<td>$n, s$</td>
<td>Indices pertaining to the normal and tangential coordinate along a streamline</td>
<td></td>
</tr>
<tr>
<td>$P/D$</td>
<td>Ratio between the pitch and the diameter of the half rods</td>
<td>–</td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$\vec{r}$</td>
<td>Vector of position</td>
<td>m</td>
</tr>
<tr>
<td>$S$</td>
<td>Lateral surface of the measured volume of fluid</td>
<td>m</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
<td>s</td>
</tr>
<tr>
<td>$u, v$</td>
<td>Transversal and vertical velocity component of the flow</td>
<td>ms$^{-1}$</td>
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<table>
<thead>
<tr>
<th>Greek letters</th>
<th>Description</th>
<th>Units</th>
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</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Bending angle of the streamline</td>
<td>°</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength of coherent structures in the stream-wise (vertical) direction</td>
<td>m</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Fluid density</td>
<td>kgm$^{-3}$</td>
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</table>

<table>
<thead>
<tr>
<th>Acronyms</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>FEP</td>
<td>Fluorinated Ethylene Propylene</td>
</tr>
<tr>
<td>FIV</td>
<td>Flow-induced vibration</td>
</tr>
<tr>
<td>FSI</td>
<td>Fluid-structure interaction</td>
</tr>
<tr>
<td>LDA</td>
<td>Laser Doppler Anemometry</td>
</tr>
<tr>
<td>PIV</td>
<td>Particle Image Velocimetry</td>
</tr>
<tr>
<td>RIM</td>
<td>Refractive index-matching</td>
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</table>

<table>
<thead>
<tr>
<th>Non dimensional groups</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$Re$</td>
<td>Reynolds number</td>
</tr>
</tbody>
</table>
Bibliography


Appendix A

This appendix shows the derivation of Equation 2.26 used in chapter 2 to estimate the refraction of the laser beam through the FEP layer. Referring to Figure A.1

\[ \Delta x \]

Figure A.1: Refraction of the laser beam as it crosses the FEP half-rod. The horizontal refraction \( \Delta x \) due to the curvature of the FEP wall is retrieved from geometrical considerations.

\[
\sin \alpha_1 = \frac{x}{R_{i,FEP}}; \quad \sin \alpha_3 = \sin \alpha_1 \frac{\eta}{\eta_{FEP}}, \quad (A.1)
\]

where \( \alpha_3 \) is the angle of the refracted light ray through the FEP, \( \eta \) and \( \eta_{FEP} \) are air and FEP’s indices of refraction. \( \alpha_3 \) is obtained as

\[
\alpha_3 = \arcsin \left( \frac{x \eta}{\eta_{FEP} R_{i,FEP}} \right). \quad (A.2)
\]

Considering the triangle AOB, the law of sine is applied twice:
\[
\frac{\overline{AB}}{\sin \alpha_2} = \frac{R_{i,FEP}}{\sin \alpha_4}; \quad \frac{\overline{AB}}{\sin \alpha_2} = \frac{R_{i,FEP} + t_{FEP}}{\sin (\pi - \alpha_3)}.
\] (A.3)

Hence,
\[
\sin \alpha_4 = \sin \alpha_3 \frac{R_{i,FEP}}{R_{i,FEP} + t_{FEP}} = \frac{x \eta}{\eta_{FEP}(R_{i,FEP} + t_{FEP})},
\] (A.4)

from which \(\alpha_4\) is obtained as
\[
\alpha_4 = \arcsin \left( \frac{x \eta}{\eta_{FEP}(R_{i,FEP} + t_{FEP})} \right).
\] (A.5)

The inner angles of the triangle \(\triangle AOB\) sum up to \(\pi\):
\[
\alpha_2 + (\pi - \alpha_3) + \alpha_4 = \pi,
\] (A.6)
meaning that the value of \(\alpha_2\) is estimated as
\[
\alpha_2 = \pi - (\pi - \alpha_3) - \alpha_4,
\] (A.7)

where \(\alpha_3\) and \(\alpha_4\) are given by Equations A.2 and A.5.

\(\overline{AB}\) is obtained by applying the law of cosine to the triangle \(\triangle AOB\):
\[
\overline{AB}^2 = R_{i,FEP}^2 + (R_{i,FEP} + t_{FEP})^2 + 2R_{i,FEP}(R_{i,FEP} + t_{FEP}) \cos \alpha_2.
\] (A.8)

Finally, the distortion due to the presence of FEP is given by
\[
\Delta x = \overline{AB} \sin (\alpha_4 - \alpha_3).
\] (A.9)
Appendix B

The refractive index matching technique (RIM) can be applied either by employing materials whose refractive index is close to the working fluid’s (S2F method, for conciseness), or by adjusting the refractive index of the fluid to the one of the material forming the experimental setup (F2S method, for conciseness) [74]. The experimental facilities described in this thesis adopt the S2F method (see section 3.2.3) because the F2S option has proven unfeasible in the current experiments. This appendix explains why the F2S method has not been followed. A possible implementation of the F2S RIM technique would consist of components (rods, casing) made of Perspex (\( \eta_{\text{PMMA}} = 1.49 \)) while the working fluid would be a solution of water and salt to adjust the refractive index to the Perspex’s. The refractive index of such a solution, \( \eta_{\text{sol}} \), can be estimated through the Looyenga or the Lorentz-Lorenz formulas [113], respectively

\[
(\eta_{\text{sol}}^{2/3} - 1) \phi_{\text{sol}} = \sum \phi_i \left( \eta_i^{2/3} - 1 \right) \tag{B.1}
\]

and

\[
\left(\eta_{\text{sol}}^2 - 1 \right) \phi_{\text{sol}} = \sum \phi_i \left( \eta_i^2 - 1 \right) \left( \eta_i^2 + 2 \right) \tag{B.2}
\]

where \( \eta_{\text{sol}} \) is the refractive index of the solution to be determined, \( \phi_{\text{sol}} \) is the molar volume of the solution, and \( \phi_i \) and \( \eta_i \) are the molar volume and the refractive index of the individual compounds (i.e. water and salt). The refractive index of a solution of NaCl and water is measured with an Abbe refractometer to test the accuracy of Equation B.1 and B.2: the results are shown in Figure B.1.
Refractive index as a function of the concentration of NaCl dissolved in water predicted by the theory and measured experimentally.

The Looyenga and Lorentz-Lorenz equations underpredict the experimental refractive index of the solution and the discrepancy grows larger as more salt is added to the solution. A possible explanation is that the two equations do not take into account the volume change between the crystal and ionic (dissolved) form of the salt, which is to be determined empirically [114]. If the components of the rod bundle would be made of Perspex, the aqueous solution should reach a refractive index of $\eta_{PMMA} = 1.49$. The considered salts are CsI, CsCl, KI and KSCN. The refractive index of the aqueous solution is estimated with Equation B.1 and B.2, and shown in Figure B.2a and B.2b, respectively. Although Figure B.2 shows that

CsI and KI are the best candidates to match the Perspex’s refractive index, the required amount of salt exceeds their solubility limit. Furthermore, a salty solution would corrode the metal parts of the experimental lopp such as the pump, hence the F2S RIM technique is discarded and the S2F strategy is followed instead.
Appendix C

Considering a control volume immersed in a fluid in steady state (Figure C.1), and neglecting gravity and frictions, a momentum balance can be set up. In an Eulerian frame of reference, along the $s$ direction, this is

$$\rho U_s \cdot U_s \, dn \hat{e}_s + \rho U_{s+ds} \cdot U_{s+ds} \, dn \hat{e}_{s+ds} + \rho U_n \cdot U_s \, dn \hat{e}_n + \rho U_n \cdot U_{s+ds} \, dn \hat{e}_{s+ds} + p_s \, dn \hat{e}_s + p_{s+ds} \, dn \hat{e}_{s+ds} = 0,$$  \hspace{1cm} (C.1)

Considering that $\hat{e}_s = -\hat{e}_{s+ds}$, the previous equation becomes

$$\rho U_s \cdot U_s \, dn \hat{e}_s - \rho U_{s+ds} \cdot U_{s+ds} \, dn \hat{e}_s + \rho U_n \cdot U_s \, dn \hat{e}_n - \rho U_n \cdot U_{s+ds} \, dn \hat{e}_s + p_s \, dn \hat{e}_s - p_{s+ds} \, dn \hat{e}_s = 0,$$  \hspace{1cm} (C.2)

from which it follows
Combining Equation C.10 and Equation C.4 together leads to

\[-2\rho U_s \frac{\partial U_s}{\partial s} \hat{e}_s - \rho U_n \frac{\partial U_n}{\partial s} \hat{e}_s - \rho U_n \frac{\partial U_n}{\partial s} \hat{e}_s - \frac{\partial p_s}{\partial s} \hat{e}_s = 0.\]  \hspace{1cm} \text{(C.3)}

If density is constant, it can be moved out of the derivatives:

\[-2\rho U_s \frac{\partial U_s}{\partial s} \hat{e}_s - \rho U_n \frac{\partial U_n}{\partial s} \hat{e}_s - \rho U_n \frac{\partial U_n}{\partial s} \hat{e}_s = 0.\]  \hspace{1cm} \text{(C.4)}

The momentum balance along the \( n \) direction is

\[\rho U_s \cdot U_n ds \hat{e}_n + \rho U_s \cdot U_{n+dn} ds \hat{e}_n + \rho + U_n \frac{\partial U_n}{\partial s} \hat{e}_n + \rho U_n U_n + dn ds \hat{e}_{n+dn} + p_n ds \hat{e}_n + p_{n+dn} ds \hat{e}_{n+dn} + \rho dnds \frac{U_s^2}{R} \hat{e}_n = 0, \]  \hspace{1cm} \text{(C.5)}

Considering that \( \hat{e}_n = -\hat{e}_{n+dn} \), the previous equation becomes

\[\rho U_s \cdot U_n ds \hat{e}_n - \rho U_s \cdot U_{n+dn} ds \hat{e}_n + \rho U_n U_n ds \hat{e}_n - \rho U_n U_{n+dn} ds \hat{e}_n +
\]

\[+ p_n ds \hat{e}_n - p_{n+dn} ds \hat{e}_n + \rho dnds \frac{U_s^2}{R} \hat{e}_n = 0, \]  \hspace{1cm} \text{(C.6)}

\[-\frac{\partial (\rho U_s \cdot U_n)}{\partial n} ds \hat{e}_n - \frac{\partial (\rho U_n \cdot U_n)}{\partial n} ds \hat{e}_n - \frac{\partial p_n}{\partial n} ds \hat{e}_n + \rho dnds \frac{U_s^2}{R} \hat{e}_n = 0, \]  \hspace{1cm} \text{(C.7)}

\[-\rho U_s \frac{\partial U_n}{\partial n} \hat{e}_n - \rho U_n \frac{\partial U_s}{\partial n} \hat{e}_n - \rho U_n \frac{\partial U_n}{\partial n} \hat{e}_n - \frac{\partial p_n}{\partial n} \hat{e}_n + \rho \frac{U_s^2}{R} \hat{e}_n = 0. \]  \hspace{1cm} \text{(C.8)}

Recalling the continuity equation, and assuming constant fluid density, it follows that

\[\frac{\partial U_n}{\partial s} \hat{e}_n = -\frac{\partial U_s}{\partial s} \hat{e}_s. \]  \hspace{1cm} \text{(C.9)}

Substituting the previous equation in Equation C.8 we obtain

\[\rho U_s \frac{\partial U_s}{\partial s} \hat{e}_s - \rho U_n \frac{\partial U_n}{\partial s} \hat{e}_n - \rho \frac{\partial (U_n \cdot U_n)}{\partial n} \hat{e}_n - \frac{\partial p_n}{\partial n} \hat{e}_n + \rho \frac{U_s^2}{R} \hat{e}_n = 0. \]  \hspace{1cm} \text{(C.10)}

Combining Equation C.10 and Equation C.4 together leads to

\[-2\rho U_s \frac{\partial U_s}{\partial s} \hat{e}_s - \rho U_s \frac{\partial U_n}{\partial s} \hat{e}_s - \rho U_n \frac{\partial U_s}{\partial s} \hat{e}_s - \frac{\partial p_s}{\partial s} \hat{e}_s + \rho U_s \frac{\partial U_s}{\partial s} \hat{e}_s +
\]

\[-\rho U_n \frac{\partial U_n}{\partial n} \hat{e}_n - \rho \frac{\partial (U_n \cdot U_n)}{\partial n} \hat{e}_n - \frac{\partial p_n}{\partial n} \hat{e}_n + \rho \frac{U_s^2}{R} \hat{e}_n = 0, \]  \hspace{1cm} \text{(C.11)}

and then
\[ -\rho \frac{U_s}{s} \frac{\partial U_s}{\partial s} \hat{e}_s - \rho U_s \frac{\partial U_n}{\partial s} \hat{e}_s - \rho \frac{\partial p_s}{\partial s} \hat{e}_s - \rho U_n \frac{\partial U_n}{\partial n} \hat{e}_n + \rho \frac{U_s^2}{R} \hat{e}_n = 0. \]  
(C.12)

Recalling the definition of streamline, the velocity vector \( \vec{v} = U_s \hat{e}_s \) is tangent to the \( s \) direction, meaning \( U_n = 0 \), so the previous equation is re-written as

\[ -\rho U \frac{\partial U_s}{\partial s} \hat{e}_s - \frac{\partial p}{\partial s} \hat{e}_s - \frac{\partial p}{\partial n} \hat{e}_n + \rho \frac{U_s^2}{R} \hat{e}_n = 0, \]  
(C.13)

which is expressed in its compact form

\[ -U_s \frac{\partial U_s}{\partial s} \hat{e}_s - \frac{1}{\rho} \nabla p + \frac{U_s^2}{R} \hat{e}_n = 0, \]  
(C.14)

with \( \nabla \equiv \left( \frac{\partial p}{\partial s} \hat{e}_s, \frac{\partial p}{\partial n} \hat{e}_n \right) \). Equation C.14 is the same as Equation 5.25 derived with a Lagrangian approach.
Appendix D

The equation
\[
\int_{-\infty}^{\sigma} \left[ 1 + \tan^2 \beta(\vec{r}) \right]^{1/2} \frac{d\beta(\vec{r})}{dy} \, dy = \int_{-\infty}^{\sigma} \frac{1}{\rho v^2(\vec{r})} \frac{\partial p(\vec{r})}{\partial n} \, dy,
\]
(D.1)
treated in Chapter 5 is solved by means of the following substitution:
\[
\kappa = \beta(\vec{r})
\]
\[
d\kappa = \frac{d\beta(\vec{r})}{dy} \, dy
\]
and assuming that the streamline far upstream of the wire is straight, that is \( \beta(y=\infty)=0 \). The left-hand side term becomes
\[
\int_{0}^{\beta(y=\sigma)} \left[ 1 + \tan^2 \kappa \right]^{1/2} \, d\kappa = \int_{0}^{\beta(y=\sigma)} \left[ 1 + \tan^2 \beta(\vec{r}) \right]^{1/2} \, d\beta(\vec{r}).
\]
(D.2)
Considering that
\[
1 + \tan^2 \beta(\vec{r}) = \frac{1}{\cos^2 \beta(\vec{r})},
\]
(D.3)
it follows that
\[
\int_{0}^{\beta(y=\sigma)} \left[ 1 + \tan^2 \beta(\vec{r}) \right]^{1/2} \, d\beta(\vec{r}) = \int_{0}^{\beta(y=\sigma)} \frac{d\beta(\vec{r})}{\cos \beta(\vec{r})}.
\]
(D.4)
The integral is now re-written as
\[
\int_{0}^{\beta(y=\sigma)} \frac{d\beta(\vec{r})}{\cos \beta(\vec{r}) \cos \beta(\vec{r})} = \int_{0}^{\beta(y=\sigma)} \frac{\cos \beta(\vec{r}) \, d\beta(\vec{r})}{\cos^2 \beta(\vec{r})} = \int_{0}^{\beta(y=\sigma)} \frac{\cos \beta(\vec{r}) \, d\beta(\vec{r})}{1 - \sin^2 \beta(\vec{r})}.
\]
(D.5)
Then the following substitution is operated:

\[ q = \sin \beta (\vec{r}) \]
\[ dq = \cos \beta (\vec{r}) d\beta (\vec{r}) \]

such that

\[ \beta (y = \sigma) \int_0^\beta \cos \beta (\vec{r}) d\beta (\vec{r}) = \int_0^{\sin \beta (y = \sigma)} \frac{1}{1 - q^2} dq. \] (D.6)

The integrand of the left-hand side term is rearranged as follows:

\[ \frac{1}{1 - q^2} = \frac{1}{(1 + q) (1 - q)} = \frac{A}{1 + q} + \frac{B}{1 - q}, \] (D.7)

where \( A = B = 1/2 \).

Therefore,

\[ \sin \beta (y = \sigma) \int_0^\beta \frac{1}{2} \int_0^1 \frac{1}{1 + q} + \frac{1}{2} \int_0^1 \frac{1}{1 - q} = \frac{1}{2} [\ln (1 + q) - \ln (1 - q)]_0^{\sin \beta (y = \sigma)}. \] (D.8)

Equation D.1 becomes

\[ \frac{1}{2} [\ln (1 + \sin \beta (y = \sigma)) - \ln (1 - \sin \beta (y = \sigma))] = \int_\sigma^{-\infty} \frac{1}{\rho v^2 (\vec{r})} \frac{\partial p(\vec{r})}{\partial n} dy. \] (D.9)

For small \( \beta (\vec{r}) \),

\[ \sin \beta (\vec{r}) \approx \beta (\vec{r}), \] (D.10)

so

\[ \ln (1 + \sin \beta (y = \sigma)) \approx \beta (y = \sigma), \] (D.11)

and

\[ \ln (1 - \sin \beta (y = \sigma)) \approx -\beta (y = \sigma), \] (D.12)

For the left-hand side of Equation D.9, it follows that

\[ \frac{1}{2} [\ln (1 + \sin \beta (y = \sigma)) - \ln (1 - \sin \beta (y = \sigma))] \approx \frac{1}{2} [\beta (y = \sigma) - (-\beta (y = \sigma))] \approx \beta (y = \sigma). \] (D.13)

For small angles, \( \beta \) is, thus, expressed as

\[ \beta (y = \sigma) = \int_\sigma^{-\infty} \frac{1}{\rho v^2 (\vec{r})} \frac{\partial p(\vec{r})}{\partial n} dy, \] (D.14)

which is equal to Equation 5.37.
Appendix E

The uncertainty quantification of the correlation for the bending angle $\beta_w$ (Equation 5.54) is presented in this appendix. The dimensional tolerances of the rod and the wire are considered (see table 3.2), as well as the uncertainty on the measured mean angle $\beta_w$.

The uncertainty $\delta \beta_w$ on the value predicted by the correlation is expressed as

$$\delta \beta_w = \left| \frac{\partial \beta_w}{\partial b} \delta b \right|,$$

(E.1)

where $\delta b$ is the uncertainty on the exponent of the correlation, and $\frac{\partial \beta}{\partial f}$ is obtained deriving Equation 5.54 with respect to $b$:

$$\frac{\partial \beta_w}{\partial b} = \frac{h}{\phi_{\text{pin}}} \ln \left( \frac{\phi_w}{\phi_h} \right) \cdot \left( \frac{\phi_w}{\phi_h} \right)^b,$$

(E.2)

being $h$, $\phi_{\text{pin}}$, and $\phi_w$ the helicoid wire pitch, the pin diameter and the wire diameter of an arbitrary hexagonal bundle. The dimensions of the SEEDS-2 bundle are substituted in the previous equation, that is $h=h$, $\phi_{\text{pin}}=D_r$ and $\phi_w=D_w$.

The $\delta b$ term is determined by the uncertainty propagation formula as

$$\delta b = \sqrt{\left( \frac{\partial b}{\partial D_w} \delta D_w \right)^2 + \left( \frac{\partial b}{\partial D_r} \delta D_r \right)^2 + \left( \frac{\partial b}{\partial \beta} \delta \beta_w \right)^2},$$

(E.3)

where $\delta D_w$, $\delta D_r$ and $\delta \beta_w$ are respectively the uncertainties on the diameter of the rod and wire of the hexagonal bundle of this research, and $\beta_w$ the mean measured angle of bending. The partial derivatives $\frac{\partial b}{\partial D_w}$, $\frac{\partial b}{\partial D_r}$ and $\frac{\partial b}{\partial \beta_w}$ are obtained by deriving Equation 5.56 with respect to $D_w$, $D_r$, and $\beta_w$:

$$\frac{\partial b}{\partial D_w} = \frac{\ln \left( \frac{D_r}{h} \tan \beta_w \right)}{\ln \left( D_w/D_h \right)^2} \left[ -\frac{P+D_w}{PD_w} + \frac{4\sqrt{3}P-2\pi D_w}{2\sqrt{3}P^2 - \pi \left( D_r^2 + D_w^2 \right)} \right],$$

(E.4)

$$\frac{\partial b}{\partial D_r} = \frac{1}{D_r \ln \left( D_w/D_h \right)} + \ln \left( \frac{D_r}{h} \tan \beta_w \right) \left[ -\frac{1}{P} + \frac{4\sqrt{3}P-2\pi D_r}{2\sqrt{3}P^2 - \pi \left( D_r^2 + D_w^2 \right)} \right],$$

(E.5)
\[
\frac{\partial b}{\partial \beta_w} = \frac{1}{\ln(D_w/D_h)} \frac{1 + \tan^2 \beta_w}{\tan \beta_w}.
\]  \hspace{1cm} (E.6)

Equation E.1 is finally used to estimate the uncertainty on the angle predicted by the correlation derived in Chapter 5, which is equal to 5\%.
Acknowledgements
List of publications

Journal papers


Book chapters


Conference papers


About the author

The author was born in Pisa (Italy) on 4\textsuperscript{th} November, 1990 (although he prefers to say Livorno is his native town). He finished his high school studies at Liceo Scientifico U. Dini in 2009. The same year he enrolled in the bachelor degree on Nuclear Engineering, Safety and Protection at the University of Pisa where he graduated on 4\textsuperscript{th} March, 2013 with the thesis “Analysis of the structural response of a nuclear waste disposal during the impact of a commercial Boeing-747 aircraft”. He continued the studies at the same university with the master degree on Nuclear Engineering. Between 2014 and 2015 he moved to SCK-CEN nuclear research centre (Mol, Belgium) where he did his master thesis project with title “CFD study of Polonium evaporation from LBE liquid mixture”. He obtained his master degree on 20\textsuperscript{th} July, 2015 and in September he became a PhD candidate on fluid-mechanics in the group of prof. Jan Leen Kloosterman, at Delft University of Technology. In October 2019 he was hired as consultant by the Licensing Team of the Nuclear Research and consultancy Group (NRG) in Arnhem.