Viscosity determination using the quasi-Scholte wave

Master Thesis

by

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In this thesis, the use of the quasi-Scholte wave for the determination of the viscosity of a fluid at high temperature, with a very low viscosity, is investigated. In the setup, a Lamb wave was first excited on a plate by a transducer, which would later convert to the quasi-Scholte wave in the fluid. The attenuation of the quasi-Scholte wave was measured in Neper per meter by changing the immersion depth of the plate. The research has been performed at room temperature and the viscosity was measured for water. The results yielded a viscosity of $\mu = 0.1 \pm 0.3$ mPa·s at 20 °C, in disagreement with the literature values. The disagreement is believed to be partly due to the temperature dependent amplitude, fluctuating by 20% in 25 min, and to interference altering the signal strength periodically by 10% for a 0.44 cm immersion depth change. Furthermore, simulations have been run using COMSOL®. However, these have not shown consistent results, and the viscosity measurement has not been

In this report, the main focus is put on the understanding of using ultrasonic attenuation for the determination of fluid properties. For example, the transfer function of the electrical to the mechanical signal by the transducer was researched. Also, the time dependence of the attenuation was thoroughly examined, and the effect of temperature changes on the sent wave, excitation of the Lamb wave and the phase velocity in the plate and in the fluid were investigated. Furthermore, interference of the reflected wave and its frequency were explored. These and other aspects are discussed in this thesis. Finally, a new setup is suggested to possibly remedy the time dependence of the attenuation, and new measurements are proposed to further develop understanding of the setup, as well as increasing the accuracy.
Before we kick off, I would like to thank a few people who helped me during my thesis. First and foremost, Sara and Martin, who showed dedication, perseverance and unprecedented optimism along the way. Without their enthusiasm during the numerous discussions we have had, I do not think I would have enjoyed doing this research as much as I did. An extra thanks to Sara for reading and correcting my thesis while on a holiday trip to Georgia.

When I first came in around September, looking for a thesis subject in the area of nuclear physics, I was surprised to hear that research was going on in the field of acoustics as well. I think we might conclude that, in retrospect, it was a courageous decision. However, the combination of using simulations and doing experiments has been highly enjoyable.

Secondly, I would like some people at Imaging Physics, as they have helped us forward greatly. I would like to thank Martin for his help in arranging the possibility to measure in their lab. Also, I would like to thank Verya and Maysam for the numerous times they have helped me when I walked into their office, even though my research did not have anything particular to do with their research. And Koen I would like to thank for the educative discussion, he came with numerous practical insights.

Lastly, my gratitude goes out to the thesis committee taking the time to read through my report, listening to my presentation and finally discussing the results during the defence. In fact, while writing down the results for this thesis report, I have gained so many new insights that I might go so far as to say: I am really looking forward to the defence.
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Introduction

With an eye on global warming, the need for sustainable and clean power supply to the world’s fast-growing population is urgent. New sources of energy are being discovered and developed at a fast pace. The generation-IV nuclear reactors are an example of these novel concepts. One of these is the molten salt reactor, and in this thesis the focus will be on the molten salt fast reactor. To prove the safety of this type of reactors, the SAMOFAR project has been set up. SAMOFAR stands for “Safety Assessment of the Molten Salt Fast Reactor” [1]. The physical properties of the salt will be investigated in order to fully understand and predict the behaviour of the reactor, to prove its safety and ultimately to design the reactor. For example, the properties need to be measured in order to understand the fuel circulation, fuel reprocessing, heat transfer, general safety installations, safety procedures in case of accidents, core design and many other aspects. In Delft, the fundamental behaviour of the salt is now researched, and in this thesis a setup is investigated to determine the viscosity of the salt at high temperatures.

1.1. Molten Salt Fast Reactor

Proudly are the results of the first molten salt reactor presented by Haubenreich and Engel [2]. However, in spite of the promising results, research in the United States was not continued by decision of the government [3]. In the molten salt fast reactor, contrastingly to the other designs of the generation IV reactors, the fuel itself is a liquid. The molten salt fast reactor is a further development of this type of reactor [1], and a schematic drawing of the reactor core is shown in figure 1.1.

With this figure, several important aspects can be introduced. First, the green indicated molten salt is shown to be pumped around. It will reach criticality in the core, and will deliver its thermal energy to the heat exchangers. However, part of the molten salt exits the loop for reprocessing, such that the uranium, actinides and the soluble fission products can be separated [1]. Also, the soluble and gaseous fission products are removed by gas bubbles injected in the core, and shown as dots in figure 1.1. An annular cylinder of thorium is added as a breeding blanket, transmuting non-fissile thorium-232 to fissile uranium-233 upon capturing a neutron [4]. Lastly, at the bottom of the core, a drainage pipe and freeze plug are shown. In case of an emergency, the freeze plug will melt and the molten salt will drain to a fail-safe storage tank [1].

The novelty of the design resides in the twofold purpose that is served by the salt. The first function is as a nuclear fuel, and the second is as a coolant. For the former function, the density as a function of temperature is an important parameter (as the density influences the neutronics greatly). For the latter, both density and viscosity are important parameters. For example, these parameters need to be known to predict and understand fluid flow in normal operations or in case of an emergency (where the salt will flow to an underground containment vessel) and to predict the heat transfer (important for the design of the heat exchanger).[5][6]

The salt of which these parameters need to be investigated is a composition of LiF-ThF4-Uf4-PuF4, consisting of lithium, thorium, uranium and plutonium fluorides. This salt melts at around 600 °C [7] depending on the composition, and the MSFR will work at approximately 750 °C [1]. The measurements will be performed up to temperatures of 1200 °C, such that in case of an accident or emergency, the fluid properties and behaviour can be predicted.
1. Introduction

Figure 1.1: In this figure, a schematic drawing of the molten salt fast reactor core is shown. The circulated molten salt is shown in green, with the dots representing gas bubbles. The red annular cylinder surrounding the molten salt is the breeding blanket. [1]

1.2. High Temperature Viscosity Measurement

Some research has gone into the temperature dependence of the viscosity of fluid salts [8][9][10], and the viscosity of the molten LiF-ThF-UF-PuF salt is expected to be relatively low, down to approximately 3 mPa·s. Current techniques are not capable of measuring such low viscosities of fluids at such high temperatures. Also, the corrosivity of the molten fluoride salt poses a problem for most materials used for the setup assembly. Furthermore, as the fuel will be highly radioactive, only a small amount may be worked with for the determination of the viscosity.

Mastromarino [6] has discussed the choice for a new method determining the viscosity and possibly density of fluids at high temperatures using acoustics. The first research on this application of acoustics dates back to research performed by Mason et al. in 1949 [11]. These experiments rely on the usage of a transducer, converting an electric signal to a mechanical displacement. These transducers make use of piezoelements, which demagnetise at their Curie temperature thus making them unusable.

A new measurement setup needs to be designed and its accuracy needs to be studied. A setup was described and tested by Cegla (see [12]), especially to measure fluid properties in harsh environments. It makes use of the same transducers and relies on the same attenuation mechanisms of acoustic waves in fluids, albeit the transducer will now be separated from the fluid, and the waves will be passed on by a waveguide. The investigation of this setup will be the starting point for this thesis.

1.3. Thesis Outline

In this research, the first step will be to recreate the measurement setup as described by Cegla. In this setup, a waveguide in the form of a strip or plate will be inserted into the fluid. A wave will be sent through the waveguide, and the amplitude of its reflection will be measured. By adjusting the immersion depth the attenuation per meter can be measured. The latter depends on the viscosity, which can then be derived. The setup discussed in this thesis will however, only be used at room temperature.

Concurrently, the setup will be simulated using COMSOL. The physical setup will be used to benchmark the simulations. The simulations, in turn, can be used to predict behaviour in conditions that are more challenging to physically set up (at high temperatures, for example). The experiments are per-
formed on water, as water is a comparable fluid (though its viscosity is predicted to be approximately a three factor lower). With the experiments, the influence of different configurations is investigated. For example, the insertion depths, strip dimensions and material, frequency and shape of the sent wave and other characteristics will be researched. Also, some light is shed on the predicted influence of increasing the temperature. When the theory and results of these aspects have been introduced, recommendations for future research and for the future setup will be given and discussed.

At the beginning of this research, it was expected that it would be possible to successfully determine the viscosity of fluids around two thirds or three quarters into the research time window set. It was envisaged to then design a setup that could measure the viscosity of fluids at temperatures up to 1200 °C. However, unfortunately, it has not been possible to correctly determine the viscosity nor has it been possible to get consistent attenuation results. As prior knowledge of these applications of acoustics lacked at the RPNM (reactor physics and nuclear materials) research group, many aspects were new and the research took longer than expected. As such, during the research, the goals were adjusted. Instead of investigating the influence of the aforementioned configurations on the temperature dependence of the measurements, the accuracy and repeatability have been researched. The setup used, recommendations for the handling of the setup, and the results, discussion and outlook have been described in this thesis.

First, the theory will be discussed in chapter 2. As the researcher was unfamiliar with continuum mechanics, the starting level is basic. Building up from that level, the wave behaviour in fluids, bulk materials and plates can later be understood. Then, the setup is introduced. Some of the results that led to clear conclusions are presented in the setup chapter 3, such that this chapter can form a guideline for doing these experiments. The results that led to preliminary conclusions subject still to discussion, or new hypotheses, have been set out in chapter 4.
2.1. Continuum Mechanics

The final goal of this research, is to develop a better understanding of the new measurement method for the viscosity of fluids at high temperature with very low viscosities. In order to do so, first some continuum mechanics need to be discussed, to understand the properties that will be examined and the measurement methods. An elementary start will be made defining some fundamental quantities for continuum mechanics, introducing stress, strain and their relation. These fundamental quantities give rise to the wave equations in bulk materials, in plates and on surfaces. Finally, the more practical implications of the fundamental physics are discussed, such as dispersion.

2.1.1. Stress

Stress is expressed in N/m$^2$. It is the force that particles exert on each other per unit surface. In figure 2.1a, an area $A$ is shown. On this surface area, a force $F$ is exerted in both directions normal to the surface. These forces must be equal, else the surface would experience an infinite acceleration as it has no mass. Volume forces, such as gravity, do not act on a plane. Shear stress is also expressed in N/m$^2$, however, now the force is exerted perpendicular to the normal of the surface. In figure 2.1b, one can observe shear stress.

In any volume, each plane ($x_1x_2$, $x_2x_3$ and $x_3x_1$) can experience both a shear and a normal stress. To express these stresses, the 3x3 stress tensor is used. In this matrix, each normal stress is filled in on the diagonal, and each shear stress on the off-diagonal points. The following matrix is found:

$$\mathbf{T} = \begin{bmatrix}
\sigma_{x_1x_1} & \tau_{x_1x_2} & \tau_{x_1x_3} \\
\tau_{x_2x_1} & \sigma_{x_2x_2} & \tau_{x_2x_3} \\
\tau_{x_3x_1} & \tau_{x_3x_2} & \sigma_{x_3x_3}
\end{bmatrix},$$ (2.1)

where $\sigma_{x_1}$ is the stress normal to the $x_2x_3$ plane (pointing outwards), $\tau_{x_1x_2}$ is the shear stress on the $x_2x_3$ plane parallel to $x_2$ and $\tau_{x_1x_3}$ is the shear stress on the $x_2x_3$ plane parallel to $x_3$. These components are graphically displayed in figure 2.2, where indices 1, 2 and 3 indicate directions $x_1$, $x_2$ and $x_3$ respectively.

One can imagine that if pressure $p$ is the only stress working on a fluid or gas, we will find the stress tensor to be

$$\mathbf{T} = \begin{bmatrix}
-p & 0 & 0 \\
0 & -p & 0 \\
0 & 0 & -p
\end{bmatrix}. \tag{2.2}$$

The minus signs arise from the fact that pressure works inwards. It is important to note that the matrix is symmetric. Although the tensor seems to depict a volume, the volume can be made arbitrarily small while the forces per unit area remain. As before, where the forces on the surface needed to be equal on both sides in order to ensure there was no infinite acceleration, we also need to ensure there is no infinite angular acceleration. Therefore, we need $\tau_{ij} = \tau_{ji}$.

2.1.2. Strain

When stress is applied to a body, the body will deform. Strain is a dimensionless measure that expresses the relative deformation of a body. If one looks at normal strain, it is defined as the deformation divided
2.1. Continuum Mechanics

(a) In this figure, normal stress on a surface A is depicted.

(b) Here, shear stress on a surface A is shown.

Figure 2.1: Normal and shear stress are shown, where the difference is whether the force F is exerted (a) perpendicular or (b) normal to the intermediate surface.

Figure 2.2: The Cauchy stress tensor is depicted, with the normal and shear stresses on the different planes indicated by $\sigma_{ij}$. 
by the original length, or

$$S = \frac{(x_2 - x_1)}{x_1} = \frac{\Delta x}{l},$$

(2.3)

where \(x_1\) and \(x_2\) are the original and new length respectively, \(\Delta x\) is the change in length and \(l\) is the original length. In figure 2.3, it can observe how this would be calculated. If one would look at the absolute deformation, different values for the same strain could be obtained. For example, in figure 2.3, the size of block a increases from 1 to 1.5. However, if one looks at the extension of ab, the size increases from 2 to 3. The absolute strains would be 0.5 and 1 respectively, whereas the actual strain is calculated by

$$S = \frac{a_2 - a_1}{a_1} = \frac{1.5 - 1}{1} = \frac{ab_2 - ab_1}{ab_1} = \frac{3 - 2}{3} = 0.5.$$

![Figure 2.3: In this figure, a rectangle before and after exerting strain is depicted.](image)

In a similar manner, one can find the shear strain. Also in this case, it is needed to divide the absolute deformation by an initial length. However, in this case, the initial length is be perpendicular to the force applied.

### 2.1.3. Strain Rate Tensor

Often one has to work with the rate of strain, giving information of how much strain is induced per unit time. The strain rate is found by differentiating strain with respect to time. A start will be made with a simple example of one dimensional normal strain. Take for example two points, \(a\) and \(b\), of which the relative distance is deforming in time. This is depicted in figure 2.4.

It is now known that the strain can be found by dividing the difference in distance at time \(t_2\) by the distance at time \(t_1\), or

$$S = \frac{(ab)_{t_2} - (ab)_{t_1}}{(ab)_{t_1}}.$$

Here, \((ab)_{t_1}\) and \((ab)_{t_2}\) are the initial and final lengths respectively. From the figure, the following

![Figure 2.4: Here, increasing normal strain is depicted at stages \(t_0\) and \(t_1\). The increase in strain is due to the velocity of the particles increasing in the \(x_1\) direction.](image)
relations can be found:

\[(ab)_1 = \delta x_1\]
\[(ab)_2 = \delta x_1 + v(x_{a1} + \delta x_1)\delta t - v(x_{a1})\delta t\]

\[S = \frac{\delta x_1 + v(x_{a1}\delta t + \delta x_1) - v(x_{a1})\delta t - \delta x_1}{\delta x_1} = \frac{\partial v(x_1)}{\partial x_1}.\]

Now, velocity \(v\) has been introduced, which depends on position \(x\), and for simplicity is assumed not to be dependent on time \(t\). The following relation for the strain can be found:

\[\frac{\partial S}{\partial t} = \frac{\partial v}{\partial x_1}.\]

The strain rate for shear strain can also be derived. Suppose that the particle only move in the \(x_1\) direction, this is depicted in figure 2.5.

The strain rate is found to be

\[\frac{\partial S}{\partial t} = \frac{(ab)_{1, t_1}/(ab)_2 - (ab)_{1, t_2}/(ab)_3}{\delta t}.\] (2.4)

The first subscript specifies the direction, the second the moment in time. In this simplified example, \(v_3\) is kept to zero, and only a gradient in \(v_1\) in the \(x_1\) direction is dealt with.

In this equation, the following can be filled in:

\[(ab)_{1, t_1} = \delta x_{1, t_1}\]
\[(ab)_{1, t_2} = \delta x_{1, t_1} + (v_1(x_{a1} + \delta x_{1, t_1}) - v_1(x_{a1}))\delta t\]
\[(ab)_3 = \delta x_3.\]

As such, it is found that

\[S = \frac{\delta x_{1, t_1}}{\delta x_3} - \frac{\delta x_{1, t_2} + (v_1(x_{a1} + \delta x_{1, t_1}) - v_1(x_{a1}))\delta t}{\delta x_3} = \frac{v_1(x_{a1} + \delta x_{1, t_1}) - v_1(x_{a1})}{\delta x_3}\delta t.\]

Finally, the shear strain rate is found, being

\[\frac{\partial S}{\partial t} = \frac{\delta v_1}{\delta x_3}.\] (2.5)
In a similar way, all other shear strain rates can be derived. For example, the equation for shear strain rate $\gamma_{13}$ can be found to be:

$$\gamma_{13} = \frac{\delta v_1}{\delta x_3} + \frac{\delta v_3}{\delta x_1}. \quad (2.6)$$

Now, the strain rate tensor can be build. As for the stress tensor, this tensor will be symmetric, as can also be observed from equation 2.6 above. To avoid overcounting, on the off-diagonal spots a factor $\frac{1}{2}$ is added. The strain rate tensor is found to be equal to:

$$E' = \begin{bmatrix}
\epsilon_{11}' & \epsilon_{12}' & \epsilon_{13}' \\
\epsilon_{12}' & \epsilon_{22}' & \epsilon_{23}' \\
\epsilon_{13}' & \epsilon_{23}' & \epsilon_{33}'
\end{bmatrix}, \quad (2.7)$$

where each $\epsilon_{ij}$ is found by:

$$\epsilon_{ij}' = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right).$$

Now, the possible compressibility of the fluid has not yet been included in the tensor. This term will only act on the diagonal of the equation, and a factor $\frac{1}{3}$ is included, again to avoid overcounting. The expression for $\epsilon_{ij}$ then changes to

$$\epsilon_{ij}' = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - \frac{1}{3} \left( \sum_k \frac{\partial v_k}{\partial x_k} \right) \delta_{ij}, \quad (2.8)$$

where $\delta$ is the Kronecker-Delta function.

### 2.1.4. Reaction to Normal Strain

When force is applied to a spring, the amount of extension or compression depends on the force and the spring constant. The same applies to volumes. Suppose pressure is applied to a body, then the body will naturally decrease in volume. The force needed for a relative deformation, so the force needed for a certain strain to occur, is expressed in the Bulk modulus $B$. The modulus is given by [13]:

$$B = \lim_{\delta V \to 0} \frac{-p}{\delta V/V_0}, \quad (2.9)$$

where $V_0$ is the original volume, $\delta V$ the change in volume and $p$ is the pressure difference from the pressure that was applied to $V_0$. The units of the bulk modulus are given in [N / m$^3$]. We can now continue to set up one of the equations that we will need later to derive the wave equation. In figure 2.6, we depict a fluid that is moving in the x direction (and is homogeneous in the $x_2$ and $x_3$ direction). We can define $\phi(x, t)$ as the displacement at time $t$ at position $x$. Then, a relative change in volume can be found as follows, where $x$ is replaced by $x_1$ as for simplicity the velocity gradient is assumed to be zero in the $x_2$ and $x_3$ directions:

$$\frac{\delta V}{V_0} = \frac{\phi(x_1 + \delta x_1, t) - \phi(x_1, t)}{\delta x_1 \delta x_2} = \frac{\phi(x_1 + \delta x_1, t) - \phi(x_1, t)}{\delta x_1}. \quad (2.10)$$

From the earlier obtained equation 2.9, it is known that this can be related to a pressure, finding:

$$\frac{p}{B} = - \lim_{\delta V \to 0} \frac{\delta V}{V_0} = \lim_{\delta x_1 \to 0} \frac{\phi(x_1 + \delta x_1, t) - \phi(x_1, t)}{\delta x_1} = \frac{\partial \phi}{\partial x_1}. \quad (2.11)$$

To find the wave equation, an expression for $\frac{\partial p}{\partial t}$ needs to be found. As it is clear that the following holds:

$$v = \frac{\partial \phi}{\partial t},$$

the relation in equation 2.11 can be differentiated with respect to time, finding that

$$\frac{\partial}{\partial t} \left( \frac{p}{B} \right) = - \frac{\partial}{\partial t} \left( \frac{\partial \phi}{\partial x} \right) = - \frac{\partial v}{\partial x}. \quad (2.12)$$
2.1. Continuum Mechanics

Figure 2.6: Here, bulk displacement in combination with compression or extension is depicted, where the compression or extension is the result of the velocity field changing with $x_1$. For the sake of a simple explanation, the velocity field is kept constant in the $x_2$ and $x_3$ directions.

As such, the following relation can be presented:

$$\frac{\partial p}{\partial t} = -B \frac{\partial v}{\partial x}. \quad (2.13)$$

2.1.5. Reaction to Shear Strain

For both liquids and solids, strain will induce stress. The amount of stress that is induced by strain, depends on the proportionality constants, such as dynamic viscosity and the shear modulus. Now, there is an important difference between non-elastic fluids and linearly elastic solids. These liquids will only give resistance, when they are being deformed. So, for non-elastic fluids there is no such thing as stationary strain. Therefore, for liquids strain rates will be dealt with.

Liquids

Suppose strain as in figure 2.1b is applied to a fluid, by dragging a surface along the fluid with velocity $u$. It is clear that a drag force will be induced by the fluid. This drag force will be stronger if the velocity $u$ with which the plate of area $A$ is dragged along is bigger. Also, if area $A$ is larger, the drag force will be larger. However, one can also imagine that the strain will be the largest close to the plate. It was discovered that in general indeed, a linear relation between these parameters exists, constituting the following relation [14]:

$$F_{drag} = \eta A \frac{v}{y}. \quad (2.14)$$

Here, $F_{drag}$ is the drag force induced by the strain rate of the fluid, and $\eta$ is a proportionality constant unique for each fluid, and called the dynamic viscosity in units [Ns / m²]. It is clear that the units for dynamic viscosity are similar to that of the bulk modulus, save the unit of time [s] included for the dynamic viscosity, indicating the use of strain rates. It must be noted that the formula above only holds for Newtonian fluids, where a Newtonian fluid constitutes a fluid of which the viscous stress increases linearly with the strain rate.

The earlier presented stress tensor in equation 2.1 can now be expanded to include stress induced by shear strain rates. Recalling that viscosity is the measure that gives the ratio of stress induced by strain, we find the stress tensor to be:

$$\mathbf{T}^i = -\begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix} + 2\eta E^i \quad (2.15)$$

Also, the earlier derived expression of Newton’s second law can now be expanded to include viscosity. In this case, not only the pressure difference should be considered, but also a difference in strain induced stress. If only a movement in the $x_1$ direction is considered, $\delta T_{11}$ can be found to be equal to
\[ \delta T_{j1} = p(x_1 + \delta x_1, t) - 2\eta E_{j1}(x_1 + \delta x_1, t) - p(x_1, t) + 2\eta E_{j1}(x_1, t) \]
\[ = \frac{\partial p}{\partial x_1} \delta x_1 - \frac{\partial E_{j1}}{\partial x_1} \delta x_1, \]

where the element of the strain tensor to be plugged in is equal to

\[ \frac{\partial E_{j1}}{\partial x_1} = \frac{\partial}{\partial x_1} \left( \frac{1}{2} \left( \frac{\partial u_1}{\partial x_1} + \frac{\partial u_1}{\partial x_1} \right) - \frac{1}{3} \frac{\partial u_1}{\partial x_1} + \frac{\partial u_1}{\partial x_2} + \frac{\partial u_1}{\partial x_3} \right) \quad (2.16) \]
\[ = \frac{2}{3} \frac{\partial^2 u_1}{\partial x_1^2}. \quad (2.17) \]

### Solids

For solids however, the contrary is true: there is no such thing as a new equilibrium position after a deformation. The solid will always return to its original shape as long as the deformation is elastic, and not so great to become plastic. In this thesis, only elastic displacements will be considered. As such, only strains and not strain rates will be dealt with for solids. The units of the moduli describing the amount of stress induced by a certain amount of strain, are thus given in \([N/m^2]\).

The stress tensor for solids will be similar to that of liquids. With the strain displacement equations and the constitutive equations, the following stress tensor is found, where two new proportionality constants are introduced [13]:

\[ T^s = \lambda \begin{bmatrix} \varepsilon_{11}^s & 0 & 0 \\ 0 & \varepsilon_{22}^s & 0 \\ 0 & 0 & \varepsilon_{33}^s \end{bmatrix} + 2\mu E^s. \quad (2.18) \]

Here, the tensor \(E^s\) has the same shape as \(E^l\) in equation 2.7. However, it does not contain the derivatives of the velocity \(v_i\) with respect to \(x_j\) as for liquids. Instead, the displacements \(u_i\) of the particles with respect to \(x_j\) should be plugged in. So, the \(\varepsilon_{ij}\) can be defined as follows:

\[ \varepsilon_{ij}^s = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right). \]

The two constants \(\lambda\) and \(\mu\) in equation 2.18 are the first and second Lamé constants, where \(\mu\) is also known as the shear modulus. It is interesting to note that other moduli depend on the Lamé constants. For example, the bulk modulus is given by \(B = \lambda + \frac{2}{3}\mu\).

### 2.1.6. Resistance to body forces

Up until now, only surface forces have been discussed, such that all expressions were in terms of \(N/m^2\) or \(Ns/m^2\). Surface forces are constituted by forces acting between neighbouring particles, for example by the elastic force between them or by a traction force on the surface. Now, body forces, acting directly on the particle, will be included in the formulas. The best example of such a body force is gravity.

** Liquids **

Newton’s second equation will now shortly be revisited, such that an expression can be derived which later will be used to derive the wave equation. In figure 2.7, a moving fluid is depicted. Here, a small volume of dimension \(\delta x_1, \delta x_2, \text{ and } \delta x_3\) is moving with velocity \(v_i(x_i, t)\). Also, a pressure \(p(x_1, t)\) is applied to this volume, where a pressure gradient in the \(x_1\) direction exists. The fluid density is indicated by \(\rho(x_1, t)\).

Then, two different expressions can be found for Newton’s second law, one in terms of the density and the acceleration and one in terms of the pressure difference and the surface area \(A\) in the \(x_2x_3\) plane. Thus, the following expressions
2.1. Continuum Mechanics

Figure 2.7: In this figure, a fluid that is displacing and its accompanying pressure field is depicted.

\[ F_1 = (\delta_1 p) A_{23} \]
\[ F_1 = ma_1, \]

both constitute Newton's second law. Note that the pressure difference, which only exists in the \( x_1 \) direction, has been indicated by \( \delta_1 p \). In these equations, one can identify

\[ m = \rho \delta_1 x_2 \delta x_3 \]
\[ a_1 = \frac{\partial v_1}{\partial t}. \]

Then, filling in these expressions in the two previously introduced variants of Newton's second law, yield that

\[ F_1 = - (\delta_1 p) \delta A_{23} = \lim_{\delta x_1 \to 0} \left( \frac{p(x_1, t) - p(x_1 + \delta x_1, t)}{\delta x_1} \right) \delta x_2 \delta x_3 \]
\[ = - \frac{\partial p}{\partial x_1} \delta x_1 \delta x_2 \delta x_3 = \rho \delta x_1 \delta x_2 \delta x_3 \frac{\partial v_1}{\partial t}. \]

The following relation can be found, where \( x_1 \) is generalised to \( x_i \):

\[ \frac{\partial p}{\partial x_i} = -\rho \frac{\partial v_i}{\partial t}. \] (2.19)

Newton's second law can be expanded to include viscosity. Looking at the previously obtained equation 2.15, a change in pressure over \( x_1 \) as a result of a viscous stress can be found to be:

\[ \frac{\partial}{\partial x_1} \left( 2\eta \frac{F_{11}^i}{x_1^2} \right) = \frac{\partial p}{\partial x_1}. \]

As the change in pressure only influences the normal stresses in the \( x_1 \) direction, the relation can be filled in, finding that

\[ \frac{\partial p}{\partial x_1} = 2 \eta \frac{2 \partial^2 u_i}{3 \partial x_1^2} \]

Here, the relation obtained in equation 2.16 was plugged in. Then finally, the viscous relation for Newton's second law is found to be as follows:

\[ - \frac{\partial p}{\partial x_1} = \rho \frac{\partial v_1}{\partial t} + \frac{4}{3} \frac{\partial^2 u_1}{\partial x_1^2} \] (2.20)
Solids
For solids, the derivation is very similar as for liquids. For example, assuming a solid body with a stress increasing in the $x_1$ direction, Newton's second law can be expressed in two ways:

$$F_1 = (\delta_1 \sigma) A_{23}$$
$$F_1 = m \frac{\partial v_1}{\partial t}.$$  

Again, mass $m$ will be rewritten in the corresponding size of the arbitrary volume and its density $\rho$. Then, following the earlier derivation, it is found that

$$F_1 = -\delta \sigma \delta A_{23} = \lim_{\delta x_1 \to 0} \left( \frac{\sigma(x_1, t) - \sigma(x_1 + \delta x_1, t)}{\delta x_1} \right) \delta x_2 \delta x_3$$
$$= -\frac{\partial \sigma}{\partial x_1} \delta x_1 \delta x_2 \delta x_3 = \rho \delta x_1 \delta x_2 \delta x_3 \frac{\partial v_1}{\partial t}.$$  

Newton’s second law will have a slightly different expression, where the viscosity term is dropped as it is irrelevant for the purely elastic materials considered. Also, the equation is generalised in all directions, where it can be noted that the velocity gradient can be non-zero in any direction, and the stress field can also vary in any direction. The generalised relation is found to be

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j}.$$  \hspace{1cm} (2.21)

Here, $\sigma_{ij}$ are the elements of the solid stress tensor $\mathbf{T}$.

2.1.7. Bulk Viscosity
Solids
The bulk modulus was already introduced in equation 2.9. For solids, no further discussion is needed, and the equation is presented in a slightly different fashion below for clarity:

$$B = -V \frac{dP}{dV}.$$  \hspace{1cm} (2.22)

Also, both “viscosity” parameters, being the two Lamé constants have been introduced before, in section 2.1.5.

Liquids
Having found equation 2.15, the fluid bulk viscosity can be derived. As there were two parameters constituting proportionality constants in the stress tensor for solids (see section 2.1.5), there will in fact be two viscosity constants for fluid as well. One of these, the dynamic viscosity, has been introduced in equation 2.14. However, when compressibility comes into play, another viscosity parameter, comparable to the first Lamé constant $\lambda$ is introduced. The fluid bulk viscosity $\kappa$ will be derived in a similar way as the bulk modulus $B$ was found. However, not the mere strain, but the strain rate will be plugged in, such that:

$$\kappa = -V \frac{dP_{\delta t}}{dV}.$$  \hspace{1cm} (2.23)

Now, in a similar fashion as before, the $\frac{-1}{V} \frac{\partial}{\partial t} dV$ term will be converted. Assuming again the volume shown in figure 2.7, it can be found that the initial volume $V_1$, at time $t_1$, and the volume an infinitesimal amount of time $\delta t$ later, $V_{1+\delta t}$, can be expressed as

$$V_1 = \delta x_1 (\delta x_2 \delta x_3)$$
$$V_{1+\delta t} = (\delta x_2 \delta x_3) \left( \delta x_1 + (v(x_1 + \delta x_1, t_1) - v(x_1, t_1)) \delta t \right),$$
such that the change in volume can be found with the following equation, where the limit of \( \lim_{\Delta x \to 0} \) is taken:

\[
\delta V = (\delta x_1 \delta x_2 \delta x_3) \left( \frac{\partial v}{\partial x_1} \delta t \right).
\]

Then, taking \( \delta t \) and \( (\delta x_2 \delta x_3) = \delta V \) to the other side, and taking the limit \( \lim_{\delta t \to 0} \), the relation

\[
1 \frac{\partial V}{\partial t} = \frac{\partial v}{\partial x_1}
\]

is found. Note that a change has been made, where the original \( \delta V \) over a change in time, has now become \( \partial V \) as \( \delta t \) has gone to zero. Subsequently, the volume fraction \( (\delta x_1, \delta x_2, \delta x_3) \) has been named \( \delta V \), where this \( \delta \) does not constitute a change over time, but rather over space. Assuming a constant gradient in velocity \( v \), the relation can be expanded for any volume \( V \), and the earlier obtained equation 2.23 for the fluid bulk modulus can be rewritten as

\[
\kappa = \frac{d P_{tot}}{\partial v/\partial \hat{x}}.
\]

Now, assume a fluid under normal pressure, such that the stress will be equal to the pressure only, indicated by \( P_1 \). Then later, an irrotational velocity field is introduced, where the \( x_2 \) dimension is omitted to save some algebraic (for the experiments performed for this thesis, one dimension will not be of interest, see section 2.2.3). The total pressure with the velocity field is indicated by \( P_2 \). Then, by making use of equation 2.15, it can be found that:

\[
\begin{align*}
P_1 &= \sigma_{11} + \sigma_{22} + \sigma_{33} = -(P_{x_1} + P_{x_2} + P_{x_3}) = -3p \\
P_2 &= \sigma_{11} + \sigma_{22} + \sigma_{33} = -3p + \lambda(3\epsilon_{11} + 3\epsilon_{33}) + 2\eta(\epsilon_{11} + \epsilon_{33}).
\end{align*}
\]

However, the earlier used single viscosity parameter \( \eta \) has now been further developed to not only reflect dynamic viscosity, but also reflect the first viscosity constant \( \lambda \). Using equation 2.8, the \( \epsilon_{ij} \) can be filled in. For example, the first \( \epsilon \) can be found to be

\[
\epsilon_{11} = \frac{1}{2} \left( 2 \frac{\partial v_1}{\partial x_1} - \frac{1}{3} \frac{\partial v_1}{\partial x_1} + \frac{\partial v_3}{\partial x_3} \right).
\]

In a similar way, \( \epsilon_{22} \) and \( \epsilon_{33} \) can be found, such that in the end \( P_2 \) is found to be equal to

\[
P_2 = -3p + \lambda \left( \frac{\partial v_1}{\partial x_1} + \frac{\partial v_3}{\partial x_3} \right) = -\frac{2}{3} \eta \left( \frac{\partial v_1}{\partial x_1} + \frac{\partial v_3}{\partial x_3} \right).
\]

Then, the change in pressure can be plugged in in the formula for \( \kappa \), such that

\[
\kappa = \frac{P_2 - P_1}{\frac{\partial v_1}{\partial x_1} + \frac{\partial v_3}{\partial x_3}}
\]

is found. Then, filling in the expressions for \( P_2 \) and \( P_1 \), \( \kappa \) is found to be equal to:

\[
\kappa = \lambda + \frac{2}{3} \eta.
\]

In this research, the scope will be limited to Stokesian fluids [15]. As such, \( \kappa \) is set to zero. The first and second viscosity parameter can then be related by:

\[
\lambda = -\frac{2}{3} \eta.
\]
Modelling a liquid as a solid

As ultrasonic waves with small displacements will be applied to the materials for this research, no plastic deformations will take place. Only time harmonic motions of the fluid will be dealt with, as for the solid. However, the solid is assumed to be perfectly linearly elastic, whilst the fluid will dissipate energy due to its viscosity. The advantage of dealing with harmonic oscillations is that, instead of working with rates of strain, the strain can be multiplied by $-i\omega$ [16]. First, the bulk modulus is revisited, which is now given by:

$$B = -\frac{p}{\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}}$$

$$-p = B \left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right) = B (\epsilon_{11} + \epsilon_{33}).$$

Then, the strain induced stress tensor can be further developed by filling in equation 2.18, finding

$$\sigma_{ij} = \left(B + \frac{2}{3}i\omega\right)\epsilon_{kk} \delta_{ij} - 2\eta_i \omega \epsilon_{ij}.$$  \hspace{1cm} (2.26)

Constructing the full matrix, it is found that:

$$\begin{bmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33}
\end{bmatrix} = \begin{bmatrix}
\frac{\lambda + 2\eta}{3} & \lambda & \lambda \\
\lambda & \frac{\lambda + 2\eta}{3} & \lambda \\
\lambda & \lambda & \frac{\lambda + 2\eta}{3}
\end{bmatrix} \begin{bmatrix}
\epsilon_{11} \\
\epsilon_{22} \\
\epsilon_{33}
\end{bmatrix}$$

$$= \begin{bmatrix}
C_{11} & C_{12} & C_{13} \\
C_{12} & C_{11} & C_{13} \\
C_{13} & C_{12} & C_{11}
\end{bmatrix} \begin{bmatrix}
\epsilon_{11} \\
\epsilon_{22} \\
\epsilon_{33}
\end{bmatrix}.$$  \hspace{1cm} (2.27)

$$\begin{bmatrix}
\lambda + 2\eta & \lambda & \lambda \\
\lambda & \frac{\lambda + 2\eta}{3} & \lambda \\
\lambda & \lambda & \frac{\lambda + 2\eta}{3}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{3} & 0 & 0 \\
0 & \frac{1}{3} & 0 \\
0 & 0 & \frac{1}{3}
\end{bmatrix} \begin{bmatrix}
C_{11} \\
C_{12} \\
C_{13}
\end{bmatrix}.$$  \hspace{1cm} (2.28)

$$\begin{bmatrix}
C_{11} & C_{12} & C_{13} \\
C_{12} & C_{11} & C_{13} \\
C_{13} & C_{12} & C_{11}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{3} & 0 & 0 \\
0 & \frac{1}{3} & 0 \\
0 & 0 & \frac{1}{3}
\end{bmatrix} \begin{bmatrix}
\frac{1}{3} & 0 & 0 \\
0 & \frac{1}{3} & 0 \\
0 & 0 & \frac{1}{3}
\end{bmatrix} \begin{bmatrix}
C_{11} \\
C_{12} \\
C_{13}
\end{bmatrix}.$$  \hspace{1cm} (2.29)

It can be observed that $C_{11}$ can be constructed from $C_{12}$ and $C_{44}$, such that in reality only two independent variables are to be dealt with, being:

$$C_{44} = \eta_i \omega$$

$$C_{11} = B + \frac{4}{3} C_{44}.$$ \hspace{1cm} (2.30)

Now, it has become clear that the stiffness constant working on shear strains, $C_{44}$, also appears in the stiffness constant working on normal strains, $C_{11}$. The shear strains bring about the viscous losses in shear waves, and the stiffness constant is the proportionality constant defining the amount of energy lost. Since exactly this proportionality constant is also applied to normal strains, there must be some viscous losses when normal strains are applied. These normal strains are brought about by compression waves, which thus also experience viscous losses.
2.2. Wave Theory

First, the general wave equation for pressure waves in bulk materials will be derived. Continuing, the wave equations for wave guides will be derived. In the experiment, plate waves will be dealt with, the most important plate waves being:

- Lamb waves: asymmetrical  Flexural or transversal mode
- Lamb waves: symmetrical  Longitudinal pressure mode
- Shear waves  In-plane mode

Apart from plate waves, interface (or surface) waves will be dealt with. The most important different surface waves are:

- Stoneley waves  Solid-Solid interface
- Scholte waves  Solid-Liquid interface
- Rayleigh waves  Solid-Vacuum interface
- Love waves  Solid-Vacuum interface - horizontal modes (interference of many shear waves)

Only the Scholte waves will be discussed explicitly. Stoneley waves will not be dealt with in this experiment. The Rayleigh and Love waves in plates are better described by the above-mentioned Lamb waves and shear waves in plates.

2.2.1. Bulk Longitudinal Wave

To find the wave equation, only one more step is needed as the two precedent equations, \ref{2.19} and \ref{2.13}, were already derived. Rearranging these equations, it can be found that

$$\delta v_1 = -\frac{1}{B} \frac{\partial p}{\partial t} \delta x = -\frac{1}{\rho} \frac{\partial p}{\partial x} \delta t = \delta v_1,$$

such that the one dimensional acoustic pressure wave equation can be found to be

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0. \quad (2.31)$$

Here, $\frac{\rho}{B} = \frac{1}{c^2}$ was defined, and where $c$ is the speed of the wave. The equation can be expanded to include viscosity. Looking at equation \ref{2.16}, it can be found that the viscous wave equation is given by

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} + \frac{4}{3} \frac{\eta}{B} \frac{\partial^3 p}{\partial x^2 \partial t} = 0. \quad (2.32)$$

2.2.2. Reflection and Transmission

Before the derivation of plate waves is introduced, a short discussion of the behaviour of acoustic waves at boundaries will be discussed. The discussion is more clear in the Fourier domain, and is limited to the non-viscous wave equation given in equation \ref{2.31}. Noting that the Fourier transform of time derivative is easily found:

$$\mathcal{F} \left\{ \frac{dp(x, t)}{dt} \right\} = i\omega \hat{p}(x, \omega).$$

The Helmholtz equation is found to be

$$\nabla^2 \hat{p}(x, \omega) + \omega^2 \hat{p}(x, \omega) = 0. \quad (2.33)$$

Plane waves - 1D

In 1D, the general solutions to equation \ref{2.33} are

$$\hat{p}(x, \omega) = \sum_k R_k e^{-ikx}. \quad (2.34)$$
where $k$ is the wavevector, or

$$|k| = \frac{\omega}{c}.$$ 

Now, the velocity can be calculated by inputting the solution for $\hat{p}$ in the Fourier transform of Newton’s second law:

$$-\nabla \hat{p}(r, \omega) = i\omega \rho \hat{v}(r, \omega).$$

(2.35)

Filling in the general solutions to equation 2.33, the velocity is

$$\hat{v}(r, \omega) = \frac{1}{\rho c} \sum_k \frac{k}{|k|} F_k e^{-i k \cdot r}.$$ 

This equation can be rewritten as

$$\hat{p}(r, \omega) - \frac{1}{\rho c} \sum_k \frac{k}{|k|} F_k e^{-i k \cdot r} = \hat{Z} \hat{v}(r, \omega).$$

(2.36)

Here, $Z$ is the acoustic impedance, giving the relation between the pressure of the particles and the velocity of the wave. In 1D, the impedance is a real number, meaning that the phase of the wave is constant (which is indeed true for a non-viscous pressure wave/plane wave).

**Plane waves - 3D**

In 3D, the general solution to equation 2.33 is:

$$\hat{p}(r, \omega) = \hat{A}(\omega) e^{-i \frac{r}{c_0} \cdot r} + \hat{B}(\omega) e^{-i \frac{r}{c_0} \cdot r}.$$ 

However, the second term indicates an inward propagating wave. Mathematically, an inward propagating wave is also a solution to the Helmholtz equation, but as this was never physically observed, it can be omitted. The acoustic impedance in 3D can be found by using the same method as before, equating

$$-\nabla \hat{p}(r, \omega) = \hat{A}(\omega) e^{-i \frac{r}{c_0} \cdot r} + \hat{A}(\omega) i \omega e^{-i \frac{r}{c_0} \cdot r} = i\omega \rho \hat{v},$$ 

such that the acoustic impedance is found to be

$$Z = \frac{c \rho}{i \omega r} + 1.$$ 

(2.37)

**Interface conditions**

At an interface, where the wave is transmitted in (and reflected of) a new material with different properties, two conditions [17] can be set up:

Continuity of pressure: $\hat{p}_1 + \hat{p}'_1 = \hat{p}'_2$ \hspace{1cm} (2.38)

Continuity of $v$: $v_1 + v'_{11} = v'_{12}$. \hspace{1cm} (2.39)

Here, with perpendicular, the direction normal to the interface is meant. An example of such an interface is depicted in figure 2.8.

Also, the slowness vector is defined for a wave travelling at velocity $c$:

$$s = \frac{1}{c} (\cos(\theta), \sin(\theta)).$$ \hspace{1cm} (2.40)

The interface behaviour for the 1D wave will be investigated. The Fourier transform of $p(r, \omega)$ is then be given by:

$$\hat{p}(r, \omega) = \hat{A}(\omega) e^{-i \omega \cdot r}.$$
2.2 Wave Theory

Consider this wave to be travelling in direction \( s_{1} \), of which the angle of incidence on material two is \( \theta_{1} \). The wave is reflected at angle \( \theta'_{1} \) and transmitted at angle \( \theta'_{2} \). The following expressions for \( \hat{p} \) can then be found:

\[
\hat{p}_{1}(r, \omega) = \hat{A}_{1}(\omega) e^{-i \frac{\omega}{c_{1}} (x_{1} \cos(\theta_{1}) + x_{3} \sin(\theta_{1}))} \\
\hat{p}'_{1}(r, \omega) = \hat{A}'_{1}(\omega) e^{-i \frac{\omega}{c_{1}} (x_{1} \cos(\theta'_{1}) + x_{3} \sin(\theta'_{1}))} \\
\hat{p}''_{2}(r, \omega) = \hat{A}''_{2}(\omega) e^{-i \frac{\omega}{c_{2}} (x_{1} \cos(\theta'_{2}) + x_{3} \sin(\theta'_{2}))}.
\]

**Continuity of pressure**

Now, if equation 2.38 is applied at \( x_{1} = 0 \), the following relation for a wave hitting on a surface can be found:

\[
\hat{A}_{1}(\omega) e^{-i \frac{\omega}{c_{1}} (x_{3} \sin(\theta_{1}))} + \hat{A}'_{1}(\omega) e^{-i \frac{\omega}{c_{1}} (x_{3} \sin(\theta'_{1}))} = \hat{A}''_{2}(\omega) e^{-i \frac{\omega}{c_{2}} (x_{3} \sin(\theta'_{2}))}.
\]

As continuity for all \( \omega \) and \( x \) is needed, where it is known that

\[
\frac{\sin(\theta_{1})}{c_{1}} = \frac{\sin(\theta'_{1})}{c_{1}} = \frac{\sin(\theta'_{2})}{c_{2}}
\]

(2.41)

from Snell’s law. The interface condition now reduces to

\[
\hat{A}_{1}(\omega) + \hat{A}'_{1}(\omega) = \hat{A}''_{2}(\omega),
\]

which, after rearranging, such that it would be of the form \( 1 + R = T \), can be expressed as

\[
R = \frac{\hat{A}_{1}(\omega)}{\hat{A}'_{1}(\omega)}
\]

(2.42)

\[
T = \frac{\hat{A}''_{2}(\omega)}{\hat{A}_{1}(\omega)}
\]

(2.43)

**Continuity of perpendicular speed**

Equation 2.39 at \( x_{1} = 0 \) yields:

\[
\hat{v}_{1} \cos(\theta_{1}) - \hat{v}'_{1} \cos(\theta'_{1}) = \hat{v}''_{2} \cos(\theta'_{2}).
\]

For a plane wave, it was earlier found that \( \hat{v} = \frac{1}{\rho c} \hat{p} \) and \( \theta_{1} = \theta'_{1} \), thus finding:

\[
\frac{\hat{A}_{1}}{\rho_{01} c_{1}} \cos(\theta_{1}) - \frac{\hat{A}'_{1}}{\rho_{01} c_{1}} \cos(\theta'_{1}) = \frac{\hat{A}''_{2}}{\rho_{02} c_{2}} \cos(\theta'_{2})
\]

\[
\frac{\cos(\theta_{1})}{\rho_{01} c_{1}} \left( 1 - \frac{\hat{A}'_{1}}{\hat{A}_{1}} \right) = \frac{\cos(\theta'_{2})}{\rho_{02} c_{2}} \left( \frac{\hat{A}''_{2}}{\hat{A}_{1}} \right).
\]
Combining these equation with the earlier defined relation of $1 + R = T$, the following relations are finally arrived upon:

\[
R = \frac{Z_2 \cos(\theta_1) - Z_1 \cos(\theta_2)}{Z_2 \cos(\theta_1) + Z_1 \cos(\theta_2)} \quad (2.44)
\]
\[
T = \frac{2Z_2 \cos(\theta_1)}{Z_2 \cos(\theta_1) + Z_1 \cos(\theta_2)} \quad (2.45)
\]

Now, to get a feeling for the equations above, it is interesting to plot $R$ for different angles of incidence, depicted in figure 2.9. Also, two different values for $Z$ will be included. In one case, the red line, $Z_1 \approx Z_2$ is taken. The reflection coefficient is relatively low in this case, and also it is clear that if the wave is incident under an angle, the reflection coefficient does not change much. However, when we $Z_1 \gg Z_2$ is plugged in, the result in blue is obtained. Not only is the reflection coefficient greater than before (i.e. the transmission coefficient will be lower), also the effect of an a wave entering at an angle will increase the reflection coefficient more greatly.

![Reflection coefficient as a function of angle $\theta$](image)

### 2.2.3. Lamb waves

Now, waves in solid materials will dealt with. Although the wave equation will look similar, there is one important difference in the derivation. Whereas in fluids the wave energy is dissipated through the viscosity, solids will be approximated as being perfectly linearly elastic materials. The wave equation will be re-derived. Then, the solutions will be split into a symmetric part and an asymmetric part, these solutions will govern the wave modes in a plate.

Also, other propagation parameters need to be reconsidered. Not necessarily because a solid is dealt with, but because a guided wave instead of a bulk wave is discussed. While the wave equation might remain, the propagation parameters will differ, as is elaborated in table 2.1.

<table>
<thead>
<tr>
<th></th>
<th>Bulk</th>
<th>Guided</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase Velocities</td>
<td>Constant</td>
<td>Function of frequency</td>
</tr>
<tr>
<td>Group Velocities</td>
<td>Same as phase velocities</td>
<td>Generally not equal to phase velocity (Generally) Dispersive</td>
</tr>
<tr>
<td>Pulse shape</td>
<td>Nondispersive</td>
<td></td>
</tr>
</tbody>
</table>

### Plate wave equation

A start is made with the equation of motion as in formula 2.21 and equation 2.18 defining the stress induced by the strain:
2.2. Wave Theory

\[ \rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j} \]

\[ T = \lambda \begin{bmatrix} e_{11} & 0 & 0 \\ 0 & e_{22} & 0 \\ 0 & 0 & e_{33} \end{bmatrix} + 2\mu \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{12} & e_{22} & e_{23} \\ e_{13} & e_{23} & e_{33} \end{bmatrix} \]

\[ = \lambda \frac{\partial u_k}{\partial x_k} \delta_{ij} + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right). \]

Now, the second equation can be filled in the first. Changing the index \( k \) to \( j \), the relation

\[ \rho \frac{\partial^2 u_i}{\partial t^2} = \lambda \frac{\partial^2 u_k}{\partial x_k \partial x_j} \delta_{ij} + \mu \left( \frac{\partial^2 u_i}{\partial x_j^2} + \frac{\partial^2 u_i}{\partial x_j \partial x_l} \right) \]

\[ = \lambda \frac{\partial^2 u_j}{\partial x_j \partial x_j} + \mu \left( \frac{\partial^2 u_i}{\partial x_j^2} + \frac{\partial^2 u_i}{\partial x_j \partial x_i} \right) \]

is found. Then, the Navier-Cauchy formulation can be found, where

\[ \rho \frac{\partial^2 u_i}{\partial t^2} = \mu \frac{\partial^2 u_i}{\partial x_j^2} + (\lambda + \mu) \frac{\partial^2 u_j}{\partial x_j \partial x_j}. \] (2.46)

Now, to continue, the displacement field \( u \) will be decomposed into a scalar field \( \phi \) and a vector field \( H \) by means of a Helmholtz decomposition of the form

\[ u = \nabla \phi - \nabla \times H. \] (2.47)

Filling this in in equation 2.46, the new expression

\[ \rho \frac{\partial^2}{\partial t^2} \left( \nabla \phi - \nabla \times H \right) = \mu \nabla^2 \left( \nabla \phi - \nabla \times H \right) + (\lambda + \mu) \nabla \left( \nabla \cdot (\nabla \phi - \nabla \times H) \right) \]

is found. The first part of the right hand side, by noting that the Laplacian is a scalar operator, and thus can be moved around freely, can be rearranged, finding that

\[ \nabla^2 \left( \nabla \phi \right) - \nabla^2 \left( \nabla \times H \right) = \nabla \left( \nabla^2 \phi \right) + \nabla \times \left( \nabla^2 H \right). \]

In the second part of the equation, note that the gradient of a curl is zero, so it is found that

\[ \nabla \left( \nabla \cdot (\nabla \phi) \right) - \nabla \left( \nabla \cdot (\nabla \times H) \right) = \nabla \left( \nabla^2 \phi \right), \]

finally arriving at:

\[ \nabla \left( \rho \frac{\partial^2 \phi}{\partial t^2} - (\lambda + 2\mu) \nabla^2 \phi \right) - \nabla \times \left( \rho \frac{\partial^2 H}{\partial t^2} - \mu \nabla^2 H \right) = 0. \]

Noting that \( \phi \) describes a purely divergent motion and \( H \) a purely rotational motion, the previous equation can be split up into two independent equations. The divergent motion will describe a longitudinal wave and the rotational motion will describe a shear wave [17]:

\[ \frac{\partial^2 \phi}{\partial t^2} = c_1^2 \nabla^2 \phi \] (2.48)

\[ \frac{\partial^2 H}{\partial t^2} = c_2^2 \nabla^2 H, \] (2.49)
are the longitudinal and the shear (rotational) velocities of the waves respectively. The general solution to these equations can be found to be

\[ \phi, H = A \phi, H(x, t) \exp(i k_x x - \omega t) \]  

(2.52)

In general, \( k \) is the complex valued wavenumber vector, or

\[ k = n \cdot k_{re} + ib \cdot k_{im}. \]

Here, \( n \) and \( b \) are the unit vectors defining the directions of the real and imaginary part of vector \( k \). Now, if this expression for \( k \) is plugged in equation 2.52, the expression

\[ \phi = A \phi_j(x, t) \exp(i (n \cdot k_{re} x - \omega t)) e^{-b \cdot k_{im} x}. \]  

(2.54)

is found. Now, it can clearly be observed that the real part of the wavenumber is the propagating term of the wave; it does not die out with \( x \) but oscillates. The phase velocity of the wave is given by

\[ c_{ph} = \frac{\omega}{|k_{re}|}. \]  

(2.55)

Also, it is evident that the imaginary part is a purely dampening term, adding an exponential decay. There are a few different possible values for \( k_{im} \), defining the propagation of different waves. For example, if \( k_{im} = 0 \), we are dealing with a homogeneous wave. If \( k_{im} \neq 0 \), but \( k_{im} \cdot n = 0 \), an inhomogeneous wave which is attenuated in the direction perpendicular to the propagation direction is obtained. For example, in section 3.1.3, the quasi-Scholte wave will be discussed. Much of the energy of this wave propagates in the surrounding fluid of the plate, and the amount of energy that propagates in the fluid decays exponentially as one moves away from the plate. Finally, there is the inhomogeneous case where \( k_{im} \cdot n \neq 0 \), describing waves that decay exponentially in the direction of the propagation of the plate wave. Summarising, the following values for the wavenumber can be distinguished:

- \( k_{im} = 0 \): Homogeneous
- \( k_{im} \neq 0, k_{im} \cdot n = 0 \): Inhomogeneous; Scholte or Rayleigh waves
- \( k_{im} \cdot n \neq 0 \): Inhomogeneous; Leaky-Lamb waves

Also, the complex bulk velocities can be found as following [16]:

\[ c_{bulk} = \frac{\omega}{k}. \]  

(2.56)

Now, the parameter describing the attenuation will be introduced, \( \kappa \), in Nepers per wavelength. This is an attenuation of \( e^{-\kappa} \) that a wave experiences per wavelength travelled. The following relations are provided:
2.2. Wave Theory

\[ e^{-\kappa} = e^{-|k_{lm}|d} \]
\[ \kappa = |k_{lm}| \lambda \]
\[ \lambda = \frac{2\pi}{|k_{Re}|} \]
\[ \frac{\kappa}{2\pi} = \frac{|k_{lm}|}{|k_{Re}|} \]

When the attenuation direction and the propagation direction are parallel, the following relations for \( c_{\text{bulk}} \) are found,

\[ c_{\text{bulk}} = \frac{\omega}{|k_{Re}| + i|k_{lm}|} = \frac{\omega}{k} \]

(2.57)

and

\[ c_{\text{bulk}} = \frac{|k_{lm}|}{1 + i \frac{k}{2\pi}} \]

(2.58)

Plane Strain

Before continuing, the plane strain assumption will be addressed. In a state of plane strain, all strain components act only in one plane. Thus in 3D, one dimension can be neglected in the analysis. This is depicted in figure 2.10.

![Figure 2.10: In this figure, the concept of plane strain is graphically made clear. As the plate is relatively large in the \( x_3 \) direction, and the propagation direction of the plate wave is perpendicular to \( x_2 \), plane strain conditions may be applied.](image)

This simplifies the calculation. The strain tensor can, for the geometry of figure 2.10, now be written as:

\[
\begin{bmatrix}
\varepsilon_{11}^s & \varepsilon_{12}^s & \varepsilon_{13}^s \\
\varepsilon_{21}^s & \varepsilon_{22}^s & \varepsilon_{23}^s \\
\varepsilon_{31}^s & \varepsilon_{32}^s & \varepsilon_{33}^s 
\end{bmatrix} =
\begin{bmatrix}
\varepsilon_{11}^s & \varepsilon_{13}^s \\
\varepsilon_{31}^s & \varepsilon_{33}^s \\
\end{bmatrix}.
\]

(2.59)

Recall that the Helmholtz decomposition (as in equation 2.47) has resulted in one curl free scalar field and one divergence free vector field. To make this more intuitive, some of the fields that might exist in the plate can be depicted, see figure 2.11. It is apparent that in the upper figure, the curl is most prominent (the arrows indicate a divergence free field), and in the lower figure, the divergence is most prominent (the arrows indicate a curl free field). As such, the displacement fields can be found to be

\[
\begin{bmatrix}
u_1 \\
u_2 \\
u_3
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial \phi}{\partial x_1} & \frac{\partial H}{\partial x_1} & \frac{\partial \phi}{\partial x_2} \\
\frac{\partial \phi}{\partial x_2} & \frac{\partial \phi}{\partial x_3} & \frac{\partial H}{\partial x_2} \\
\frac{\partial \phi}{\partial x_3} & \frac{\partial \phi}{\partial x_1} & \frac{\partial H}{\partial x_3}
\end{bmatrix} +
\begin{bmatrix}
\frac{\partial H}{\partial x_1} & -\frac{\partial \phi}{\partial x_1} & \frac{\partial \phi}{\partial x_2} \\
\frac{\partial \phi}{\partial x_2} & \frac{\partial \phi}{\partial x_3} & \frac{\partial \phi}{\partial x_1} \\
\frac{\partial \phi}{\partial x_3} & \frac{\partial \phi}{\partial x_1} & \frac{\partial \phi}{\partial x_2}
\end{bmatrix}
\]

(2.60)
Because of the plane strain condition, the following holds:

\[
\begin{align*}
    u_1 &= \frac{\partial \phi}{\partial x_1} + \frac{\partial H}{\partial x_3} \\
    u_2 &= 0 \\
    u_3 &= \frac{\partial \phi}{\partial x_3} - \frac{\partial H}{\partial x_1}.
\end{align*}
\]

(2.61)

To continue the search for the \(u_i\)'s, the expressions for \(\phi\) and \(H\) must be further developed. Previously, there were found as in the form of equation 2.52. Now, the geometry will be defined such that the wave will be propagating in the \(x_1\) direction. Due to the plane strain condition, the constants \(A_{\phi,H}\) can depend only on \(x_3\). Then, also recalling equation 2.48, the relation

\[
\phi = A_\phi(x_3)e^{i(k_1x_1-\omega t)}
\]

\[
\frac{\partial^2 \phi}{\partial \tau^2} = c_1^2 \left( \frac{\partial^2 \phi}{\partial x_1^2} + \frac{\partial^2 \phi}{\partial x_3^2} \right)
\]

can be constituted. Filling in, and taking out the common term, yields that

\[
-\omega^2 A_\phi(x_3)e^{i(k_1x_1-\omega t)} = c_1^2 \left( -k_1^2 e^{i(k_1x_1-\omega t)} + \frac{\partial^2 A_\phi(x_3)}{\partial x_3^2} e^{i(k_1x_1-\omega t)} \right).
\]

The same method can be applied for \(H\). Two new variables are introduced:

\[
p = \frac{\omega^2}{c_1^2} - k_1^2
\]
\[
q = \frac{\omega^2}{c_2^2} - k_2^2,
\]

such that

\[
A_\phi(x_3)(-p) = \frac{\partial^2 A_\phi}{\partial x_3^2}
\]
\[
A_H(x_3)(-q) = \frac{\partial^2 A_H}{\partial x_3^2}.
\]
The solutions are found to be

\[
A_\phi(x_3) = A_{\phi_1}(x_3) \sin(p x_3) + A_{\phi_2}(x_3) \cos(p x_3)
\]
\[
A_H(x_3) = A_{H_1}(x_3) \sin(q x_3) + A_{H_2}(x_3) \cos(q x_3).
\]

**Splitting the solutions**

Then finally, the equation can be split into two modes as is done by Rose [17], and the following modes are found:

**Symmetric modes**

\[
A_\phi = A_{\phi_2} \cos(p x_3)
A_H = A_{H_1} \sin(q x_3)
\]
\[
u_1 = i k A_{\phi_2} \cos(p x_3) + q A_{H_1} \cos(q x_3)
\]
\[
u_3 = -p A_{\phi_2} \sin(p x_3) - i k A_{H_1} \sin(q x_3)
\]

**Antisymmetric modes**

\[
A_\phi = A_{\phi_1} \sin(p x_3)
A_H = A_{H_2} \cos(q x_3)
\]
\[
u_1 = i k A_{\phi_1} \sin(p x_3) - q A_{H_2} \sin(q x_3)
\]
\[
u_3 = p A_{\phi_1} \cos(p x_3) - i k A_{H_2} \cos(q x_3)
\]

These are the Lamb wave modes. They are the waves that move in the plane consisting of the wave propagation vector and the vector normal to the plate. These waves can be symmetrical (\(S_1\) modes) about the median plane of the plate. The waves stretch and compress the plate in the wave direction, effectively (though slightly) compressing and stretching the plate in the perpendicular direction. That is why this mode is called the extensional mode. This mode is also known as the longitudinal mode, describing a pressure wave-like motion. Or they can be anti-symmetrical (\(A_1\) modes), the flexural mode or transversal mode. Lamb waves stand out from sinusoidal wave modes in unbounded media in the fact that they are described by two entire, infinite, families of wave modes. In infinite media (in every direction), there are just two wave modes, the bulk longitudinal and the bulk transverse wave. These modes are depicted in figures 2.12 and 2.13 respectively.

Figure 2.12: \(S_0\) mode and the exaggerated displacement. Note the symmetry around the \(x_1\),\(x_2\) plane. The figure was obtained from a simulation result in COMSOL®.

Figure 2.13: \(A_0\) mode and the exaggerated displacement. Note the symmetry around the \(x_1\),\(x_2\) plane. The figure was obtained from a simulation result in COMSOL®.
2.2.4. Viscoelastic Waves
As stated in section 2.1.7, the fluid is modelled as a solid. Now, if the correct stiffness parameter found in equation 2.30 is inputted in the wave velocity relations 2.50, and noting the relation between the complex wavevector and the bulk velocity in equation 2.57, it can found that

\[ \frac{\omega}{k^l} = c^l_{\text{bulk}} = \sqrt{\frac{\lambda + 2\mu}{\rho}} = \sqrt{\frac{C_{11}}{\rho}} = \sqrt{\frac{B - \frac{s}{2} i \omega \eta}{\rho}} \] (2.66)

\[ \frac{\omega}{k^s} = c^s_{\text{bulk}} = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{C_{44}}{\rho}} = \sqrt{\frac{-i \omega \eta}{\rho}}. \] (2.67)

Then, starting of with the longitudinal components, the angular frequency can be found as

\[ \omega = k^l \sqrt{-\frac{\eta \omega}{ip}} \]

\[ \omega^2 = \left( (k^l_{Re})^2 + 2ik^l_{Im} k^l_{Re} - (k^l_{Im})^2 \right) \frac{-\eta \omega}{ip}. \]

Looking at the real parts first, so

\[ \text{Re}\{\omega^2\} = \left( (k^l_{Re})^2 - (k^l_{Im})^2 \right) \frac{B}{\rho} + 2k^l_{Im} k^l_{Re} \frac{4 \omega \eta}{3 \rho}, \]

the following expression for the longitudinal phase velocity can be found:

\[ \text{Re}\{\omega\} = c^l_{\text{ph}} = \sqrt{1 - \left( \frac{k^l_{Im}}{k^l_{Re}} \right)^2} \frac{B}{\rho} + 2 \frac{k^l_{Im}}{k^l_{Re}} \frac{4 \omega \eta}{3 \rho} \]

\[ = \sqrt{1 - \left( \frac{k}{2\pi} \right)^2} \frac{B}{\rho} + 2 \frac{k}{2\pi} \frac{4 \omega \eta}{3 \rho}. \]

Where in fact, if the attenuation coefficient is small, this equation can be simplified considerably [16]. As for this research, only fluids with low viscosities will be dealt with, the equation may be simplified indeed, finding that

\[ (c^l_{ph})^2 = \frac{B}{\rho}. \] (2.68)

The imaginary part of the angular frequency remains untouched, and will be equated to zero, such that

\[ \text{Im}\{\omega\} = 0 = 2ik^s_{Im} k^s_{Re} \frac{B}{\rho} - \left( (k^s_{Re})^2 - (k^s_{Im})^2 \right) \frac{4 \omega \eta}{3 \rho}. \]

Then, the ratio between the imaginary and real parts of the wavenumber can be found and rearranged

\[ \frac{k^s_{Im}}{k^s_{Re}} = \left( 1 - \left( \frac{k^s_{Re}}{k^s_{Im}} \right)^2 \right) \frac{2\omega \eta}{3B} \]

\[ 0 = \frac{2\omega \eta}{3B} \left( \frac{k^s_{Im}}{k^s_{Re}} \right)^2 + \frac{k^s_{Im}}{k^s_{Re}} - \frac{2\omega \eta}{3B}. \]
Solving, and keeping only the positive root (the physical solution), yields:

\[
\frac{k_s^s}{k_{Re}^s} = \frac{-1}{2} \frac{3B}{2\omega \eta} + \sqrt{1 + \left(\frac{4 \omega \eta}{3 B}\right)^2 \frac{3}{4 \omega \eta}}
\]

\[
\kappa^I = \frac{3B \pi}{2\omega \eta} \left(1 + \left(\frac{4 \omega \eta}{3 B}\right)^2 - 1\right).
\]

Now, the factor \(x = \frac{4 \omega \eta}{3 B}\) will be small, and using a Taylor series (just the first two terms), the following expression for the attenuation per wavelength is found:

\[
\kappa^I \approx \frac{2\pi}{x} \left(1 + \frac{x^2}{2} - 1\right) \approx \frac{4\pi \omega \eta}{3B},
\]

or simpler

\[
\kappa^I \approx \frac{4\pi \omega \eta \rho}{3\left(c_{ph}^I\right)^2}.
\]

The same derivation can be followed for shear waves, finding that

\[
k_s^s = \sqrt{-\frac{\rho \omega}{i \eta}}
\]

\[
(k_s^s)^2 = (k_{Re}^s)^2 + 2ik_{Im}^s k_{Re}^s - (k_{Im}^s)^2 = \frac{i \rho \omega}{\eta},
\]

and that,

\[
\text{Re}((k_s^s)^2) = 0 \rightarrow |k_s^s| = |k_{Im}^s|
\]

\[
\text{Im}((k_s^s)^2) = 2k_{Im}^s k_s^s = \frac{\rho \omega}{\eta} \rightarrow |k_s^s| = \sqrt{\frac{\rho \omega}{2\eta}}.
\]

So that finally, one finds:

\[
c_{ph}^s = \frac{\omega}{|k_{Re}^s|} = \sqrt{\frac{2\eta \omega}{\rho}}
\]

\[
\kappa = 2\pi \frac{|k_{Re}^s|}{|k_{Im}^s|} = 2\pi
\]

**2.2.5. Leaky Lamb Wave**

Lamb waves generally exist on solid-vacuum interfaces, but can also propagate along a solid-fluid interface, as depicted in figure 2.14 from [18]. In this figure, the solid is located in the positive \(y\) direction, and the liquid in the negative \(y\) direction. However, when propagating along a solid-fluid interface, the Lamb wave propagates highly attenuated. In this case, the wave energy is not merely attenuated due to the viscosity constants. The wave will leak away along the propagation direction. In the solid, the wave energy will also attenuate exponentially normal to the interface. In the liquid, if one looks along the propagation direction of the longitudinal wave that is leaking away from the Lamb wave, the amplitude stays constant [18], perfectly so if it concerns a non-viscous fluid. The fact however, that the wave leaking away from the waveguide has an imaginary term \(k_{Im}^s\) perpendicular to the guided wave propagation direction \(x_s\), indicates the leakage of the wave.

The phase velocity \(c_{ph}\) of the leaky Lamb wave is similar to that of the Lamb wave [18]. However, as the Lamb wave generally has a wave velocity higher than that of the velocity of sound of the surrounding
fluid, the waves are not well supported and therefore attenuate strongly. In the figure, Figure 2.14, the
difference in velocity can be observed. The propagation direction of the wave in the surrounding fluid
changes as the velocity cannot be supported. This effect can be compared to a sonic boom.

2.2.6. (Quasi-)Scholte Waves
The derivation for the matrices needed for theoretical modelling of the quasi-Scholte wave is described
by Rose [17]. The derivation relies on the partial wave technique, where the waves in the plate are
modelled as a set of bulk transversal and bulk longitudinal waves. By inputting the correct boundary
conditions, the waves can be modelled for plates, where the waves are constantly reflected and trans-
mitted. However, neither the theoretical model nor the resulting characteristic equation yield any good
insights in the wave behaviour, and are therefore not discussed in this report.

The implications are however, important for the research. First, the difference between the Scholte
and quasi-Scholte wave will be explained. In many dispersion diagrams, the "frequency-thickness" is
used on the x-axis. This gives more information, because the thickness of the plate is only relative
to the wavelength, and thus the frequency. For higher frequencies, the wavelength becomes smaller,
which means the plate becomes relatively thick. Real Scholte waves are defined as being surface
waves, on plates one thus speaks of quasi-Scholte waves. In the limit of the frequency going to infin-
ity, the relative plate thickness will go to infinity as well. The quasi-Scholte will then become Scholte
waves. The same applies to the quasi-Rayleigh waves, becoming Rayleigh waves if the frequency goes
to infinity.

Also, one must differentiate between Scholte waves and the leaky Lamb wave (also sometimes
called the leaky Rayleigh waves). Both waves are well described in [18]. The Scholte waves are waves
that can only exist on a liquid-solid interface, a Scholte wave is depicted in figure 2.15. The fluid is
located in the negative y direction, and the solid in the positive y direction. If the liquid is non-viscous,
there is no attenuation along the propagation direction. However, the wave amplitude attenuates ex-
ponentially away from the interface (so it attenuates perpendicularly to the propagation direction).

The velocity of the quasi-Scholte wave is similar to, but slightly lower than, the velocity of sound of
the surrounding fluid [19]. In the figure, it can be observed that indeed, the propagation direction of the
fluid bulk longitudinal wave is parallel to that of the guided wave in the solid. As such, $k_{\text{lin}}$ will have no
component perpendicular the propagation direction of the guided wave, and the wave will not leak away.
Note that $k_{\text{re}}$ has a relatively strong component in the perpendicular to the propagation direction of the
guided wave, although this component will not influence its attenuation in the propagation direction.
The attenuation in the propagation direction will only be brought about by the viscosity constants, as
shown in equation 3.1.
2.2. Wave Theory

2.2.7. Wave Velocity

The velocity of the different plate waves can be theoretically found. Disperse was used to obtain these curves, and the curves for a tungsten plate are shown in figure 2.16. These curves are also referred to as dispersion curves. Both the quasi-Scholte wave, and the flexural A0 mode, exhibit dispersion [17]. In fact, the symmetric mode also shows dispersion, although it is harder to distinguish in figure 2.16.

Dispersion means that the propagation velocity depends on the wavelength. When the wavelength is very small (compared to the thickness of the plate), more waves modes will fit in the plate and depending on the measurement resolution different modes will melt together. Depending on the frequency spectrum of the wave, the wave can be more dispersive for a wide range of frequencies or can retain its shape for a more sharply peaked amplitude around a center frequency.

A frequency should be chosen such that the different modes do not melt together but also such that the attenuation is prominent enough. Generally, frequencies of the order 1 MHz - 3 MHz are used. Only the zeroth order waves do not have a cutoff frequency. It separates them from the other modes as they are the only modes that can exist for any frequency. Also, in general, they carry more energy than higher-order modes [17].
In this chapter, the experimental setup will be introduced. First, the physical setup will be discussed, and then the simulations. The physical setup together with the theory can be used to benchmark the simulations. The simulations, in turn, can be used to test different setups without having to buy the materials or measurement devices, and can help in the error estimation. After these two sections, the data analysis will be discussed. For both the physical setup and the simulations, the data analysis is equal. The analysis does however, depend on other parameters such as dispersion of the wave and increases in temperature.

3.1. Physical Setup
First, the design of the setup will be discussed. Then, the distinction between two wave excitation methods will be introduced. After that, the theoretical topics will be discussed with an eye on the physical setup.

3.1.1. Design
In this thesis, the focus is on the flexural modes, although the setup is designed such that both flexural and shear modes can be excited (separately and concurrently). Both modes are excited through a traction force working on the plane of the thin side of the plate. In the plate itself, this surface force will continue as an elastic force between particles.

First, an ultrasonic wave is generated by a transducer. The transducer converts the electrical signal into a mechanical signal, a displacement wave, which is transferred to the plate. A schematic drawing of the setup, see figure 3.1a, shows how this is done. The transducer at the top is clamped to the plate right below. Note how the axes have changed, this is the result of the wave propagating in the $x_1$ direction. For the flexural wave, the displacements will be in the $x_1x_2$ plane, analogous to the explanations in section 2.2.3. The shear wave has its displacements in the $x_1x_3$ plane. By turning the transducer around the $x_1$ axis, the polarisation of the waves can be chosen.

A shear transducer with a peak frequency around 5 MHz and a diameter of 13 mm was used. The signal sent out by the transducer is discussed in section 3.1.2. After transmission to the plate, when the sent wave moves along the $x_1$ direction, it should be noted that this path should be clear of obstructions which can cause scattering. As such, the clamps (indicated by “Clamping” in figure 3.1a) are put at some distance $\Delta x_2$ from the middle of the plate to prevent interference. It was confirmed by putting pressure at the edges and the center of the plate respectively, that the wave indeed propagated along the center line of the plate along the $x_1$ direction, as in the latter case no change in amplitude was observed and in the former the reflected wave was completely attenuated.

A more detailed drawing of the setup can be observed in figure 3.1b. To install the transducer, pressure is first applied downwards on top of the transducer, whereafter it is clamped on the sides and the pressure on the top is released. To enhance traction between the transducer and the plate, a shear couplant was used.

The immersion depth of the plate in the fluid can be altered by moving the fluid container, resting on an elevator that can be steered with a wheel. A ruler can be used to gauge insertion depth, although the number of turns of the wheel is more accurate. The wheel has six spokes, and five full turns (i.e.
3.1. Physical Setup

(a) Schematic setup, with the axes defined in the top left corner
(b) Technical drawing of setup with ruler and wheel

Figure 3.1: In this figure, the schematic and technical drawings of the setup can be found. The transducer is clamped on the top.

30 spokes) of the wheel correspond to a 1 cm difference in insertion depth. As such, steps of less than 0.3 mm insertion depth can easily be differentiated. The correct fluid height was one of the biggest sources of error for Cegla (see [12]), moving the elevator by counting the stokes greatly reduces the error. However, it must be noted that hysteresis was observed, where the fluid could differ in height by approximately 0.6 cm depending on whether the plate was inserted in or extracted from the water.

It is the aim to measure the material properties of corrosive salts at temperatures of up to 1200 °C. Therefore, the transducer must be at some distance from the to be measured salt. The transducer will send an \( A_0 \) wave through the waveguide, and upon reaching the fluid most of the energy is transmitted. In the liquid, the \( A_0 \) mode is lost quickly by leakage in the liquid; only the Scholte mode will remain.

To ensure low temperatures at the transducer, the plate will need to be sufficiently long. The actual length was not researched into depth, as it depends on the final design. For example, the plate thickness influences the heat transfer strongly. However, the preliminary results for some finite element simulations indicated that the plate lengths needed would be acceptable, with an eye on dispersion (refer to section 2.2.7) and manageability of the setup.

Dimensions
The most important dimensions are shown in figure 3.2. In this figure, one can find the maximum plate length \( d_1 \) and the maximum width \( d_2 \). Note that the plate must always be shorter than \( d_1 \), as otherwise no room for raising the elevator is left. The recommended plate dimensions are listed in in table 3.1.

Also, the dimensions of the fluid container are listed. Two different containers were used. The one that was created later (and is both “thicker” in \( x_3 \) and higher in \( x_1 \)) is indicated by an asterisk. The height of the fluid container is indicated by \( d_1 \) and the “thickness” by \( d_3 \). The width \( d_2 \) is fixed and not indicated. The dimensions are summarised in table 3.2.

The plates that were used were made of either stainless steel, or tungsten. The thicknesses ranged between 0.05 and 0.2 mm. Apart from the plate size, the physical properties such as the longitudinal
3. Experimental Setup

(a) Technical drawing of setup, with max length and width of plate included.

(b) Technical drawing of fluid container, with widths and heights for two different containers listed.

Figure 3.2: Technical drawings with the most important dimensions included.

<table>
<thead>
<tr>
<th>Dim</th>
<th>Size</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_2$</td>
<td>$d_2 &gt; 60$ mm</td>
<td>Plate width should be at least 60 mm, as else it will not be clamped correctly.</td>
</tr>
<tr>
<td>$d_2$</td>
<td>$d_2 &lt; 100$ mm</td>
<td>Plate width should not exceed 100 mm, as else it cannot fit.</td>
</tr>
<tr>
<td>$d_1$</td>
<td>$d_1 &gt; 45$ mm</td>
<td>Plate should be at least 45 mm long, as else no insertion depth in fluid can be measured. In fact, to have some range of depths, at least 60 mm is advised.</td>
</tr>
<tr>
<td>$d_1$</td>
<td>$d_1 &lt; 200$ mm</td>
<td>Plate length should not exceed 200 mm, as else no fluid container will fit underneath.</td>
</tr>
</tbody>
</table>

Table 3.1: Table listing the maximum and minimum plate dimensions (apart from thickness, which can be chosen freely).

<table>
<thead>
<tr>
<th>Dim</th>
<th>Size</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_3$</td>
<td>$d_3 = 15$ mm</td>
<td>Thickness of fluid container 1.</td>
</tr>
<tr>
<td>$d_3$</td>
<td>$d_3 = 55$ mm</td>
<td>Thickness of fluid container 2.</td>
</tr>
<tr>
<td>$d_1$</td>
<td>$d_1 = 80$ mm</td>
<td>Height of fluid container 1.</td>
</tr>
<tr>
<td>$d_1$</td>
<td>$d_1 = 120$ mm</td>
<td>Height of fluid container 2.</td>
</tr>
</tbody>
</table>

Table 3.2: Table listing the sizes of the fluid container. Two different containers were used, the one that was created later during the research is indicated with an asterisk.
and shear velocity, were not verified due to time constraints. Several methods exist for the measurement of these velocities, one of which is the “amplitude spectrum method” explained by Pialucha in [20]. Now, numbers from literature or as given by the manufacturer were used instead. For the actual viscosity measurements, it is important to accurately measure these velocities as they will influence the amount of energy travelling in the plate versus in the fluid.

### 3.1.2. Single Pulse versus Arbitrary Wave

The experiments were started with the setup available at the Reactor Institute Delft. In this setup, the electrical signal that was used to excite the transducer was a single pulse from a pulser/receiver. The transducer would then ring at its center frequency around 5 MHz, for around 5 cycles, with a Hanning-like window. In fact, although the setup was simpler, the signal to be analysed was more complex due to the wide amplitude spectrum.

In later measurements, a different setup provided by the acoustics group at Imaging Physics was used. Here, an arbitrary waveform generator in combination with a radio frequency amplifier and a delimiter was used to excite a more steerable wave. With the freedom of the waveform shape, different effects could be studied more accurately and exclusively than with the single pulse setup. Also, the results were easier to reproduce qualitatively. However, with the combination of the waveform generator, the amplifier and the delimiter, new sources of error were also introduced.

In figures 3.3a and 3.3b, the time domain signals of the reflected wave from the single pulse setup and the arbitrary wave setup respectively are plotted. Also, in figures 3.4a and 3.4b, the frequency spectra of the reflected wave are plotted. Note that the difference in amplitude is due to amplification (20 dB, factor 10) by the pulser/receiver in the single pulse setup. As one can see, albeit the sent out wave was centred around 5 MHz, the center frequency of the reflected pulse is lower (this is discussed in section 4.1.1). More importantly here, is the peak wideness. As one can observe, for the single pulse wave (about 5 cycles), the amplitude spectrum has a Gaussian-like shape and with a relatively high variance. Whereas for the arbitrary wave (a 60 cycled 3 MHz pulse was used), the peak is very well centred around 3 MHz, albeit some spectral leakage is apparent as well. The difference in these frequency spectra has many implications for the experiments.

In table 3.3, one can find all the components that were used in either one of the setups, with the corresponding brand and product code (or similar).

![Reflected signal from single pulse setup wave](image1)

![Reflected signal 60-cycled wave](image2)

(a) Reflected signal for single pulse setup. (b) Reflected signal for 60-cycled wave.

Figure 3.3: Here, the reflected signals of the two different setups can be compared. Only the reflection of the Lamb wave is shown, as that is the relevant wave. Note that even though the signal that was sent out in (a) was much shorter (around 3 cycles), the reflected signal is much longer than in (b), where the signal was 60-cycled.
3. Experimental Setup

![Amplitude spectrum reflected signal from single pulse setup wave](image1)

(a) Frequency spectrum single pulse setup.

![Amplitude spectrum reflected signal from 60-cycled wave](image2)

(b) Frequency spectrum for 60 cycled wave of arbitrary wave setup.

Figure 3.4: In these figures, the amplitude spectra of the two different setups can be compared. (a) Part of the lower frequencies are cut-off due to the limited size of the oscilloscope window, also, 1/f noise is visible. (b) The peak is sharply centred around 3 MHz. Note that the lower amplitude in (b) is due to the 20 dB amplification by the pulser/receiver for (a).

<table>
<thead>
<tr>
<th>Part</th>
<th>Brand</th>
<th>Product no.</th>
<th>Description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transducer</td>
<td>Olympus</td>
<td>V155-RM</td>
<td>Shear contact transducer, with diameter 13 mm and peak frequency 5 MHz [21]</td>
<td></td>
</tr>
<tr>
<td>Couplant</td>
<td>Olympus</td>
<td>SWC-2</td>
<td>Shear wave couplant (very high viscosity) [22]</td>
<td></td>
</tr>
<tr>
<td>Pulser/Receiver</td>
<td>JSR</td>
<td>DPR300</td>
<td>Pulser/Receiver sending out a single pulse to the transducer [23]</td>
<td></td>
</tr>
<tr>
<td>Oscilloscope 1</td>
<td>Keysight</td>
<td>DSOX2024A</td>
<td>Oscilloscope used for Single Pulse Setup [24]</td>
<td></td>
</tr>
<tr>
<td>Arbitrary Waveform Generator</td>
<td>Rigol</td>
<td>DG1022A</td>
<td>Waveform generator sending out sine waves with a selectable amount of cycles, an amplitude up to 20 Vpp, in a frequency range of 5 µHz - 25 Hzv [25]</td>
<td></td>
</tr>
<tr>
<td>Amplifier</td>
<td>E&amp;I</td>
<td>2100L</td>
<td>Amplifier, 50 ± 1.5 dB power gain, in range of 10 kHz - 12 MHz [26]</td>
<td></td>
</tr>
<tr>
<td>Delimiter</td>
<td>TU Delft</td>
<td>Custom</td>
<td>Delimiter to split the input and output signals</td>
<td></td>
</tr>
<tr>
<td>Oscilloscope 2</td>
<td>Keysight</td>
<td>DSOX1102G</td>
<td>Oscilloscope used for Arbitrary Wave Setup [27]</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.3: Table listing all the parts and instruments used in the experiments. In the references, one is referred to a website where the corresponding part is sold for more information.

3.1.3. Plate Waves

As was discussed before in chapter 2.2, the three wave modes that will be excited with are the shear wave, the flexural wave and the symmetrical wave. By turning (polarising) the transducer, a choice can be made between the excitation of the flexural or the shear wave. Although the transducer is reportedly polarised such that “the direction of the polarization of shear wave is nominally in line with the right angle connector” (see [21]), in reality the polarisation is approximately 10° off. As such, the correct polarisation is found by looking at the reflected signal. The excitation of the symmetric wave cannot be changed by polarising the transducer in the $x_2$-$x_3$ plane, but only by turning it around its $x_2$ axis; the used setup however, did not allow to do so.

In figure 3.5, one can observe a full signal that was reflected. Although the transducer was polarised such that the flexural wave was the strongest to be excited, it is impossible to completely exclude excitement of other modes. As such, not only the flexural A0 mode will be excited, but also the
symmetric S0 and shear SH0 modes. Furthermore, the A0 mode will undergo partial mode conversion when reflecting at the lower boundary of the plate (see figure 3.1a), exciting a new S0 mode (and the converse also happens, the initial S0 mode will partially convert to a new A0 mode). In the figure, the following signals can be distinguished (note that as the transducer was polarised to excite the A0 mode, this mode has the highest amplitude).

- **S0** - The symmetrical mode, showing little dispersion and a high velocity.
- **S0+A0** - The wave that has travelled one way as a symmetrical mode and the other way as a flexural mode (or the other way around), showing more dispersion than the pure S0 mode, as the A0 mode is highly dispersive.
- **SH0** - The shear mode, of which the tail arrives at the same time as the head of the next wave mode, making them inseparable. However, by inserting the plate into the fluid, the shear mode can be distinguished as the velocity remains practically unaffected, whereas the quasi-Scholte wave is up to a factor 2 slower than the A0 mode.
- **A0** - The Lamb mode, highly dispersed. The high frequencies have a similar velocity to the shear mode.
- **S0+A0\textsubscript{r2}** - The second reflection of the aforementioned S0+A0 wave superimposed over the lower frequency range of the A0 mode and more dispersed.
- **SH0\textsubscript{r2}** - The second reflection of the aforementioned SH0 wave, mixed with the second reflection of the A0 mode as well.

![Reflected signal from single pulse](image)

Figure 3.5: Figure depicting all the wave modes for a plate suspended in air. It is clear that the flexural mode is of much higher amplitude than the shear or symmetrical mode.

It is tempting to assume that the S0+A0 mode reflection is the SH0 mode, as the SH0 is otherwise hidden. Carefully analysing the wave velocities, as well as looking at the dispersion will prove the
contrary. Also, by turning the transducer, one can check where the SH0 mode should arrive. Lastly, by looking at the signal after inserting in water (see figure 3.6), it can be verified as well. Due to the slowness of the quasi-Scholte wave, and the complete attenuation of the leaky Lamb mode, the shear and flexural signals can be distinguished.

In the subsequent subsections, the Lamb, leaky Lamb and quasi-Scholte wave modes will be discussed in more depth.

**Lamb Waves**

From figure 3.5, it is not possible to retrieve information about the amplitude ratios of the S0 to the A0 mode, or the SH0 to the A0 mode. However, as the shear wave does not lose energy upon entering the water (no wave conversion takes place, no reflection was observed), the signal strength of the shear wave at 1.0 cm can be taken. This signal will only be off because of the attenuation due to viscosity, which is in the order of 1%.

Analysing computationally, it was found that both the shear and the symmetrical waves were on average a factor 100 (or 40dB) lower in amplitude. The Lamb waves will, upon hitting the fluid surface, partly reflect as a Lamb wave, and partly continue as either a leaky Lamb wave or a quasi-Scholte wave. The reflection however, was too weak to be distinguishable from the noise.

**Leaky Lamb Wave**

As previously discussed in section 2.2.5, the leaky Lamb wave is highly attenuated. It is unclear whether the leaky Lamb wave will also be excited at other points than at the first entry of the guided wave in the fluid. Possibly, the leaky Lamb wave is excited again at the bottom of the plate. It is not possible to measure the leaky Lamb wave separately. As the wave is so highly attenuated, the propagation distance would need to be so low, that the leaky Lamb wave will always interfere with the quasi-Scholte wave.

In figure 3.6, the attenuations as obtained from Disperse of the leaky Lamb and quasi-Scholte wave can be compared. Note the different scaling on the y-axis. It was not possibly to obtain the attenuation curves for the leaky Lamb wave for the lower frequencies. In an attempt to approximate the possible behaviour for lower frequencies, the curve has been extended with a dotted line. In the worst case scenario, the attenuation will remain above 300 Np/m for the frequency range in which the experiments are performed. As such, to ascertain sufficient attenuation, the lowest insertion depth was kept to 1 cm, implying at least 95% damping. Compared to the approximate 1 Np/m attenuation for the quasi-Scholte wave (or 1% damping), and assuming the conversion to the quasi-Scholte mode is sufficient, the effect of any damping of the leaky Lamb wave should be negligible for the attenuation measurements.

This could be confirmed by looking at the arrival times of the waves. As can be observed from the earlier depicted dispersion curves in figure 2.16, the reflection of the wave converted to the leaky Lamb mode will arrive before the quasi-Scholte mode. In figure 3.7, one can observe the reflections for insertion depths 0.0 cm, 0.5 cm and 1.0 cm respectively. In the 0.5 cm figure, the leaky Lamb wave reflection is still visible. In the 1.0 cm figure, the leaky Lamb wave reflection is not visible anymore, nor is it measurable.

A more in depth discussion of the leaky Lamb wave can be found in the appendix, in section A. For example, the statement “assuming the conversion to the quasi-Scholte mode is sufficient” is considered in greater detail.

**Quasi-Scholte Wave**

To finally find the correct viscosity using the quasi-Scholte wave, the attenuation and the group velocity should be measured. In the next section, section 3.3.1, the different methods for determining the group velocity have been set forward. Here, it will be discussed how the attenuation can be found.
3.1. Physical Setup

Figure 3.6: Here, one can compare the theoretical attenuation of the leaky Lamb wave and the quasi-Scholte wave for a 0.1 mm tungsten plate in 20 °C water. Note the difference in scaling.

Figure 3.7: In this figure, the reflected waves can be compared for different insertion depths. These results are for a 0.2 mm steel plate in water. It is clear that the insertion depth should be at least 1.0 cm.
First, the signal strength of the reflected wave must be found. These are

\[ S_1(\omega) = S_0(\omega)e^{-\alpha x_1} \]
\[ S_2(\omega) = S_0(\omega)e^{-\alpha x_2}, \]

where the following variables have been introduced:
- \( S_1 \) - Signal strength at insertion depth \( i \) [arb.]
- \( \omega \) - Angular frequency (frequency can be used as well) [rad/s]
- \( S_0 \) - Signal strength after transmission, but at insertion depth 0 [arb.]
- \( \alpha \) - Attenuation [Np/m]
- \( x_i \) - Insertion depth \( i \) [m]

For the signal amplitudes, a Fourier transform must be obtained in order to correct for frequency dispersion. It is then possible to solve for attenuation \( \alpha \), finding

\[ \alpha = \frac{1}{2(x_2 - x_1)} \ln \left( \frac{S_1(\omega)}{S_2(\omega)} \right), \quad (3.1) \]

where insertion depth \( x_1 < x_2 \).

Now, the attenuation can seemingly change due to dispersion. In figure 3.7, the change in amplitude due to dispersion is not yet clear. However, if we compare the signal strength at 1.0 cm and 4.0 cm insertion depth in figure 3.8, the amplitude appears to be increasing.

This increase in amplitude can be explained by looking at the group velocity of the leaky Lamb and the quasi-Scholte wave. The group velocity curves as obtained from Disperse for a 0.2 mm thick steel plate in vacuum and in water have been plotted in figure 3.9. In this figure, it can be seen that from around 1 MHz and on, the group velocity of the leaky Lamb wave rises while that of the quasi-Scholte mode drops. For the leaky Lamb wave, the wave package will spread out due to the group velocity for high frequencies being higher than for low frequencies. The head of the wave package, where there will be mostly high frequencies (see section 2.2.7), will move faster than the tail of the wave, where there will be mostly low frequencies, thus distorting the wave package. For the quasi-Scholte mode, the reverse is true, and the dispersion is partly reversed. In figure 3.8, it is clear that although the wave for the 4.0 cm insertion depth has a higher amplitude, it is also shorter indicating that the amount of energy in the wave did not actually increase.

### 3.1.4. Frequency

Frequency tuning has many implications on the experiments. The amount of energy of the wave that will be propelled through the fluid depends on the thickness of the plate, where the thicker the plate, the more energy will travel in the fluid. Or, equivalently, the higher the frequency, the more energy will
travel in the fluid. Secondly, the attenuation of a bulk longitudinal wave in fluids will rise as function of frequency, as can be seen in equation 3.1. As such, the attenuation of the quasi-Scholte wave will, in general, rise with the frequency. Lastly, while increasing the frequency, the dispersion will also decrease. This will not only increase temporal resolution, but also the signal to noise ratio.

Higher attenuations are generally favourable, as this will decrease the error in the viscosity determination. Along with the decrease of dispersion, it is advised to use high frequencies where possible. It has been observed that using a thinner plate will in general allow for higher frequencies. In 4.1.5, it will become clear how the maximum frequency is limited depending on the setup. However, in its turn this will adversely affect the ratio of energy moving in the fluid to energy moving in the plate, as well as increase dispersion. Nonetheless, the latter two effects are of lesser impact than the increase in attenuation in the bulk longitudinal wave in the fluid. Thus using thinner plates with higher frequencies is advisable. Plates can be handled well down to 0.05 mm.

3.2. Simulations

In this section, the settings for the simulations will be discussed. The section is kept shorter than for the physical setup, as the focus of the experiments was more on the results of the physical setup. As was previously mentioned at the beginning of chapter 3, it was attempted to recreate the physical experiments with the simulations in order to benchmark the simulations. The goal was to thenceforth use the simulations to test difficultly built new setups.

3.2.1. Design

The simulations were done two-dimensionally. This is depicted in figure 3.10, where the plate is highlighted red and the surrounding fluid is grey. Note that the plate is now infinitely wide in the $x_2$ direction. The transducer was modelled as a prescribed displacement at the top boundary. The prescribed displacement simulates the displacement that would normally have been imposed by the transducer with perfect coupling. A sine wave, with an amplitude that has a Gaussian shape over time, was used to simulate the displacement. The displacement for a three cycled wave is depicted in figure 3.11.

The dimensions of the simulations were mostly kept equal to the dimensions previously introduced in section 3.1.1. The important difference is the change in dimensionality. With the assumption of plane strain as discussed in section 2.2.3, this is a valid simplification that should not influence the mode shapes. However, the overall amplitude is influenced. As the plates and the transducer had a
finite width in the physical setup, the wave was spread in the plate. Moreover, at the sides of the plate interfering wave modes were excited. This was confirmed in the simulations performed by de Reuver [28]. In the simulations, the plate and the transducer are simulated to be infinitely wide, whereby no spreading nor boundary interference is involved.

It was not possible to perform three dimensional simulations, as the computational burden and the size of the output files would be too large. When scaling up the simulations to the physical dimensions, the file sizes reached 100 GB. It is expected that three dimensional simulations could reach sizes of up to 10,000 GB or more. An important driver for these file sizes are the high frequencies simulated, thus indicating short wavelengths. Because of these short wavelengths, not only must the number of elements increase, but also the number of time steps. More information is given in section 3.2.3.

3.2.2. Physics

COMSOL Multiphysics® offers different sets of physics to simulate the physical setup with. For the simulations run for this experiment, only the physics set named “Acoustic-Solid Interaction, Frequency
3.2. Simulations

Domain” have been used. In this set, the fluid is modelled using pressure acoustic variations, and the solid is modelled as a solid with its elastic deformations. In table 3.4, besides the physics set used for the simulations, two different sets of physics offered by COMSOL® are listed. The latter two, indicated with the brackets, have not been used but could in future be implemented as a reference. It must be noted that in a previous investigation performed by Schuringa [29], an incorrect set of physics was used where the fluid was modelled without viscosity, and the obtained results are therefore not deemed trustworthy.

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acoustic-Solid Interaction, Frequency Domain</td>
<td>Fluid: acoustic pressure variations</td>
</tr>
<tr>
<td></td>
<td>Solid: structural deformation</td>
</tr>
<tr>
<td>(Solid Mechanics)</td>
<td>Fluid: structural deformation, modelled as solid (harmonic movements)</td>
</tr>
<tr>
<td></td>
<td>Solid: structural deformation</td>
</tr>
<tr>
<td>(Thermoviscous Acoustic-Solid Interaction, Frequency Domain)</td>
<td>Fluid: acoustic waves including thermal and viscous losses</td>
</tr>
<tr>
<td></td>
<td>Solid: structural deformation</td>
</tr>
</tbody>
</table>

Table 3.4: Table listing the different physics sets offered by COMSOL® that could be used to simulate the experiments. The first has been implemented, while the second and third sets of physics are proposed.

3.2.3. Parametric Studies

The parametric studies are discussed in detail in annex B. Convergence studies were performed to determine the correct size of the mesh and time steps. As the simulations were run in COMSOL®, all the parameters have been given abbreviated names, namely:

- dF - Thickness of fluid (distance plate-walls) - plane wave radiation
- dF - Thickness of fluid (distance plate-walls) - sound hard boundary
- hF - Height of fluid under plate
- hMWAp - Number of mesh elements per wavelength in strip in air (x, direction)
- hMWFp - Number of mesh elements per wavelength in strip in fluid (x, direction)
- hdMWFp - Number of mesh elements per wavelength in fluid (x, and x3 direction)
- TSp - Timestep size

The results, as presented in table 3.5. The parameters were found such that the uncertainty in the found viscosity falls within ±1%. Also, the parameters found for the determination of the group velocity fall within ±0.1% (or, within ±2 m/s where the group velocity will be in the order of 2000 m/s). The bracketed results are assumptions for when a fluid is involved (for the quasi-Scholtewave for example), whilst the non-bracketed results are those found for a plate in air.

Table 3.5: Results for all parametric studies

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Choice for $\alpha$</th>
<th>$\Delta \eta$</th>
<th>Choice for $c_g$</th>
<th>$\Delta c_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>dF - PWR</td>
<td>1.0 cm</td>
<td>0%</td>
<td>(1.0 cm)</td>
<td>(&lt;0.01%)</td>
</tr>
<tr>
<td>dF - SHB</td>
<td>1.0 cm</td>
<td>0%</td>
<td>(1.0 cm)</td>
<td>(&lt;0.01%)</td>
</tr>
<tr>
<td>hF</td>
<td>0.2 cm</td>
<td>0%</td>
<td>(0.2 cm)</td>
<td>(&lt;0.01%)</td>
</tr>
<tr>
<td>hMWAp</td>
<td>5</td>
<td>0%</td>
<td>7</td>
<td>0.05%</td>
</tr>
<tr>
<td>hMWFp</td>
<td>5</td>
<td>0%</td>
<td>(7)</td>
<td>(0.04%)</td>
</tr>
<tr>
<td>hdMWFp</td>
<td>3</td>
<td>0%</td>
<td>(5)</td>
<td>(0.01%)</td>
</tr>
<tr>
<td>TSp</td>
<td>8</td>
<td>1%</td>
<td>8</td>
<td>0.06%</td>
</tr>
</tbody>
</table>
3.3. Data Analysis

Finally, the methods for analysing the data will be introduced. Especially for finding the wave velocity, or more specifically, the group velocity, this is not straightforward. Also, for the attenuation, some problems occurred while analysing the data from the single pulse setup.

3.3.1. Wave Velocity

There are different ways to determine the group velocity. Cegla [12] has investigated three different methods and chose the “zero slope method” as the preferred procedure. Without further investigation, the same method was applied in this research. The other two approaches, the “method of cosine interpolation” and the “amplitude spectrum method” both had drawbacks. Also, a more direct method by cross-correlation is shortly discussed.

Zero phase slope method

The zero phase slope method can be understood from mathematics. The derivation as proposed by Ceglá is closely followed. The standard solution to the wave equation is of the form given in equation 2.52. This solution can be split to its time and space dependence. The Fourier transform for the time dependence is defined as

\[ \mathcal{F}\{u(t)\} = \mathcal{F}\{e^{-i\omega t}\} = F(\omega). \]  

(3.2)

Using the time shift properties in the Fourier spectrum, it can be found that

\[ \mathcal{F}\{u(t - t_{\text{shift}})\} = e^{-i\omega t_{\text{shift}} F(\omega)}. \]  

(3.3)

As the Fourier spectra will always be taken in the time domain, the dependence on \( x \) can be added as follows (where we neglect the subscripts given in 2.52, as they are not relevant here):

\[ \mathcal{F}\{u(x, t - t_{\text{shift}})\} = e^{-ikx e^{-i\phi_0} e^{-i\omega t_{\text{shift}}}} F(\omega). \]  

(3.4)

Here, the initial phase \( \phi_0 \) has also been added for completeness. The overall phase \( \phi \) can thus be found to be:

\[ \phi = - (kx + \phi_0 + \omega t_{\text{shift}}). \]  

(3.5)

Differentiating with respect to the frequency, one can find

\[ \frac{d\phi}{d\omega} = - \frac{dk}{d\omega} x - k \frac{d\phi_0}{d\omega} - t_{\text{shift}} - \omega \frac{dt_{\text{shift}}}{d\omega}. \]

We can now fill in \( C_g = d\omega/dk \), we find that

\[ \frac{d\phi}{d\omega} = - \frac{x}{C_g} - t_{\text{shift}}. \]  

(3.6)

If the window of the oscilloscope is kept the same, while measuring the phase slope at two different path lengths \( x \), the relation

\[ \frac{d\phi_1}{d\omega} - \frac{d\phi_2}{d\omega} = \frac{x_2 - x_1}{C_g} \]  

(3.7)

can be found, such that the group velocity is found with the equation

\[ C_g = \frac{x_2 - x_1}{\frac{d\phi_1}{d\omega} - \frac{d\phi_2}{d\omega}}. \]  

(3.8)
Thus, what needs to be done to find the group velocity is measure the signal in the same time window but at two propagation distances $x$. Then, the phase can be determined by a discrete Fourier transform. The phase should be unwrapped and the phase slope can then be determined. It is advised to use the highest temporal resolution possible, as this will decrease the error and increase the frequency range for the determination of the group velocity. This can be seen in figure 3.12. In this figure, the phase slope has been determined for the reflected quasi-Scholte wave from two signals. In one signal, the full reflected wave was obtained, thus decreasing the temporal resolution. In the other signal, only the quasi-Scholte wave was in the window of the oscilloscope; it can be seen in the figure that the obtained results are much better.

![Phase slopes](image)

Figure 3.12: In this figure (where the quasi-Scholte only slope has been offset downwards), it is clear that the frequency range at which the group velocity can be determined is much larger when the temporal resolution is better.

**Direct method**

When a signal with a narrow bandwidth and with little dispersion is used, the group velocity can be found with a more direct method. Instead of using the zero slope method, the signals can be cross correlated over time. For example, assume that the group velocity of the A0 mode has previously been found using the zero slope method. Then, the arrival times $T_1$ and $T_2$ for two signals having travelled distances $x_1$ and $x_2$ respectively, are:

$$T_1 = 2 \left( \frac{(l - x_1)}{C_{g,A_0}} + \frac{(x_1)}{C_{gScholte}} \right)$$

$$T_2 = 2 \left( \frac{(l - x_2)}{C_{g,A_0}} + \frac{(x_2)}{C_{gScholte}} \right)$$

Then, by cross correlating the time signals, the difference between these times $T_1$ and $T_2$ can be found, where the maximum of the cross correlation function corresponds to the time shift. The group velocity can thus be found to be

$$C_{gScholte} = \frac{2(x_2 - x_1)}{T_2 - T_1 + \frac{2(l - x_2 + x_1)}{C_{g,A_0}}}.$$  \hspace{1cm} (3.9)

**Different procedures to find $C_{gScholte}$**

It is also possible measure the group velocity of the quasi-Scholte wave directly using the zero phase slope method, where it is not necessary to determine the group velocity of the A0 mode. The following three different procedures for determining the group velocity of the quasi-Scholte wave will be introduced:

- Cegla’s method
• Fluid only method
• Fluid-Air method

Cegla [12] proposes to first measure the group velocity of the plate without the fluid (by the zero phase slope method). Then, he takes measurements at insertion depth $x_1$ at different center frequencies $f_1$. He shifts the plate slightly to insertion depth $x_2$. Again, he sends out signals at the same center frequencies $f_1$ as before. He then cross-correlates the signal at the first insertion depth and at the second insertion depth for each frequency $f_1$ apart. Doing so minimises the effects of dispersion, due to the centralisation of the frequency and the small difference in insertion depth.

For this method, it must be noted that the single pulse setup cannot be used, due to its broad bandwidth. As such, an attempt was made to determine the group velocity for the quasi-Scholte wave using the same procedure, but by applying the zero phase slope method for the velocity determination in the fluid as well. Then, knowing the propagation distances in vacuum and in the fluid, the overall group velocity and the group velocity of the A0 mode, the group of the quasi-Scholte mode was calculated. However, based on repeatability, it was determined that this method was not usable.

More importantly, both the method proposed by Cegla and its adjustment as explained above, will pose problems when determining the group velocity of the quasi-Scholte wave for fluids at higher temperatures. For both methods, the group velocity of the A0 mode must be measured in advance. However, as the plate is heated when inserted in the fluid, the group velocity will change accordingly. However, the heating will be non-uniform, and the group velocity must be determined for that exact heating pattern, as it will be off otherwise. If the reflection were to be visible in the reflected signal, it could be determined correctly. As this is not the case, a third procedure has been proposed.

Although not thoroughly tested, as the focus of the research was more on the attenuation measurements, the following method has been developed. It is expected that the most accurate method to find the group velocity of the quasi-Scholte wave is by using the zero slope method for the fluid only. This can be done by measuring the signal twice for two different insertion depths $x_1$ and $x_2$. However, the strip length in vacuum, above the fluid must be kept equal, such that the group velocity of the Lamb wave in air does not play part. Doing so will thus take away sources of error. And, assuming that the fluid itself is uniform in temperature, this method will be independent of the unknown temperature gradient in the plate. It is recommended to use one plate to ensure the same thickness, but turning the plate by 90°. This does require a plate of certain dimensions, as can be read in section 3.1.1.

### 3.3.2. Dispersion and Interference Compensation

Determining the amplitude spectrum was done though a fast Fourier transform algorithm. As shown before in section 3.1.2, the amplitude differed greatly for the single pulse setup and the arbitrary wave setup. Furthermore, it was mentioned that the results for the arbitrary wave setup were more reproducible. Although the amplitude spectrum for the reflected A0 wave from the single pulse setup develops smoothly over a large frequency range, this is in general not true for the reflection of the quasi-Scholte mode. For example, the amplitude spectrum of a measurement with the single pulse setup is shown in figure 3.13. It is tempting to smooth the signal by convolving the signal with an array of ones. For example, in figure 3.14, the result for smoothing with an array of ones with a length of 15% of the signal length in that frequency window is plotted.

It is evident that the signal analysis, by eye at least, is easier when the signal is smoothed. However, smoothing also means, that information of one frequency is spread out over several frequencies. This, theoretically, is wrong. However, it was also verified that in the big majority of cases, the smoothing of the signal did not change the found attenuation by means of a least squares fit with curves obtained from the Disperse software. Only when the array of ones is chosen too large, will the signal also change in its more gradual behaviour. For example, the resulting peak can be lower, and such alterations should always be avoided.

Another operation that was performed, but later stopped, was dispersion compensation. As was explained in section 3.1.3, due to the dispersion being reversed when the plate is inserted in the water, the amplitude can seemingly rise. If the window in which the reflected wave is analysed moves or
3.3. Data Analysis

Figure 3.13: Non-smoothed (directly from raw data) amplitude spectrum and corresponding attenuation for a reflected signal from the single pulse setup.

Figure 3.14: Smooothed amplitude spectrum and corresponding attenuation for a reflected signal from the single pulse setup.

changes in size with the wave, the amplitude in the frequency spectrum will be affected. An algorithm was written to find the reflected wave computationally to ensure consistency. However, in a private discussion [30], it was concluded that such a compensation can negatively influence the accuracy. The decision was made to measure in one fixed window, such that any compensation would not influence the amplitude. Also, any arbitrariness in the window chosen will be taken away. However, as the lower frequencies disperse greatly, they might fall outside the window if it is kept in one place, as can be observed in figure 3.7. As such, caution must be taken when choosing the frequency range over which the attenuation is determined.
4 Results and Discussion

The goals that were set at the beginning of the investigation, encompassed the measurements and fine tuning of the physical setup, the creation and analysing of a COMSOL® model, and to start with the design of the hot setup. However, with the challenges faced, most of the goals were not attained. The measurements with the physical setup were better understood iterating through different settings, but finally some challenges remained to be researched. The COMSOL® model showed qualitatively understandable behaviour, although the attenuation and velocities were off. The design of the hot setup was not touched upon, although some discussion will be included here for possible future work.

4.1. Physical Setup

First, the single pulse setup results will be considered. Although the results were harder to reproduce, the wide range of frequencies sent out also yielded in informative results. As the results for the arbitrary waveform setup were more consistent, and seemed more promising for the determination of the viscosity, these results will be discussed in more depth.

4.1.1. Single Pulse versus Arbitrary Wave

Part of the reason why the results were harder to reproduce was due to the complex wave composition with its broad frequency spectrum. Recall figures 3.4, where the frequency spectra for the single pulse and the arbitrary waveform setup are compared. Then, recall figure 3.5, where all the different reflected are shown. It is clear that with the differences in velocity per wave mode, and the frequency dependence of the velocity of the A0 mode, the analysis of the reflected wave for a wide frequency spectrum is complex and can pose challenges.

With the arbitrary wave setup, the frequencies contained in the sent wave were strongly centred, save the inevitable spectral leakage. As such, not only is there more information available at one certain frequency, also, the pulse shape is generally much better conserved. As the 60-cycled wave travels, only the head and the tail are dispersed, as these parts contain different frequencies. However, the sine wave in between shows no dispersion, and can be analysed accurately.

4.1.2. Attenuation: Single Pulse

It has not been possible to determine the viscosity correctly using the single pulse setup. The viscosity measured for water was \( \mu = 0.1 \pm 0.3 \text{ mPa}\cdot\text{s} \) at 20 °C, is not in agreement with the literature values. The irreproducibility of the results along with the average attenuation to be off has made a correct attenuation determination impossible. The different challenges that were encountered are presented and discussed below. First, a few interference patters are shown and their impact is considered. Then, measurements yielding a negative attenuation are examined. Lastly, some results are summarised and compared, and some sources of error are discussed.

Interference: filter settings

While measuring, some interference was observed. The JSR pulser receiver made it possible to alter the (frequency) filter. Depending on the thickness and material of the plate, and on the insertion depth in the fluid, some filter settings worked better than others. Using no filter, the signal was at times noisy and the reflected wave was hard, if not impossible, to distinguish. However, it has also been observed that incorrect filter settings, albeit possibly creating a smooth signal, could greatly influence the signal.
Figure 4.1: The reflected quasi-Scholte wave as obtained for a 0.2 mm thick steel plate inserted 0.7 cm in water. Assuming that the interference arises from the pulser/receiver filter settings, it is clear that these settings are incorrect.

Table 4.1: The different settings for the pulser/receiver

<table>
<thead>
<tr>
<th>Pulser/receiver settings</th>
<th>Heavy filter</th>
<th>Medium filter</th>
<th>Light filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Pass</td>
<td>2.5</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Low Pass</td>
<td>7.5</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Gain</td>
<td>66</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>Voltage</td>
<td>475</td>
<td>475</td>
<td>475</td>
</tr>
</tbody>
</table>

at other insertion depths. For example, in figure 4.1, the signal is plotted for the “heavy filter” settings as listed in table 4.1. Although these filter settings showed little noise, from the figure it is clear that the results are unusable. The signal is therefore also not trustworthy at other insertion depths, even if it seems smooth. As such, caution must be taken when choosing the filter settings in order not to alter the relevant frequency components of the signal. The “medium filter” and “light filter” settings (especially the latter) as listed in table 4.1 are advised.

Interference: quasi-Scholte wave

Another interference pattern was observed as well, with different period. In figure 4.2, an interference pattern that strengthens with the insertion depth. It is unclear what gives rise to this interference. The interference did not seem to be linked to the filter settings. The interference did seem to change with the thickness of the plate, where the interference became stronger with the plate thickness. Also, the interference always grew stronger with the insertion depth in water. As the interfering wave seems to move at a similar velocity as the quasi-Scholte wave, and due to its similar amplitude (part of the 6 cm depth signal from figure 4.2 has destructively interfered to a factor 0.3 of its initial strength), it is suspected that the wave mode might have been interfering with itself. Another possibility is that the leaky Lamb wave is reflected off the walls of the fluid container back to the plate, thus interfering with quasi-Scholte wave; the fast arrival time and high amplitude after that many mode conversions however, leads to believe that this is not the case.

Negative Attenuation

In a number of cases, negative attenuation was observed. In figure 4.3, the top of the peak of the amplitude spectrum is shown. Note that the signal was smoothed (see section 3.3.2), as the spectra are otherwise not comparable. It is clear that the insertion depths for the amplitude in decreasing order, are: 2, 3/1, 5, 6, 4. No good explanation was found for this arbitrary behaviour of the amplitude of the reflected wave as a function of the insertion depth. The light filter settings from table 4.1 were used, it is thus not expected that the filter brought about the interference (see subsection 4.1.2). Possibly, the interference as explained in subsection 4.1.2 could have brought about these results. Although that would not explain the negative attenuation for the first few insertion depths, where the interference
Figure 4.2: In this figure, the interference of the wave can be seen. It is clear that the interference worsens with increasing insertion depth. These results were obtained for a 0.2 mm thick steel plate in water.

was the most weak. The negative attenuation for the first few insertion depths could be the symptom of a nonlinear transfer of the plate displacement to the electrical signal (see section 4.1.5). If very small displacements are not picked up by the transducer, then reversing dispersion hence increasing the amplitude of the wave might increase the signal strength non linearly.

Results and comparison
Some of the results have been combined and depicted in figures 4.4a and 4.4b. These are the results for a 0.2 mm thick steel plate in water, with the medium and light filter settings respectively (see table 4.1). Also, the attenuations were least squares fitted with the attenuation curves from disperse. For the medium filter settings, a viscosity of \( \tau = 0.0 \pm 0.2 \text{ mPa-s} \) is found, and for the light filter settings, a viscosity of \( \tau = 0.1 \pm 0.3 \text{ mPa-s} \) is found. For the former, this was also the lowest attenuation curve extracted from disperse, it is likely that if a lower curve was possible this would be found. The error bars are based on repeatability, and are taken as the standard error of the mean. One single attenuation curve holds the information of on average 10 measurements at different insertion depths. The fact that the uncertainty based on the repeatability causes the results to disagree with the literature value of the viscosity of water (1.0 mPa-s [31]) shows that the experiments have not correctly been performed.

There are many possible sources of error. As discussed in sections 4.1.2, the transfer function of the mechanical energy to an electrical signal could be nonlinear. The material properties could be different from standard settings in disperse (less stiff material experience a higher attenuation). The thickness of the plate could be unevenly spread, such that the multiple measurements performed per plate were still off (the measurement error of the thickness gauging device is in the order of 0.001 mm); if the real plate is thicker than measured, the attenuation curves from disperse would shift both down and to the left (due to the frequency thickness product).

The enumeration of the possible sources of error could go on for a while. In order to rule out most of the uncertainty with respect to the signal shape, not every aspect will be discussed here. In the next part of this chapter, in section 4.1.3, the results of the arbitrary wave setup will be discussed. As the
4.1. Physical Setup

Figure 4.3: In this figure, the frequency spectrum is shown for the reflection of the quasi-Scholte wave. Only the top of the peak is shown for clarity. The amplitudes do not seem to attenuate with increasing insertion depth. These results were obtained for a 0.2 mm thick steel plate in water, the same data set was used for figure 4.2.

wave is much better steerable, different sources of error can be investigated separately more easily. It will become clear that some sizeable errors must be dealt with before the research with either one of the setups can be continued.
In these figures, the results of different measurements with the same configuration have been plotted. Both results are for a 0.2 mm thick steel plate in water. Here, the pulser/receiver settings were (a) the medium filter settings and (b) the light filter settings (see table 4.1). Also, the theoretical attenuation curve for $\eta = 1.002$ mPa·s is shown, and the curve obtained by least squares fitting.
4.1.3. Attenuation: Arbitrary Wave

The results for the arbitrary wave attenuation will be given in a different format. As the amplitude spectrum will be sharply peaked as shown in figure 3.4b, an attenuation graph such as presented for the single peak setup cannot be presented. However, much more information will be contained in that sharp peak, and it can be measured at different frequencies, so as to discretely find similar attenuation curves.

With the amount of information contained per frequency range, the results were expected to be more consistent. However, the results varied considerably per measurement. Some different causes for the varying signal will be discussed. First, some digital errors in the amplifier and the delimiter are discussed, in section 4.1.3. It was discovered that the results varied over time, which is elaborately discussed in the subsequent section 4.1.3. Thirdly, interference such as previously discussed in sections 4.1.2 and 4.1.2 was also observed, which is discussed in section 4.1.3.

Digital Errors

Measuring with the new setup, new errors in the form of distortion were introduced. The signal that is sent is by the amplifier, first goes through a delimiter as was previously mentioned in section 3.1.2. As the transducer is connected with a single cable for sent and received signals, a delimiter was needed to split the signals. The signal that is sent out by the amplifier is therefore not shown on the oscilloscope. The signal sent out by the transducer is. The amplitude of the sent out wave by the transducer compared to that of the reflected signal is up to a factor 160 larger, or 44 dB. Note that the signal sent by the amplifier is 50 times larger than that sent by the transducer, or 8000 times larger than that received by the transducer (that is why the delimiter is used). Also, it was relatively close in time, for 60-cycled signals on the smaller plates of 8 cm length, the wave group itself would fit 4.7 times between the sent and reflected signal.

The exact nature of the distortion is unknown. However, it has become clear that the closer the sent and received signal were, the stronger the distortion was. Also, when the pulse repetition frequency is too high, the distortion will intensify. In figure 4.5, the distortion can clearly be observed. In this figure, the distortion was due to the pulse frequency being chosen too high, at 300 µs. The wave sent was a 60-cycled 3 MHz sine wave, on a 0.1 mm thick 11.5 cm long tungsten plate. The distortion will not influence the frequency spectrum, or only for very low frequencies which fall out of the window used for amplitude evaluation. However, it is expected that the distortion will negatively impact the accuracy of the measurements. Changing the repetition frequency, lowering the number of cycles and increasing the plate length can help dampen distortion.

![Reflected signal from arbitrary wave setup](Image)

Figure 4.5: In this figure, the distortion of the delimiter and the oscilloscope can clearly be observed. Results obtained on a tungsten plate of 11.5 cm length, 0.1 mm thickness. Sent out signal was a 60-cycled 3 MHz wave, repeated at 300 µs.

Time Dependence - Introduction

When measurements were performed with a fixed time interval, but without altering the setup, a time dependent signal was found. For example, in figure 4.6, the signal strength at the center frequency as
well as the DC component of the signal are shown. These results were obtained for a 0.05 mm thick tungsten plate of 8 cm length, at a center frequency of 3 MHz with a 60-cycled signal sent out. Note that the plate was not inserted in a fluid and the results were obtained by averaging the signal 4096 times.

Figure 4.6: The signal strength of the reflected wave at the center frequency is plotted over time. The plate was not immersed in a fluid, and the setup was not touched.

The first change in amplitude, up to 15 minutes, was observed for all time interval measurements. Therefore, it is expected that this might be due to the shear couplant “settling” or heating up. With settling, the effect of the couplant being smeared out by the vibrations is meant. Or heating up could be caused by the vibrations, up to a point where it reaches its equilibrium. The change was more vigorous for the smaller tungsten plate (0.05 mm thick, 80 mm long), where it took 15 minutes and a factor 0.79 of the amplitude remained. Compare this to the change for the bigger tungsten plate (0.2 mm thick, 200 mm long), where it took between 40 and 100 minutes, changing to a factor 0.95 amplitude on average. It is therefore believed that, if the first time dependence does indeed arise from the settling of the couplant, the thickness of the plate is the most influential. However, more experiments are needed in order to confirm this.

These fluctuations cause great errors. For example, take the loss of a factor 0.2 of the primary amplitude as shown in figure 4.6. This settling loss, if measurements were performed for insertion depths 1.0 to 4.0 cm, would translate to an attenuation of 3.7 Np/m. Compare that to the attenuation curve presented in figure 3.6, and one will find that the attenuation is off by a factor 3.7. There is little correlation between the DC signal and the amplitude of the reflected signal in the Fourier domain. In other measurements, there was no correlation at all.

The second change in amplitude is more arbitrary. As can be observed in figure 3.6, the researcher might expect the signal to have become stable enough after 15 minutes, which is not the case. The signal has not shown good stabilisation within 200 minutes after installing the plate. It has however been confirmed that the output signal of the transducer is very constant (up to a factor $5 \times 10^{-5}$). The DC signal after the sent out wave did show higher fluctuations, up to a factor $2.5 \times 10^{-2}$, although it is unknown whether these DC effects might influence the peak to peak amplitude of the reflected wave. Many different hypotheses can be made for these possible fluctuations. The most important will be discussed here.
4.1. Physical Setup

**Time Dependence - Viscosity change of couplant**

The signal sent out by the transducer is transferred to the plate using the shear couplant as introduced in section 3.1.1. From Recondo et al. [32], the temperature dependence of the viscosity of honey and of carbohydrate systems can be found. It is depicted for a range of possible ambient temperatures in figure 4.7. Although the agreement of the discussed models and the temperature dependence of the viscosity of the shear couplant (see [22]) was not researched, it is expected that qualitatively they will concur. Both are organic materials with a high viscosity and well soluble in water. Also, due to its behaviour, looks and smell, all similar to thick syrup, it is expected by the researcher that the couplant consists of carbohydrates as well. From the models proposed by Recondo et al., the relative change in the viscosity of honey for a 1 K temperature change was found to be $\Delta \eta/\eta|_{293K} = 0.11, 0.11$ and 0.12 Pa·s (for the Arrhenius, WLF and VTF models respectively). Cheng [31] reports similar viscosity changes ($\Delta \eta/\eta|_{293K} = 0.09$) as a function of temperature for glycerol from empirically setup formulas. Then, knowing that the the velocity of the transducer is just the derivation of its displacement and that the shear stress goes linearly with the shear strain according to Hooke’s law, and that for periodic behaviour the fluid can be modelled as a solid by multiplying strains with $-i\omega$ (see section 2.1.7), it is found that the traction force of the couplant will linearly follow its viscosity. As such, it can be concluded that the traction force could decrease by a factor 0.1 approximately for a 1 K temperature change of the couplant, depending on the (yet unknown) viscosity as a function of temperature of the couplant.

![Viscosity of honey as a function of temperature](image)

**Figure 4.7:** In this figure, the viscosity of honey as a function of temperature has been depicted. The results were obtained with the parameters found by [32], inputted in the VTF model.

**Time Dependence - Phase velocity change in plate**

As the temperature increases, it can be expected that the wave velocities decrease as the material softens. If the phase of the wave shifts while the oscilloscope is averaging, the wave might interfere with itself altering the signal strength. The crystal structure of the used plates is unknown, but the temperature dependence of the phase velocity of the shear and flexural waves for different structures were found in literature. Lee et al. [33] have found the wave velocities in pure tungsten to decrease with approximately 0.2 m/(sK) and 0.18 m/(sK) for the longitudinal and the shear wave respectively. Bernstein at al. [34] have found the temperature dependence to be 0.2 m/(sK) and 0.19 m/sK for the longitudinal and shear waves respectively. Lastly, de Reuver [28] has found the change in phase velocity of the flexural wave to be approximately 0.1 m/(sK) with simulations. The experimental results are in very well agreement, though they do not necessarily give information about phase velocity of the flexural wave. The simulated results for the flexural wave however, had a greater uncertainty. As such, the most conservative option, i.e. the experimental results, will be worked with.

In the experiments, an average factor of 0.05 of fluctuations in the amplitude were observed. The change in temperature needed to change the amplitude of the reflected wave by a factor 0.05 can be calculated. Knowing that the wavelength $\lambda$ can be found by equating
\[ \lambda_1 = \frac{c_{\text{ph},1}}{f_0}, \]

where \( \lambda_1 \) and \( c_{\text{ph},1} \) are the first wavelength and phase velocity respectively, and \( f_0 \) the frequency of the sent wave. Then, the number of wavelengths \( N_1 \) that fit in a plate of length \( l \) twice (as we are working with the reflection) is

\[ N_1 = \frac{2l}{\lambda_1}. \]

Then, the shift in the number of wavelengths that fit in the plate is

\[ N_2 - N_1 = \frac{2l}{c_{\text{ph},2}/f_0} - \frac{2l}{c_{\text{ph},1}/f_0} \quad (4.1) \]

The phase velocity in a 0.1 mm tungsten plate at 3 MHz is approximately 1200 m/s, and the plate length used was 0.2 m (this was the longest plate, where the influence of the temperature would be the greatest). The influence on the amplitude of the reflected wave of a shift in the number of wavelengths that fit in the plate was found by analysing the amplitude spectrum of two sine waves of which one was shifted in time. In figure 4.8, the relative amplitude can be found for a change in temperature. It can be found that a 0.6 K increase in temperature will lower the signal strength by a factor 0.05 and that for a 1.0 K increase a factor 0.13 is lost.

![Amplitude reflected wave as a function of temperature change](image)

Figure 4.8: The amplitude of the reflected wave as a function of temperature shift between two measurements that are averaged. As the wave velocity changes, the wave will destructively interfere with itself in the measurements of the oscilloscope.

It must be noted that the temperature dependence of the quasi-Scholte wave velocity as a function of the fluid temperature might be higher. Noting that the quasi-Scholte wave phase velocity is slightly lower than the bulk longitudinal velocity of the surrounding fluid (see section 2.2.6), and noting that the bulk longitudinal velocity of water increases with 3.0 m/(sK) at 20 °C [35], it is found that the quasi-Scholte wave phase velocity increases with a factor 15 faster with a temperature change than that of the plate velocities. In spite of this significant velocity change, the effect on the amplitude of the reflected signal is expected to be lower than for the plate wave velocity change, as the water acts as a temperature buffer. However, when the setup at high temperatures is taken into use, this effect must be taken into account as these temperature are then not merely brought about by the ambient temperature.

**Time Dependence - Averaging**

The effect of the wave interfering with itself due to a change in temperature, depends on the change in temperature per unit time and on the amount of time it takes to perform one measurement. The
signal of the reflected wave was very unsteady when not averaged. This did not seem to be due to random noise with a short period, but to a slower moving noise that would completely alter the DC component of the signal. It is expected that the noise arises from the non-linearity in both the delimiter and the oscilloscope used. As such, standard averaging was used, where multiple time signal captures are averaged together.

First, it is important to note the waiting time needed for the signal to stabilise after moving the setup. If the pulse repetition frequency is 500 Hz, and the number of averages is 1024, one might expect that after two seconds, the signal will have stabilised. This was however, not the case as can be observed in figure 4.9. In this figure, the average amplitude of the complete frequency spectrum whole frequency spectrum of the There, it is clear that only after 30 seconds will the signal reach its final stability. Possibly, this is due to the fact that the oscilloscope does not actually save every waveform separately for the averaging, but applies a weight to the newly measured signal (such that the first measured signal will always slightly influence the latest measurement). It was confirmed that the stabilisation time scaled linearly with the number of averages.

Now, the second question is how many times the signal should be averaged in order to have the lowest deviations between two measurements. On the one hand one might expect that increasing the number of averages will always reduce random errors in strength. On the other hand, it was previously confirmed that small temperature changes might significantly impact the signal strength. And indeed, this interplay resulted in contradictory measurements.

In figure 4.10, the effect of increasing the number of averages can be examined. The measurements were done using a 0.2 m tungsten plate, of 0.1 mm thickness. The height of the peak at the center frequency of the reflected wave was captured. The measurements were performed with a two second interval. The skew per minute is defined as the normalised change in amplitude per minute, which increased with the number of averages. The standard deviation \( \sigma \) (normalised as well, thus represented as a factor) was determined with the effect of the skew taken out. The skew adjusted standard deviation \( \sigma \) decreased with the number of averages either.

Although it is unsure whether the skewness is brought about by the temperature or some other effect, it is clear that the influence is significant. If the skew is in one direction for multiple measurements, the attenuation measurements can be strongly impacted. This was the case for at least 25 min in figure 4.6 for example. Note that in this measurement, the signal was averaged 4096 times on a plate of 8 cm length and 0.05 mm thickness. If a measurement is done with one spoke height resolution (see section 3.1.1), the measurements can easily take more than 20 minutes, implying an
amplitude loss of a factor of 0.04 for a high number of averages. Such a skew is unacceptable. The standard deviation is of lesser importance, especially as the number of measurements increases, as these will generally cancel out.

It is recommended to keep the number of averages to 4 to 32. Not only will the skew be lower, also time between two measurements can be kept lower as was explained around figure 4.9. The number of averages depends on the pulse repetition frequency and the length of the plate. The shorter the plate, the higher the number of averages can be chosen. This is both needed due to the stronger non-linearity in the oscilloscope and the delimiter, and more acceptable as the effect of velocity fluctuations on a smaller plate is lower. Also, as the pulse repetition frequency is higher, the number of averages may rise accordingly, as the time for one measurement will remain equal. However, a long term measurement with a lower amount of averages should be performed to confirm the hypotheses.

Another averaging method is available as well, but has not been investigated. The high resolution acquisition method samples the signal within one window at more points than required, and subsequently averages neighbouring data points onto one required point. As the DC signal was altered because of noise, it was expected that the high resolution method would in fact not help. However, this method can be used in combination with standard averaging [36]. The stability of the signal when measuring with a combination of both methods can be investigated in the future. It is expected that extra noise reduction might be possible, but that the skew which poses the greatest challenges will not be alleviated. Also, the bandwidth of the oscilloscope will be decreased, and the signal might lose resolution in the Fourier domain.

Interference
Interference was observed for the arbitrary wave setup as well. Measurements were performed with one or two spokes (thus 0.3 or 0.6 mm) height difference in the immersion depth to further investigate the interference. First, the interference for a significantly distorted wave with low amplitude will be shown. Then, the results for less distorted waves with a higher amplitude will be discussed.

As was previously introduced in section section 4.1.3, the reflected wave can be distorted strongly as a result of the electrical instrumentation. In the figure 4.11a, such a signal is shown. It was obtained on a 0.1 mm thick 0.2 m long tungsten plate, with the sent out signal having a center frequency of 1 MHz. The wave was so strongly distorted due to its frequency and therefore amplitude being low,
4.1. Physical Setup

as 1 MHz is relatively far from the center frequency of the transducer. In figure 4.11b, it is clearly visible that the wave oscillates consistently in its amplitude when the immersion depth is increased. The immersion depth is expressed in spokes, where one spoke corresponds to 0.3 mm.

Figure 4.11: Shown in these figures are (a) the reflected signal and (b) the peak amplitude in its frequency domain on a 0.1 mm thick 0.2 m long tungsten plate.

However, not only the amplitude but also the center frequency is plotted. It was interesting to find out that the oscillations correlated with the center frequency. It must be noted that although the changes in frequency are relatively small, they are still more than a factor 10 higher than the deviations observed in the reflected wave for a 3 MHz signal over time (the results in figure 4.6). The change in amplitude lagged the change in frequency by approximately one measurement.

Another interference pattern was obtained, for a signal of which the distortion was less strong. It was for both a different amplitude (3 MHz) and a different plate (0.05 mm thickness and 80 mm length tungsten). The interference is slightly less consistent than previously observed in figure 4.11, but it shows that this interference is most likely not the result of the signal being distorted by the electronics.

Also for the latter interference pattern, the change in amplitude is preceded by a change in frequency, again by around one or two measurements. Note that all measurements were done at a fixed time interval. Due to the transfer function of the signal from the transducer to the plate, it can be expected that a change in frequency of the sent wave can provoke a change in amplitude as well. However, the transfer function as a function of frequency has not been determined. The converse might however, also be true, where the frequency of the mechanical signal will also influence the amplitude of the electrical signal sent to the oscilloscope as a result of the transfer function. The frequency for which the transfer was highest depended on the plate thickness and material (where thinner plates support high frequencies better). For both results plotted above, the frequency of the top of the transfer function was higher than the center frequency of the sent out wave, such that an increase in amplitude with frequency was to be expected.

The fact that the change in frequency preceded the change in amplitude, leads to believe that the amplitude spectrum is in fact not altered due to interference in the fluid, but that the sent wave was unstable in its amplitude spectrum. However, the fluctuations in the frequency were consistently more vigorous (more than factor 10) doing measurements while increasing immersion depth than for the
measurements performed over time while keeping the setup untouched. However, the measurements over time were only performed in vacuum. In order to determine whether it is the change in immersion depth, or the immersion itself that instigates the frequency instability, a measurement of the center frequency of the reflected wave over time must be performed while keeping the immersion depth constant.

A few hypotheses have been checked. For example, it was confirmed that the pulse repetition frequency did not influence the relative size of the changes, although it did seem to influence the consistency. It must be noted that for the above two measurements, the pulse repetition frequency was relatively high, where the pulse itself (the 60-cycled sine wave) would fit 5 and 15 times respectively between two pulses for the results plotted in figures 4.11 and 4.12. However, despite the change in consistency of the fluctuations, this would not explain the absence of fluctuations in the measurement shown in figure 4.6, where the 60-cycled sine also fitted 15 times between two pulses. Furthermore, it has been confirmed that the time between switching on the instrumentation (including the amplifier) did not influence the fluctuations. Also, the width of the fluid container did not influence the fluctuations.

The fact that the change in the pulse repetition frequency did seem to have an effect on the form of the fluctuations (changing the consistency) confirms the suspicion that they arise from the waveform generator or amplifier. The fact that when the wave is not immersed the frequency is practically constant, contradicts this suspicion. Some investigation has gone into the “wavelength” of the fluctuations, to see whether a correlation with the strip length or wavelength of the sent out wave can be found. The results are combined in table 4.2. In this table, $\lambda$ is the wavelength of the fluctuations (not of the wave in the pulse itself), $f_c$ is the center frequency of the wave sent, $d$ is the thickness of the plate. The only correlation found is that of the length of the plate and the wavelength of the fluctuations, of which the outcomes are listed in the last column as length $l$ divided by wavelength $\lambda$. It must be noted that the periodic interference patterns were not observed in a tungsten plate of 11.5 cm length and 0.1 mm thickness, but strong more random fluctuations were observed. Although deemed unlikely, it could be possible that the fluctuations happen on a much smaller length scale, and that aliasing has caused the previously discussed interferences at the scale observed. However, more measurements should be performed to draw conclusions, as for example a measurement with an even smaller immersion depth resolution should be considered.
4.1. Physical Setup

| Measurement | \( \lambda \) [cm] | \( f_0 \) [MHz] | d [mm] | l [mm] | l/\( \lambda \) [

Figure 4.11 | 1.08 ± 0.01 | 1 ± 3e-3 | 0.1 ± 0.005 | 200 ± 0.5 | 18.5 ± 0.2
Figure 4.12 | 0.439 ± 0.002 | 3 ± 7e-7 | 0.05 ± 0.005 | 80 ± 0.5 | 18.2 ± 0.2

Table 4.2: This table lists data on the interference patterns observed in figures 4.11 and 4.12.

4.1.4. Wave Velocity

The group velocity of water was obtained for a 0.2 mm thick steel plate immersed in water by making use of the zero phase slope method described in section 3.3.1. In the figure it can be seen that the group velocity is off by approximately 20%. As the curve qualitatively followed the curve obtained from Disperse, no more effort was put into the investigation of the error at the time.

First, some sensitivity analyses should be performed in order to determine the possible sources of error. For example, when making use of the “fluid only” method as described in section 3.3.1, it was unknown how an uncertainty in the immersion depth or plate length might influence the group velocity measurements, as it was a new technique. The sensitivity was retrieved by altering the immersion depth “hStrF” by 1 mm, as if an error was made while measuring. The difference in the determined group velocity is plotted in figure 4.13b. From the plot, it becomes clear that, for example, the hysteresis of the water of 0.6 cm as explained in section 3.1.1 can bring about an error in the measured group velocity curve of approximately 70 m/s.

![Group velocity measurement](image1)

(a) In this figure, the found and theoretically predicted group velocities have been depicted for a 0.2 mm thick steel plate immersed in water.

![Sensitivity of measured group velocity for 1 mm error in hStrF](image2)

(b) And in this figure, the sensitivity of the group velocity \( C_g \) as a function of an error in the measured immersion depth hStrF can be found.

Figure 4.13: In the two figures above, the found group velocity and the sensitivity of the measured group velocity as a function of an error in the immersion depth are depicted.

4.1.5. Transfer function

The transfer function has not been researched thoroughly. However some preliminary results are included. The signal strength of the first and second reflection of the sent wave at its center frequency has been plotted in figures 4.14a and 4.14b respectively. In the former figure, it can be observed that the signal strength of the first reflection does not linearly grow with the output voltage when the voltage used is high. It is deemed most likely that the transfer of the wave energy from the transducer to the plate does not go equally well for these high voltages as for somewhat lower voltages. Alternatively, the electric signal could not be well transferred to a mechanical signal by the transducer. Either way, it is expected that the sending direction gives rise to the nonlinearity at high voltages.
Furthermore, in the second figure 4.14b, nonlinear behaviour towards the lower voltages can be noted. Here, it is expected that the transfer of energy from the plate to the transducer does not go well, or possibly that the energy conversion from mechanical to electrical in the transducer does not go linearly. As such, it is believed that the nonlinearity is introduced by in the receiving direction.

More research should go into the transfer function. The influence of the plate thickness, as well as the influence of the frequency should be investigated. Also, if possible, it should be determined whether the nonlinearity arises in the sending or the receiving of the signal.

![Graph](image1)

(a) Signal strength first reflection as a function of voltage. Note the non-linear behaviour towards the higher output voltages.

![Graph](image2)

(b) Signal strength second reflection as a function of voltage. Note the non-linear behaviour towards the lower output voltages.

Figure 4.14: In the two figures above, the signal strength of the (a) first and (b) second reflection of the sent out wave at its center frequency as a function of voltage have been plotted. It is clear that towards both higher voltages and lower voltages, non-linear behaviour occur.

4.1.6. Discussion and conclusion

In conclusion, it is clear that there are still some challenges to be tackled in the future. To be able to accurately investigate each of the questions introduced in the previous sections, it is important to have a stable signal over time. That should be the first goal looking forward. In order to take out the time dependence, one could for example try to keep the temperature constant. However, as the temperature fluctuations may not exceed approximately $0.1\, ^\circ C$, and as the amplifier is 1000 W strong and will most likely heat up any room, this is not a sensible solution. Or, one could glue the plate to the transducer, such that any influence of the temperature would be taken out at the coupling; however, this is not considered a useful option as it takes away the freedom of testing different plates.

As such, two new setups are proposed, which can be constructed with minor modifications. In both of the two new setups, depicted in figure 4.15, a new scattering boundary is created. These new boundaries are created such that a reflection can be obtained of a wave mode that did not travel through the fluid. One option is the addition of a scattering mass, which can be attached to the plate (left drawing in figure 4.15). This is the easiest to implement, but the other setup is likely to yield better results. The other option (right drawing in figure 4.15) is to create a gully in the plate, such that part of the guided wave does not enter the fluid.

The new reflection of the wave that did not travel through the fluid is obtained concurrently, albeit separated in the oscilloscope window, with the reflection of the wave that did travel through the fluid. The former reflection can be used to take out any time dependent effects, assuming they arise at the sending or receiving of the signal, and not in the plate itself (because of a change in phase velocity for

![Graph](image3)
4.2. Simulations

The results of the simulations will be discussed here, albeit not in great detail. No consistent results were obtained, and it is advised to change the model to any of the other models proposed in table 3.4. First, the mode shapes obtained will be shown. Then the attenuation will be presented and discussed.

4.2.1. Mode Shapes

To confirm whether the quasi-Scholte wave is accurately modelled by COMSOL®, the wave was first analysed graphically. In figure 4.16, the results of a COMSOL simulation for a 0.1 mm thick tungsten plate in water have been depicted. The figure shows the \( x_1, x_3 \). In this figure, the fluid in rest is white and the pressure is shown in red and blue for higher and lower pressures than at rest respectively. Also, the plate is shown in green, where the displacement is also shown in red for positive displacements and blue for negative displacements. Note that although the displacements have been plotted out of plane, in the \( x_2 \) direction, this is only done so for clarity, the actual model remains two dimensional.

Indeed, if the plot 2.15 is compared to the simulation results depicted in figure 4.16, where the same displacement (\( x_3 \) displacement, where the wave propagates in the \( x_1 \) direction) is plotted, the

Figure 4.15: The two new proposed setups. On the left, a mass is attached to the plate to create a new scattering spot outside the fluid, whereas on the right the plate is split into one part that enters the fluid, and another part that does not enter the fluid.
resemblance is clear. The quasi-Scholte wave attenuates in the $x_3$ direction, perpendicular to the propagation direction ($x_1$ of the guided wave). Also, some remainders of the leaky Lamb wave can be observed closer to the boundaries. Although transmitting boundaries have been set up such that the reflection coefficient is low, part of the leaky Lamb mode travels along the fluid container walls in the simulations. Using the results of the parametric studies presented in annex B, the correct container size can be confirmed such that these wall-modes do not interfere with the obtained attenuation.

![Figure 4.16: Results of the simulation, clear Quasi-Scholte waves at the fluid-plate interface](image)

The results for a reflected wave are shown in figure 4.17. These results are for a 0.1 mm thick tungsten plate inserted 2 cm in water. The results for the simulations are different from those in physical experiments in a few ways. For example, recalling 3.1.3, other wave modes can be observed (although it must be noted that in figure 3.5 the signal was obtained without fluid immersion). Apart from the SH0 mode, which cannot exist in the two dimensional model, the S0 mode is also not excited. That is due to the perfect excitation of the flexural mode.

These contrasts make the analysis easier. However, with the simulations, the reflection coefficient on the fluid surface has been observed to be high. For example, in figure 4.17, the A0$_w$ mode, is the reflection of the A0 mode on the water surface, this wave mode has not travelled as a quasi-Scholte wave. Also, the QSch$_{1,w}$ mode can be observed, which interferes with the lower frequency components of the QSch mode. The QSch reflection is the mode that has reflected at the plate bottom. The QSch$_w$ is the quasi-Scholte mode that first reflected at the plate bottom, the reflected at the water surface, back towards the plate bottom again. The QSch$_{1,w}2$ is a higher order mode, following the extra path covered by QSch$_{1,w}$ twice.

The wave of interest, the first reflection of the quasi-Scholte wave, shows strong interference. Just like shown in figure 4.2, the interference for the simulations grows with increasing fluid depth insertion. However, the interference for the simulations observed was greater than for the experiments, especially at lower insertion depths, making the analysis harder.

The wave velocity of the quasi-Scholte mode however, did not match up with the theoretically predicted values. In fact, the velocity of the wave seemingly depended on the insertion depth, which should not be the case. As the files were too big to open, an in depth analysis was not possible. Possibly, the mode conversion of the quasi-Scholte mode back to the A0 does not go well. It has been observed that if the second A0 mode is assumed to travel at the same velocity as the quasi-Scholte mode, the velocities seem to be more consistent. However, more research is needed. A model where displacements at every point in the plate along the propagation direction of the guided wave can be extracted, would greatly enhance the accuracy and understanding of the simulations.
4.2. Simulations

Figure 4.17: In this figure, the different wave modes that were excited in the simulations are depicted. The results are obtained for a 0.1 mm thick tungsten plate inserted in water.

4.2.2. Attenuation
The results for the measurement of attenuation in water and glycerine are shown in figure 4.18. Note the rescaled y axis for the results for glycerine. Also, it must be noted that the results have been smoothed with a mask as explained in 3.3.2. For both results, the theoretically predicted attenuation curve as obtained from Disperse is plotted as well. It is clear that the curves obtained from COMSOL® do not follow the attenuation curves well.

One improvement going forward from the COMSOL® model from Schuringa [29], is that the attenuation for glycerine is higher than for water. Before, similar attenuation curves were obtained for fluids with different viscosities. However, more work on the model is needed, and the other fluid models proposed in table 3.4 should be tested for accuracy and comparison.

Figure 4.18: Shown in these figures are the attenuation curves as obtained from the simulations in COMSOL®. It is clear that the attenuation for glycerine is in general higher than for water. However, the results for both setups do not agree with the theoretically predicted attenuation curves.
Conclusion

Although the viscosity for water was not determined accurately yet, much knowledge was gained. In this thesis, it is endeavoured to pass on the relevant elements of this knowledge. First, the single pulse setup was discussed. The minor information density in the time domain as a function of frequency, together with errors later understood, led to the incorrect determination of the viscosity of water at 20 °C of \( \eta = 0.1 \pm 0.3 \text{ mPa}\cdot\text{s} \).

The research was continued using a more steerable wave using the arbitrary waveset up. Now, the viscosity was not determined due to the ongoing investigation of unexpected behaviour. Most importantly, it has been discovered that the signal strength was highly dependent on time, changing up to 20% in 25 minutes. As the viscosity induced attenuation would theoretically be only around 5%, this would introduce unacceptable errors. Also, strong fluctuations in the amplitude of the reflected signal as a function of insertion depth were observed. On a smaller plate, the signal strength would fluctuate by approximately 10% for a change in insertion depth of 0.44 cm. It has been discovered that these fluctuations are preceded by approximately one or two measurements (10 to 20 seconds time, or 0.3 to 1.2 mm immersion depth) by a corresponding fluctuation in the frequency spectrum of the signal; the frequency at the peak amplitude in the Fourier spectrum was shifted.

The main recommendation for future work is to establish a consistent signal strength over time, or to design an alteration to the setup that enables the simultaneous measurement of the signal strength of the sent wave. The signal strength of the sent wave depends strongly on many different parameters, such as for example the temperature, where a one °C change in temperature of the couplant may provoke a 10% change in signal strength. Therefore, the simultaneous measurement of the sent signal is strongly recommended. This can be done by incorporating a new scatter source in the plate; the favoured method would be by splitting the plate to effectively form two wave guides, where one will be immersed in the water and where the other will solely propagate the leaky Lamb wave.

Besides the experiments, simulations were run using COMSOL®. However, it was found out that the fluid model used was incorrect. After altering the fluid model, the attenuations showed qualitatively much better behaviour. However, the velocity of the quasi-Scholte wave was off by a factor of almost 2. As the velocity influences the attenuation, the results were not deemed trustworthy. The fluctuations as a function of immersion depth were not observed, but strong fluctuations in the amplitude spectrum evident. A few recommendation are proposed for new models, such that the simulations can further be enhanced.
Appendices
A

Leaky Lamb wave

In subsection 3.1.3, it was hand-wavingly argued that assuming the conversion to the quasi-Scholte mode was sufficient, such that the effect of any damping of the leaky Lamb wave would be negligible. These unfounded assumptions will be discussed in further depth here.

First, it is important to know why sufficient conversion is needed and what sufficient is. Earlier, we had found the following formula (see equation 3.1) for the attenuation, being:

$$\alpha = \frac{1}{2(x_2 - x_1)} \ln \left( \frac{S_1(\omega)}{S_2(\omega)} \right).$$  \hfill (A.1)

Now, let us assume we have the following variables:

- $S_{A0}$ - Flexural wave initial signal strength [arb.]
- $f_{LL}, f_{QS}$ - Fraction of transmission to leaky Lamb and quasi-Scholte modes respectively (where $f_{LL} + f_{QS} = 1$)
- $\alpha_{LL}, \alpha_{QS}$ - Attenuations for leaky Lamb and quasi-Scholte waves respectively [Np/m]
- $x_1, x_2$ - Insertion depths one and two respectively (where $x_1 < x_2$)

Then, the correct reflected signal strength $S_C$ to be found would be

$$S_C = f_{QS} S_{A0} e^{-\alpha_{QS}2(x_2-x_1)}.$$ \hfill (A.2)

However, in reality we will have an influence of the leaky Lamb wave as well, and for the signal including the error $S^*$ the following strength will be found:

$$S^* = f_{QS} S_{A0} e^{-\alpha_{QS}2(x_2-x_1)} + f_{LL} S_{A0} e^{-\alpha_{LL}2(x_2-x_1)}.$$ \hfill (A.3)

The attenuation that will be found including the influence of the leaky Lamb wave is equal to:

$$\alpha^* = \frac{1}{2(x_2 - x_1)} \ln \left( \frac{f_{QS} e^{-\alpha_{QS}2x_1} + f_{LL} e^{-\alpha_{LL}2x_1}}{f_{QS} e^{-\alpha_{QS}2x_2} + f_{LL} e^{-\alpha_{LL}2x_2}} \right),$$ \hfill (A.4)

where the error $\epsilon(\alpha)$ will be

$$\epsilon(\alpha) = \alpha^* - \alpha_{QS}.$$

Now we can plot the error as a function of insertion depth $x_2$ and quasi-Scholte fraction $f_{QS}$, where we keep insertion depth $x_1 = 0.01$ m as was discussed in section 3.1.3. Also, we will take $\alpha_{LL} = 300$ Np/m and $\alpha_{QS} = 1$ Np/m. However, it is important to note that whether or not the leaky Lamb wave is excited at the bottom end of the plate as well has a big influence, as it cuts the distance to be travelled in half. For the quasi-Scholte wave, the difference in amplitude for travelling 1 or 2 cm is 1%; for the leaky Lamb wave, the difference in amplitude is 2000%. This behaviour is plotted in figure A.1, where the error is plotted (where signal strength 1 indicates no error).

Thus, it is necessary to research the fraction of the wave that is converted to the leaky Lamb mode. Whilst practically impossible to do this 100% correct, as the leaky-Lamb and the quasi-Scholte wave...
A. Leaky Lamb wave

Figure A.1: In this figure, the error can be observed as a deviation from the correct signal strength 1. It is clear that excitation of the leaky-Lamb wave at the bottom end of the plate can have a detrimental effect on the measurements, whereas the error induced by entry excitement is acceptable.

are not completely separable, a qualitative approach can be taken. In figure A.2, the amplitude spectra of the reflected flexural modes in vacuum, 0.5 cm insertion depth and 4.0 cm insertion depth (in water) have been plotted. Also, the transmission ratios have been plotted.

It is clear that going from 0.5 cm to 4.0 cm, just a small fraction of the wave is lost (which is partly due to the normal attenuation of the quasi-Scholte wave). In the experiments, the first insertion depth is always held to 1.0 cm. The transmitted fraction of the wave going from 1.0 cm to 4.0 cm was computationally found to be above 93%, where the expected fraction for the quasi-Scholte wave should be around 94% (for 1 Np/m). It was concluded that a fraction of at least 99% of the transmitted wave at either one of the excitation points, is in the quasi-Scholte mode. The error depends on the second insertion depth as well, as the further the wave travels the smaller the influence of the leaky Lamb wave becomes. The average error for a second insertion depth range of 1.5 cm to 5.0 cm was lower than 2.5%. For some of the plates used in the experiments, much higher insertion depth could be attained and the error would go down.

Finally, in the experiments of the single pulse setup, it could be confirmed that there was no higher attenuation for the first insertion depths as compared to the higher insertion depths. If the leaky Lamb wave were to have an influence, there should be a higher attenuation followed by a change in slope and a lower attenuation when increasing the insertion depth. Such behaviour has been observed in some of the experiments in the arbitrary wave setup, see section 4.1.2. It is however believed, that this is due to other circumstances rather than the influence of the leaky Lamb wave.
Figure A.2: The amplitude spectra and transmission ratios for vacuum to 0.5 insertion depth, and vacuum to 4.0 insertion depth.
In order to check how fine our meshes should be when simulating the signal, several parametric studies were performed with COMSOL®. Below we will list the results. The following parametric studies were performed:

- $dF$ - Thickness of fluid (distance plate-walls) - plane wave radiation
- $dF$ - Thickness of fluid (distance plate-walls) - sound hard boundary
- $hF$ - Height of fluid under plate
- $hMWAp$ - Number of mesh elements per wavelength in strip in air ($x_1$ direction)
- $hMWFp$ - Number of mesh elements per wavelength in strip in fluid ($x_1$ direction)
- $hdMWFp$ - Number of mesh elements per wavelength in fluid ($x_1$ and $x_3$ direction)
- $TSp$ - Timestep size

**B.1. Thickness of fluid - plane wave radiation**

In the 2D simulation, we assume a certain thickness of the fluid. This “thickness” is the size of the fluid sample in the same direction as the thickness of the plate. As the number of fluid elements has a big influence on the computation time, this is an important parameter.

At first, we impose a plane wave radiation boundary condition on the fluid boundaries. We can then see how the waves behave if we could model the fluid as being infinite. Then, decreasing the fluid size, we can see how the boundaries start influencing the solutions. The results in the time and frequency domain are plotted in figure B.1. If we zoom in at the reflected wave, we can see that especially below a fluid thickness of 0.6cm, the solution changes substantially. In the frequency domain, we see similar substantial changes.

As we are interested in the frequency domain information, this is what we will analyse. If we assume that we are aiming for a maximum error of 1% in the viscosity, then from Disperse we can find the relative error we can have in our attenuation data.

The errors from the data are calculated by assuming that the results with the lowest uncertainty are exact (0.0% error). Then, the relative error can be found by dividing the results of a certain measurement by the results with the lowest uncertainty. These errors are plotted in figure B.2a. The allowed error can be found from the results from Disperse. For example, for water, we find that if we allow the viscosity to vary by ±1%, the errors in the attenuation data may vary by ±0.45%, as can be observed in figure B.2b.

However, it is not yet clear how an error in the dataset translates to an error in the measured attenuation. We recall equation 3.1:

$$\alpha = \frac{1}{2(x_2 - x_1)} \ln \left( \frac{S_1(\omega)}{S_2(\omega)} \right)$$  \hspace{1cm} \text{ (B.1)}$$

We can see that the transfer of an error in the signal does not linearly translate to an error in the attenuation. The difficulty resides in the fact that the viscosity is found by fitting manually. We thus have to come up with a new method to find the allowed uncertainty.
(a) The reflected signal in the time domain
(b) The reflected signal in the frequency domain

Figure B.1: The reflected wave in time and frequency domain, for different fluid thicknesses dF

What is proposed, is to find the error in the attenuation due to the error in the measurement. We can do this by taking, again, the measurement with the lowest error to be correct. We take this as the measurement data at insertion depth \( x_1 \). Then, we can calculate the error in the attenuation that would arise if we would have an error in our data at insertion depth \( x_2 \), which we call the single sided error. To be more conservative, we also propose a double sided error, where the error at \( x_2 \) is taken in exactly the other direction at \( x_1 \), imposing an error on both measurement results that both increase the error in the attenuation data.

Note, that the exact signal plus the error and the exact signal minus the error are respectively given by:

\[
S_1^{\text{err}} = S_1 + (S_2^{\text{err}} - S_1) = S_2^{\text{err}} \\
S_1^{\text{err}} = S_1 + (S_2^{\text{err}} - S_1) = 2S_1^{\text{err}} - S_1
\]

Where we have:
- \( S_1 \) Signal without error at insertion depth 1
- \( S_2 \) Signal with error at insertion depth 2
We can then find the attenuations including the errors as follows:

\[ \alpha^{SE} = \frac{1}{2(x_2 - x_1)} \ln \left( \frac{S_1(\omega)}{S_2(\omega)} \right) \]  
(B.2)

\[ \alpha^{DE} = \frac{1}{2(x_2 - x_1)} \ln \left( \frac{2S_1^e(\omega) - S_1^e(\omega)}{S_2^e(\omega)} \right) \]  
(B.3)

Where we have:
• \( \alpha^{SE} \) Single sided error
• \( \alpha^{DE} \) Double sided error

However, there is one important unknown in the equation: \( x_2 - x_1 \). Although more measurements at different insertion depths can be performed keeping this low, the errors will also grow greatly. For example, say we can choose insertion depths anywhere between 1 cm and 2 cm. If we do only two measurement, \( 1/(x_2 - x_1) \) will equal 1 cm\(^{-1}\). If we do three measurements, \( 1/(x_2 - x_1) \) will equal 2 cm\(^{-1}\), effectively doubling our error (whereas the increased number of measurements itself will only decrease our error by \( \sqrt{2} \)). We will therefore assume that in general, \( x_2 - x_1 \) will equal 3 cm, as this is a reasonable difference in insertion depth for this research.

Then finally, we can plot the attenuations including the errors for \( \Delta x = 3 \) cm. We find the the plot in figure B.3a. We can clearly see that the results from \( dF < 0.8 \) cm have big errors. So, we will keep to the measurements with \( dF \leq 0.8 \) cm. These results, including the correct attenuation and the attenuation when an error in the measured viscosity is introduced, are plotted in figure B.3b.

\[ E = \sum_{\omega=\omega_1}^{\omega=N} \frac{\alpha^{DE}(\omega) - \alpha(\omega)}{N \alpha(\omega)} \]  
(B.4)

Where we have:
• \( E \) The relative, average error
• \( \alpha \) The attenuation that is fitted
• \( N \) Number of frequency data points

First, it can be observed that the relative errors for a smaller fluid thickness, are not larger at a certain frequency range. In figure B.4, the relative errors as a function of frequency are plotted.
B.2. Thickness of fluid - sound hard boundary

In the previous section, the minimum fluid size for the simulations was found where for the boundaries, a plane wave radiation boundary condition was imposed. Although this boundary condition is useful when one wants to keep the fluid size as small as possible (to save on computation time), in the real experiments there will at least be some reflection of the waves. Therefore, a sound hard boundary condition was also imposed to check the minimum fluid thickness if we have complete reflection. Although in reality we would not have perfect reflection, this is the most safe option.

The parametric study will not be set out in as many steps as the previous section. The reflected waves are depicted in figure B.5a and the relative error is depicted in figure B.5b. In the latter, the results for dF=0.4 cm were so far off that they have been left out.

Then, in table B.2, we can see that also for the sound hard boundary condition, a fluid thickness of just 0.8 cm was enough to determine the viscosity with an error lower than 1%. However, also here, the deviations were relatively large and it is advisable to work with a fluid thickness of at least 1.0 cm. Note that although imposing the sound hard boundaries increased the errors for small fluid thicknesses substantially, for the larger fluids the effect is negligible. This is beneficial for the

![Relative error in $\alpha^DE$](image)

**Figure B.4**: Relative double sided error $\alpha^DE$ as a function of frequency for the four different fluid thicknesses

**Table B.1**: Results for fluid thickness for plane wave radiation

<table>
<thead>
<tr>
<th>dF</th>
<th>Error in found viscosity</th>
<th>Relative deviation from fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4 cm</td>
<td>-20%</td>
<td>0.88</td>
</tr>
<tr>
<td>0.6 cm</td>
<td>-20%</td>
<td>0.23</td>
</tr>
<tr>
<td>0.8 cm</td>
<td>0%</td>
<td>0.02</td>
</tr>
<tr>
<td>1.0 cm</td>
<td>0%</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

The fit with the lowest relative error is chosen as the correct fit. As each fit has to be taken from Disperse manually, only the lines with an error of: 0%, ±1%, ±5%, ±10%, ±20% are included. The results are given in table B.1. We can see, that for the boundary condition of plane wave radiation, a fluid size of 0.8 cm (or in fact, 0.79 cm if we take into account the strip size) would be large enough to find the correct viscosity. However, it must be noted that the relative deviation from the fit is still substantial with 2% on average. Also, in figure B.3b one can observe that the line for dF=0.8 cm fluctuates outside the ±10% error lines. However, as it fluctuates on both sides more or less equally, the correct viscosity could be derived. However, in order to not deal with these fluctuations, which could make future work harder, it is advisable to work with a fluid thickness of 1 cm, to stay largely within the ±1% error margins. Remember that we have been conservative as we took the error to be twice in the same direction, where in practice it is expected that the error would compensate for itself at least partially.
(a) The reflected signal in the time domain

(b) Relative errors with respect to largest fluid

Table B.2: Results for fluid thickness for sound hard boundary

<table>
<thead>
<tr>
<th>dF</th>
<th>Error in found viscosity</th>
<th>Relative deviation from fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6 cm</td>
<td>-20%</td>
<td>2</td>
</tr>
<tr>
<td>0.8 cm</td>
<td>0%</td>
<td>0.03</td>
</tr>
<tr>
<td>1.0 cm</td>
<td>0%</td>
<td>0.0006</td>
</tr>
<tr>
<td>1.2 cm</td>
<td>0%</td>
<td>0.0006</td>
</tr>
</tbody>
</table>

experimental setup as well, as the fluid size can then be kept smaller.

**B.3. Depth of fluid**

Another (less important) parameter, is the depth of the fluid under the plate. In figure B.6a, one can observe the reflected wave and in figure B.6b one can find the relative errors.

In table B.3 the results are given. Again, it is best to opt for the depth where the deviation from the fit is low. While a fluid depth of 0.1 cm would be enough, we choose 0.2 cm as here the deviations are lower than 1%.
B.4. Mesh width of strip in air

Generally, the mesh width should be chosen such that there are about six or more mesh elements per wavelength. However, it has become apparent that the mesh width of the strip in air influences the results substantially, even when a large number of elements per wavelength is chosen. This can clearly be observed in the amplitude spectrum of the reflected wave, depicted in figure B.7a. Also, the errors converge relatively slowly as can be seen in figure B.7b.

As we are dealing with a systematic error (as opposed to the random errors before), the number of mesh elements would need to increase greatly still to obtain the correct viscosity. However, we are now still assuming the double sided error where both biases would be in opposite direction therefore increasing the final error. It could be so, that although the amplitude spectra of the different numbers of mesh elements cannot be compared with each other, their attenuation spectra can be. That is, the damping as observed in figure B.7a arises solely as a function of the mesh width parameter and not of the insertion depth. To test this, we need two insertion depths per number of mesh elements and take the attenuation from these two measurements, and can only then can compare the results. This was not done with the previous studies as more simulations would then have been needed.

Then, again it is needed to propose new techniques to benchmark the correctness of the obtained signals. First, it must be stated that here, we are not interested yet in the actually measured viscosity. We investigate the error in the measured viscosity that would have arisen due to the choice of parameters. As such, the assumption will always be made that the viscosity as measured at the computationally most intensive settings, is the correct viscosity. Then, we will continue as follows. First, the attenuation at the different number of mesh elements (hMWAp) is calculated:

\[
\alpha^9 = \frac{1}{2(x_2 - x_1)} \ln \left( \frac{S_2^9(\omega)}{S_2^9(\omega)} \right) \\
\alpha^{hMWAp} = \frac{1}{2(x_2 - x_1)} \ln \left( \frac{S_2^{hMWAp}(\omega)}{S_2^{hMWAp}(\omega)} \right)
\]

(B.5) \hspace{1cm} (B.6)

Where we have:
- \(\alpha^9\) The attenuation at 9 mesh elements per wavelength
- \(\alpha^{hMWAp}\) The attenuation at hMWAp mesh elements per wavelength

Table B.3: Results for fluid depth

<table>
<thead>
<tr>
<th>dF</th>
<th>Error in found viscosity</th>
<th>Relative deviation from fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1 cm</td>
<td>0%</td>
<td>0.02</td>
</tr>
<tr>
<td>0.2 cm</td>
<td>0%</td>
<td>0.002</td>
</tr>
<tr>
<td>0.3 cm</td>
<td>0%</td>
<td>0.000004</td>
</tr>
</tbody>
</table>

(a) The reflected signal in the frequency domain

(b) Relative errors with respect to largest fluid
Then, we can calculate the change (error) in attenuation by the formulas listed below. Either the absolute and the attenuation including this error by respectively:

\[
\Delta \alpha = \frac{\alpha^{h_{MWAp}}}{\alpha^9} - 1
\]

(B.7)

\[
\alpha^* = \alpha^* + \Delta \alpha
\]

(B.8)

Here, \( \alpha^* \) is the theoretical curve for the attenuation. In the former equation, as the attenuations reach zero, the ratio at some points is very large. By taking these points out (less than 5% of the points), these local maxima and minima can be avoided. Then, finally, we can plot the results of the differences in the attenuations in figure B.8a, and the relative errors in B.8b. In table B.4 one can find the errors in the obtained viscosity.

![Attenuation for sweep hMWAp](image1)

![Error in attenuation for sweep hMWAp](image2)

(a) The reflected signal in the frequency domain (b) Relative errors with respect to largest fluid

From the results in table B.4, one can see that taking 5 mesh elements per wavelength is enough. Also, in figure B.8a, it is apparent that these measurements do not fluctuate much around the theoretical value. The deviation from the fit is still relatively large, which is mostly due to the behaviour of the ratios at attenuations close to 0 (the effect of which can be accounted for by taking out additional points). As attenuations around zero are not expected to arise in the actual simulations (and if found, would be unsubstantiated), 5 elements is a safe choice.

**B.5. Mesh width of strip in fluid**

For the number of mesh elements per wavelength, another systematic error was observed. A magnified image of the frequency spectrum is shown in figure B.9a, where the bias can clearly be observed. However, the bias is much weaker than for the mesh width of the strip in air, as can be observed from the relative errors in figure B.9b.

Although the systematic error was lower, the same method to find the correct number of mesh elements per wavelength is applied here as in the previous section. The attenuation including the
### B.6. Mesh width of fluid

For the mesh width of the fluid, a slight systematic error was observed. In figure B.11a one can find the attenuation spectra and in figure B.11b the relative errors. Although in general we have observed that 5 elements per wavelength were enough, one can see below that less elements still yield correct results. It is probable that this due to the following reason. The model was chosen such that the...
density of mesh elements was higher closer to the strip (as this is where most of the energy of the Scholte wave travels). As the wave travels away from the strip in the $x_3$ direction towards higher mesh widths, the influence on the reflected signal decreases. The results for the found viscosity are listed in table B.6.

From table B.6, it can be seen that 3 would be the correct parameter to choose. As this parameter is important for the computational cost as well, choosing 3 decreases the computation time by a factor of almost 2 compared to choosing 6 elements per wavelength.

**B.7. Size of time step**

The time stepping size to be applied depends on the mesh width in the model. In general, it may be assumed that the following criterion must be fulfilled [37]:

$$\Delta t < h_{\min} \frac{S}{c}$$  \hspace{1cm} (B.9)

Where we have:
- $\Delta t$  Time step size [s]
- $h$  Mesh width [m]
- $c$  Velocity of travelling wave [m/s]

To find the correct size for the time step, we will take the factor 5 in formula B.10 to vary. This parameter will be called $T_{Sp}$ (time step parameter). In our model, both $h$ and $c$ will vary due to the absence and presence of water. Filling in equation B.10, we find that:

$$\Delta t < \frac{1}{T_{Sp}} \min \left\{ \frac{1}{f_0 \cdot h_{MWAp}}, \frac{1}{f_0 \cdot h_{MWFp}}, \frac{1}{1} \right\}$$  \hspace{1cm} (B.10)
Table B.7: Results for mesh width of strip in fluid

<table>
<thead>
<tr>
<th>hMWFp</th>
<th>Error in found viscosity</th>
<th>Relative deviation from fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>+10%</td>
<td>0.05</td>
</tr>
<tr>
<td>6</td>
<td>+5%</td>
<td>0.03</td>
</tr>
<tr>
<td>7</td>
<td>+1%</td>
<td>0.02</td>
</tr>
<tr>
<td>8</td>
<td>+1%</td>
<td>0.007</td>
</tr>
<tr>
<td>9</td>
<td>+1%</td>
<td>0.004</td>
</tr>
</tbody>
</table>

From the simulations, it was apparent that the systematic error for a change in time step size was very large compared to the other studies. Going from 9 to 5, the signal was changed by a factor of around 2. However, as before, when comparing two studies with the same time step but a different insertion depth, the attenuations were more closely correlated. In figure B.12a attenuation spectra can be found, and in B.12b the relative error can be found. The results for the calculation of the viscosity can be found in table B.7.

![Attenuation for sweep TSp](image)

(a) The reflected signal in the frequency domain

![Error in attenuation for sweep TSp](image)

(b) Relative errors with respect to largest fluid

As the time stepping parameter is an important parameter for the computational cost, we will opt for 8. Albeit all parameters listed before were chosen such that the error would fall within ±0.5% error, and it could be worthwhile to see where the error for the time stepping error would fall below this figure as well, here we have to choose a higher error. At 7, the errors for a found viscosity of +5% and +1% are still comparable (0.016 and 0.015 respectively, or differing by a factor 1.1). However, at 8, the errors for a found viscosity of +5% and +1% are further apart (0.020 and 0.007 respectively, or differing by a factor 2.7), and the error of around 1% is more thoroughly confirmed.

B.8. Conclusion

The parametric studies have shown results that were generally according to expectations. Only for the mesh width of the fluid, the results were different than expected, and time can be saved by a factor of almost 2. The results of the parametric studies have been set forward in table B.8.

For the determination of the group velocity, the arrival time of the waves is important. As the focus was on attenuation in this research, the parametric studies for the group velocity will not be discussed in the same detail. Also, it has only been researched without a fluid. The non-bracketed results in table B.8 list the found parameters for the group velocity determination in vacuum. The bracketed results are those that are expected to be needed for the group velocity determination of the quasi-Scholte wave.

As no fluid is involved for the determination of the group velocity, the computational cost is much lower. Therefore, the parametric studies will not be discussed in the same depth as for those for the attenuations. In the same table below, one can find the chosen parameters and the results for the relative in $c_\delta$. Note that the group velocity will be in the order of 2000 m/s and that we want the
Table B.8: Results for all parametric studies

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Choice for $\alpha$</th>
<th>$\Delta \eta$</th>
<th>Choice for $\epsilon_{\eta}$</th>
<th>$\Delta \epsilon_{\eta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>dF - PWR</td>
<td>1.0 cm</td>
<td>0%</td>
<td>(1.0 cm)</td>
<td>(&lt;0.01%)</td>
</tr>
<tr>
<td>dF - SHB</td>
<td>1.0 cm</td>
<td>0%</td>
<td>(1.0 cm)</td>
<td>(&lt;0.01%)</td>
</tr>
<tr>
<td>hF</td>
<td>0.2 cm</td>
<td>0%</td>
<td>(0.2 cm)</td>
<td>(&lt;0.01%)</td>
</tr>
<tr>
<td>hMWAp</td>
<td>5</td>
<td>0%</td>
<td>7</td>
<td>0.05%</td>
</tr>
<tr>
<td>hMWfp</td>
<td>5</td>
<td>0%</td>
<td>(7)</td>
<td>(0.04%)</td>
</tr>
<tr>
<td>hdMWfp</td>
<td>3</td>
<td>0%</td>
<td>(5)</td>
<td>(0.01%)</td>
</tr>
<tr>
<td>TSp</td>
<td>8</td>
<td>1%</td>
<td>8</td>
<td>0.06%</td>
</tr>
</tbody>
</table>

uncertainty to be within $\pm \ 2 \ m/s$ [38]. All errors fall within this uncertainty.
Bibliography


