# Reactor Physics and Design Aspects of High Temperature Reactors

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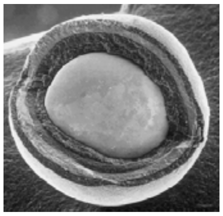
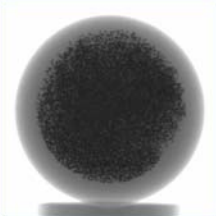
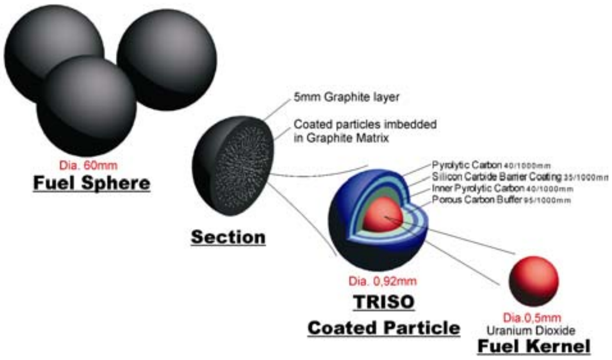
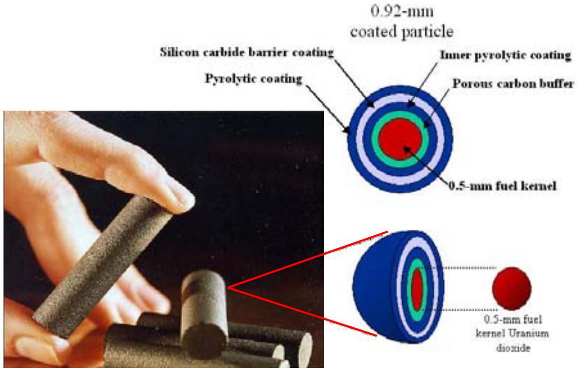
# Pebble-bed fuel







# Prismatic fuel



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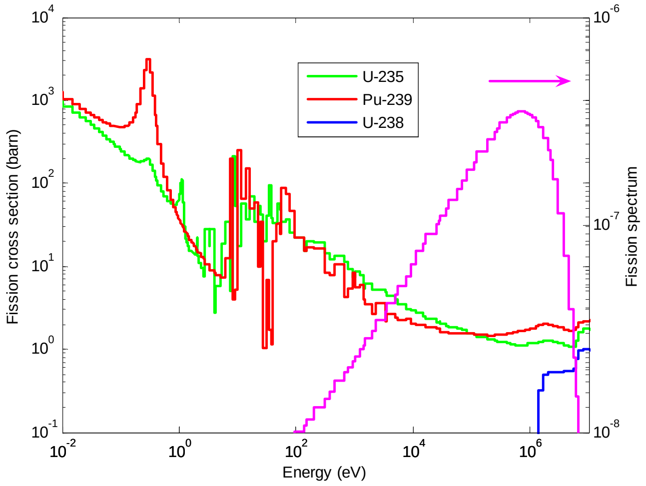
# Differences between LWR and HTR

| | LWR | HTR |
|---|---|---|
| Fuel | $UO_2$ pin | $UO_2$ sphere |
| Moderator | Water | Graphite |
| Coolant | Water | Helium |
| Temperature ($^oC$) | 300 | 900 |
| Enrichment (%) | 5 | 10 |
| Burnup (MWd/kgU) | 60 | 120 |
| Specific power (kW/kgU) | 40 | 80 |
| Power density (kW/l) | 100 | 6 |

# Moderation of neutrons



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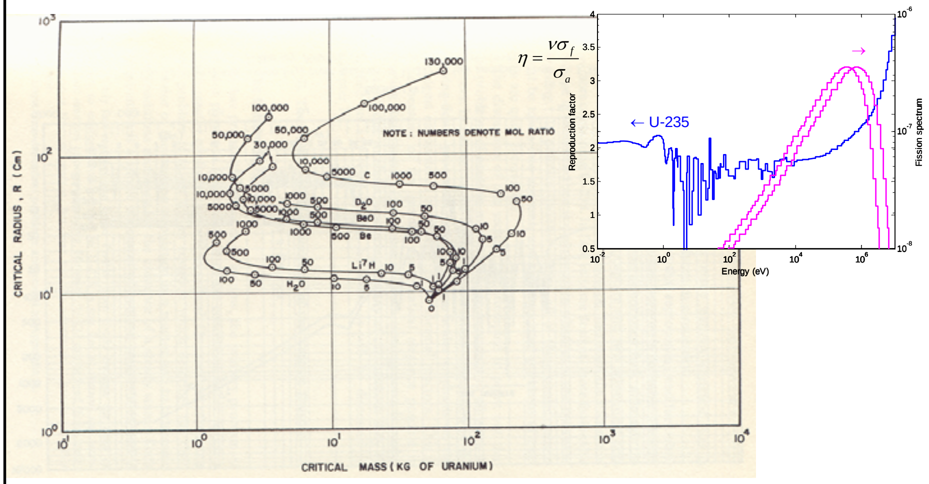
# Moderation of neutrons

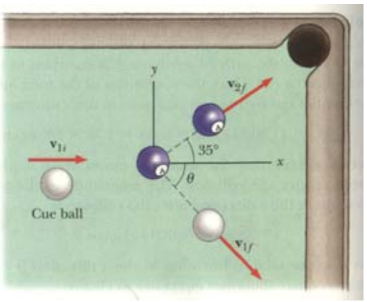


$$\eta = \frac{\nu\sigma_f}{\sigma_a}$$

Ferziger&Zweifel, Theory of neutron slowing down in nuclear reactors, 1966

# Energy transfer in collisions

- Elastic scattering most important
- Conservation of energy and momentum
- Large energy transfer in collisions at light nuclei
- Hydrogen same mass as a neutron* ⇒ largest E-transfer



$$A = \frac{M}{m} = \frac{\text{mass nucleus}}{\text{mass neutron}} \qquad \alpha = \left(\frac{A-1}{A+1}\right)^2$$

*a neutron is 0.1% heavier. Think over the consequences.

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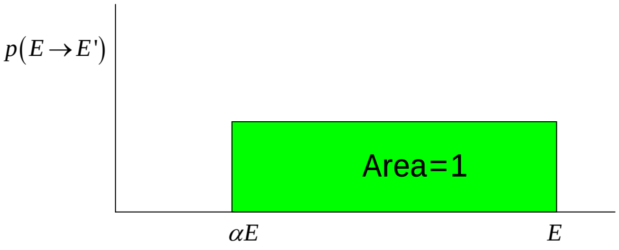
# Energy transfer in collisions

$$p(E \to E') = \begin{cases} \dfrac{1}{(1-\alpha)E} & \text{for } \alpha E < E' < E \\ 0 & \text{elsewhere} \end{cases}$$



# Energy transfer in collisions

Average energy:

$$\overline{E'} = \int_{\alpha E}^{E} E' \, p\left(E \to E'\right) dE' = \frac{1}{2} E\left(1+\alpha\right)$$

Average energy loss:

Proportional to *E*

$$\overline{\Delta E} = E - \overline{E'} = \frac{1}{2} E\left(1-\alpha\right)$$

# Lethargy

Transform Energy to a new variable that changes linearly in each collision $\Rightarrow$ Lethargy

$$u = \log\left(\frac{E_H}{E}\right)$$

Average lethargy increase per collision:

$$\xi = \int_{\alpha E_H}^{E_H} \log\left(\frac{E_H}{E}\right) \cdot \frac{1}{\left(1-\alpha\right) E_H} dE$$

$$= 1 + \frac{\alpha}{\left(1-\alpha\right)} \log\left(\alpha\right)$$

# Number of collisions

Number of collisions to slow down a neutron from $E_H$ =1 Mev to $E_L$ =1 eV in various media:

$$n = \frac{\log\left(E_H / E_L\right)}{\xi}$$

| Element | A | $\alpha$ | $\xi$ | $n$ |
|---|---|---|---|---|
| H | 1 | 0 | 1.000 | 14 |
| D | 2 | 0.111 | 0.725 | 19 |
| Be | 9 | 0.640 | 0.207 | 67 |
| C | 12 | 0.716 | 0.158 | 88 |
| U | 238 | 0.983 | 0.00838 | 1649 |

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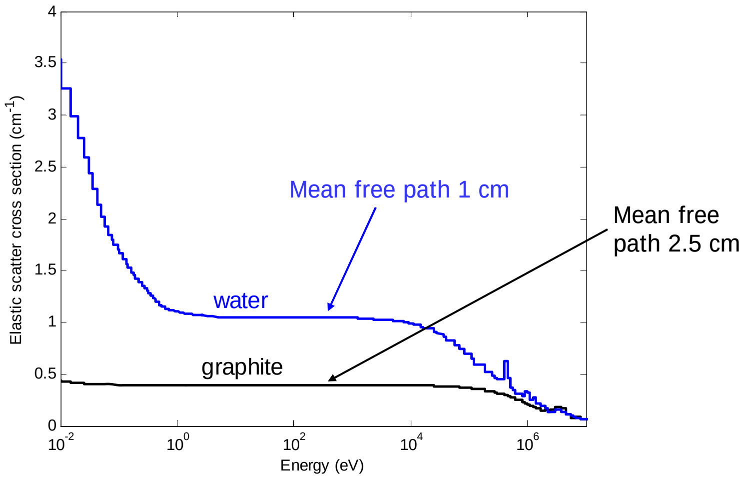
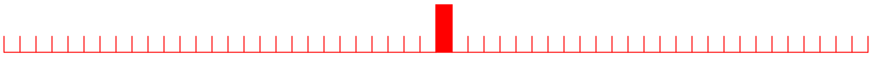
# Elastic scatter cross section



# Space-dependent slowing down

- Moderating power $\equiv \xi \Sigma_s$
- Energy transfer to the moderator per unit path length = lethargy gain by the neutron: $du = \xi \Sigma_s dx$



Monte Carlo game:

- Start particles at isotropic plane source
- Follow the particles from interval to interval
- In each interval, certain probability to scatter
- When scattering, particles can reverse direction
- When scattering, particles gain $\xi$ lethargy

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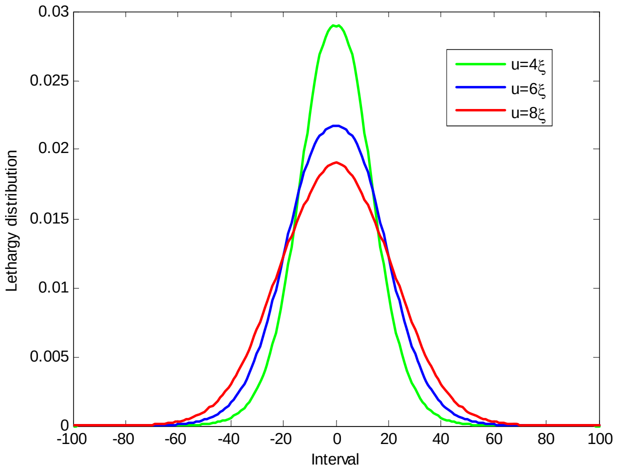
# Space-dependent slowing down



# Space-dependent slowing down

Fermi-age model
Age-diffusion equation
Continuous slowing down model

Takes the form of time-dependent diffusion eq. without absorption

$$\boxed{\nabla^2 q(\mathbf{r},\tau) = \frac{\partial q(\mathbf{r},\tau)}{\partial \tau}}$$

where $q(\mathbf{r},u) = \xi \Sigma_s(u)\phi(\mathbf{r},u)$ (slowing down density) and $\tau(u)$ is the Fermi age:

$$\tau = \frac{1}{6}\langle r^2 \rangle$$

$\tau$ is proportional to the mean squared distance a source neutron travels until it reaches $\tau$

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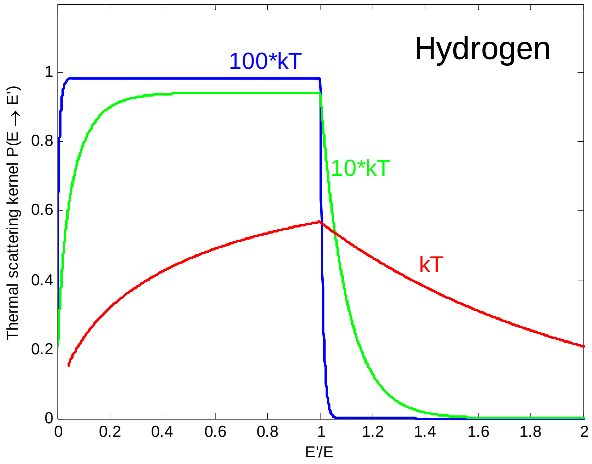
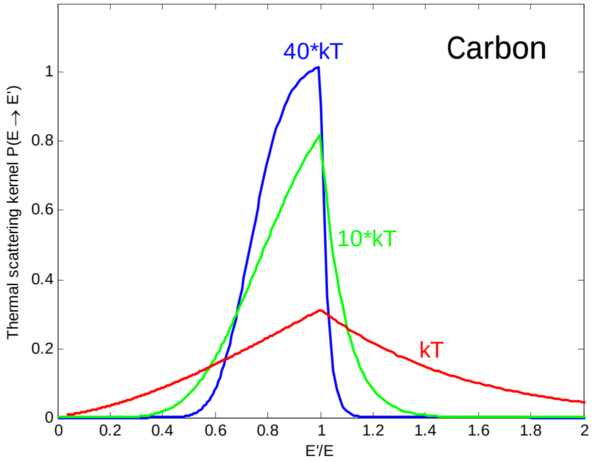
# Thermal scattering kernel



Lamarsh, Introduction to Nuclear Reactor Theory, 1966

# Thermal scattering kernel



Massimo, Physics of High Temperature Reactors, 1976

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# Thermal neutron spectrum

$$M(E) = \frac{2\pi n_0}{(\pi kT)^{3/2}} \sqrt{E} \exp\left(\frac{-E}{kT}\right)$$

Average neutron energy:

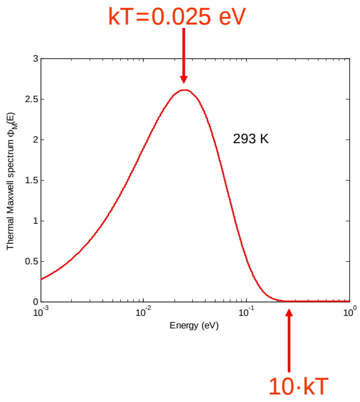
$$\overline{E} = \int_0^\infty E\, M(E)\, dE = \frac{3}{2} kT$$

$$\phi_M(E) = \frac{2\pi n_0}{(\pi kT)^{3/2}} \left(\frac{2}{m}\right)^{1/2} E \exp\left(\frac{-E}{kT}\right)$$

Most probable energy:

$$\frac{\partial \phi_M(E)}{\partial E} = 0 \Rightarrow E_T = kT$$

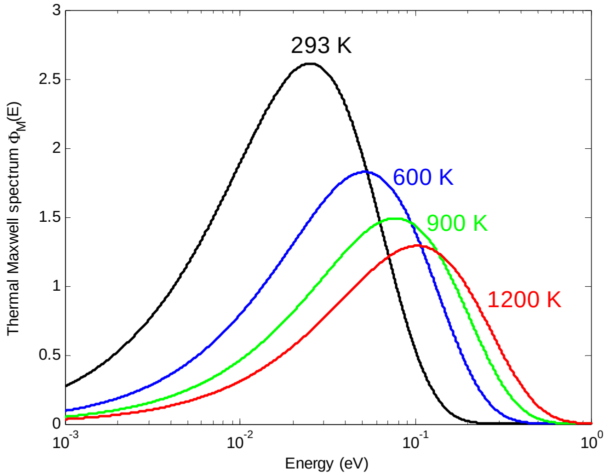
Corresponding velocity: $v = \sqrt{\frac{2kT}{m}}$ (2200 m/s for T=293 K)



kT=0.025 eV

10·kT

## Thermal neutron spectrum



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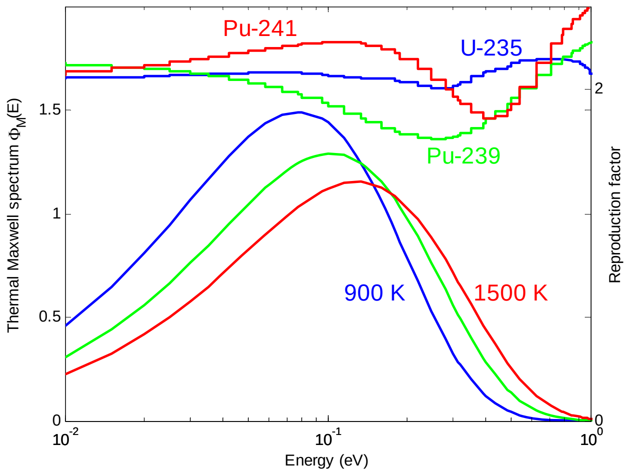
## Moderator temperature effect



$$\eta = \frac{\nu \Sigma_f}{\Sigma_a}$$

# Moderator temperature effect

|  | $T_{PEBBLE}$ | $T_{REFLEC}$ | $k$ | $\alpha\left(\text{pcm/K}\right)$ |
|---|---|---|---|---|
|  | 1100 | 1100 | 1.32553 |  |
| Pebble | 1400 | 1100 | 1.30006 | -4.9 |
| Reflec | 1100 | 1400 | 1.33318 | 1.4 |

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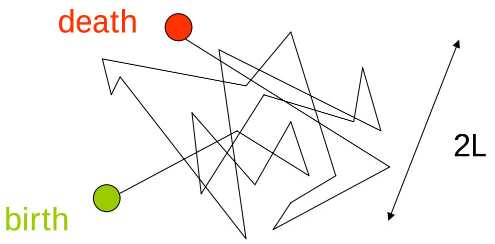
# Space-dependent slowing down

Recall from diffusion theory $\left(\text{assumed to be known}\right)$

$L$ is diffusion length

$$L^2 = \frac{1}{6}\left\langle r^2 \right\rangle$$

$L^2$ is mean squared distance a thermal neutron travels until absorption



# Space-dependent slowing down

Fermi age $\tau = \frac{1}{6}\langle r^2 \rangle$  Diffusion length $L^2 = \frac{1}{6}\langle r^2 \rangle$

Migration area $\boxed{M^2 = \tau + L^2}$

Migration length $M$ is $1/\sqrt{6}$ of the rms distance a neutron travels between birth as a fission neutron and absorption in thermal range

**TABLE 8-7: Diffusion Parameters for Some Common Moderators**

| Moderator | Density (g/cm$^3$) | D(cm) | $\Sigma_a$(cm$^{-1}$) | $L$(cm) | $\tau_{th}$(cm$^2$) | $M$(cm) |
|---|---|---|---|---|---|---|
| $H_2O$ | 1.00 | 0.16 | 0.0197 | 2.85 | 26 | 5.84 |
| $D_2O$ | 1.10 | 0.87 | $2.9\times10^{-5}$ | 170 | 131 | 170 |
| Be | 1.85 | 0.50 | $1.0\times10^{-3}$ | 21 | 102 | 23 |
| Graphite | 1.60 | 0.84 | $2.4\times10^{-4}$ | 59 | 368 | 62 |

Duderstadt&Hamilton, Nuclear Reactor Analysis, 1976

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# Safety design philosophy

Fundamental safety functions

- Control of reactivity
- Removal of (decay) heat from the core
- Confinement of radioactive material

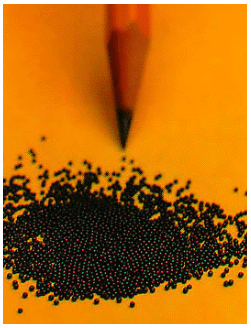
Defense in depth

- Prevention of off-normal event
- Control of off-normal event
- Mitigation of off-normal event

# Fission product containment

HTR

LWR



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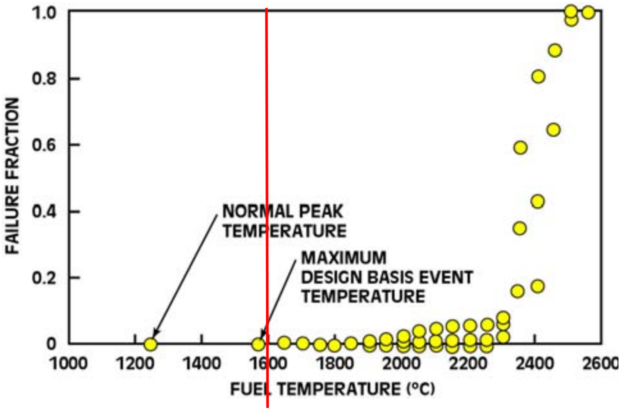
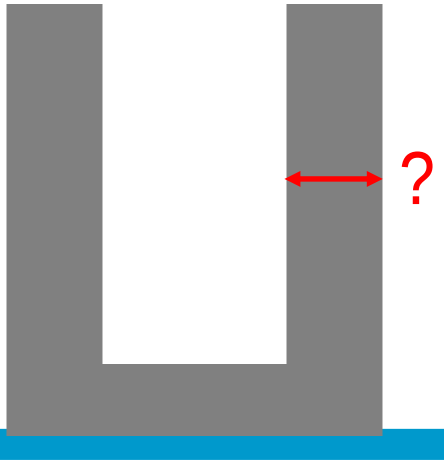
# Fission product containment



Considered to be a safe lower temperature limit

# Pebble-bed reactor core design



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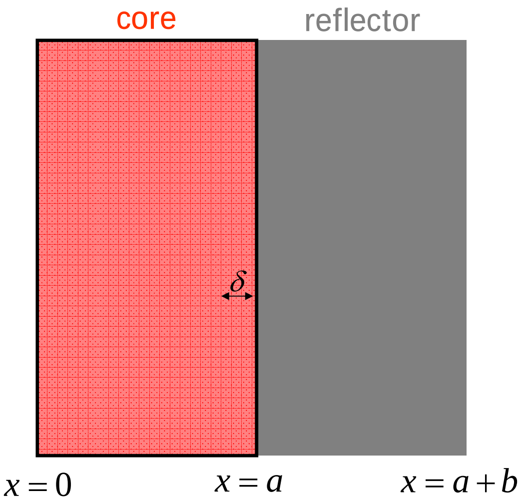
# Reflector savings



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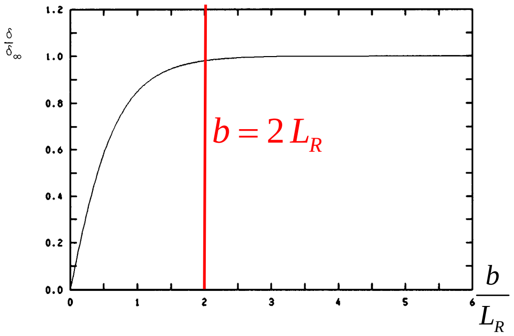

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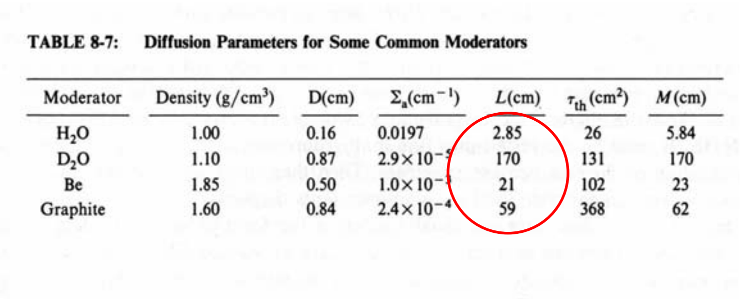

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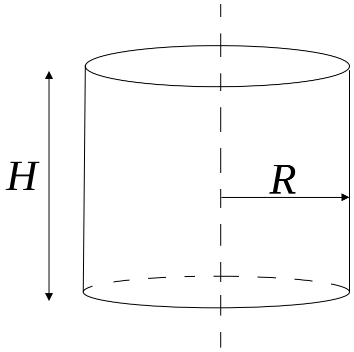
Duderstadt and Hamilton, Nuclear Reactor Analysis, 1976

# Low leakage core design



$$P_{NL} = P_{FNL} \cdot P_{TNL}$$

$$= \frac{1}{1+B_g^2\tau} \cdot \frac{1}{1+B_g^2L^2}$$

$$\approx \frac{1}{1+B_g^2M^2}$$

$\tau$ is a measure for the rms distance travelled until the neutron reaches the thermal energy range

$L^2$ is a measure for the rms distance a thermal neutron travels until is is absorbed

$M^2$ is migration area

$$\boxed{H = 1.85\,R}$$

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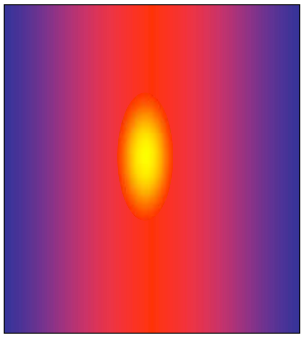
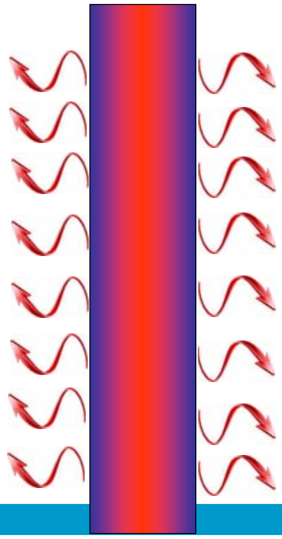
# Reactor core design

DLOFC





Passive decay heat removal needs long tall reactor cores

# Reactor core design

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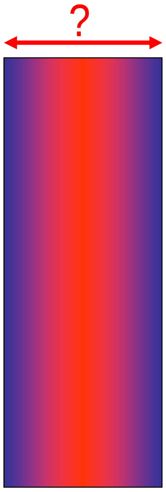
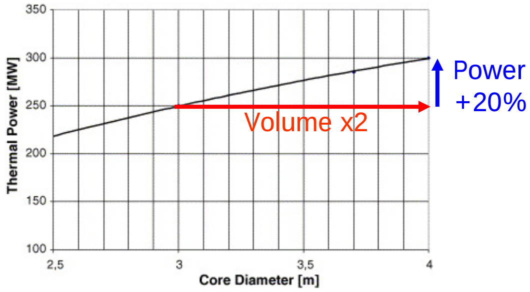


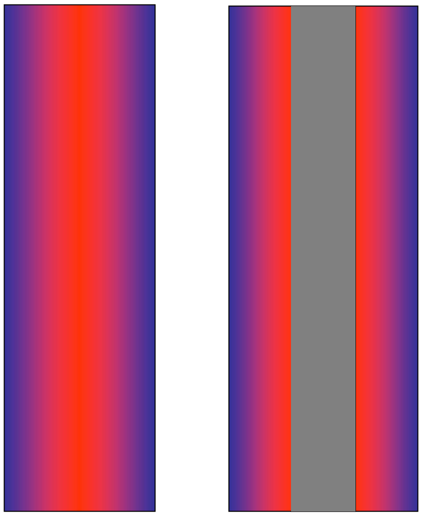


Ben Said etal, Nucl Eng Des, 236(2006) 648

Core diameter limited by

- shutdown capability
- RPV diameter
- decay heat removal

# Reactor core design



Passive decay heat removal needs long tall reactor cores

*Or*

Cores with a fuel-free central column

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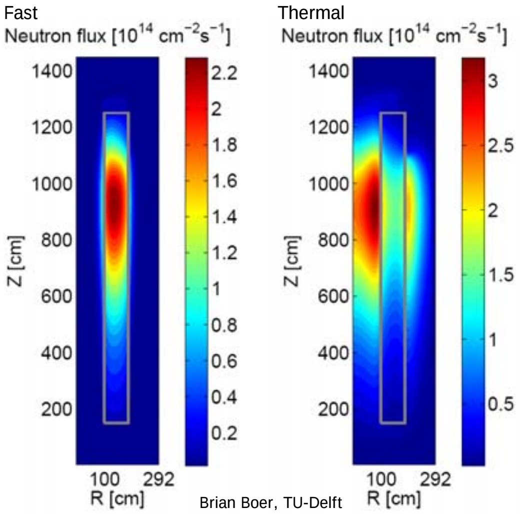
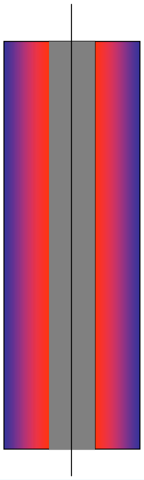
# Neutron flux profile



Brian Boer, TU-Delft

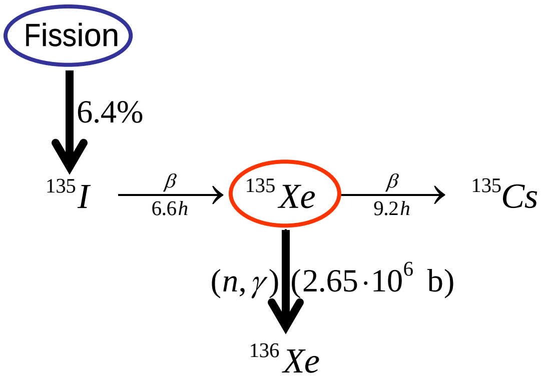


# Reactor core design

| *Design* | | | | |
|---|---|---|---|---|
| Core height [m] | 11 | | 11 | |
| Inner core diameter [m] | - | | 2 | |
| Outer core diameter [m] | 3 | | 3.7 | |
| RPV outer diameter [m] | 5.7 | | 6.4 | |
| *Variant* | *Brayton* | *Rankine* | *Brayton* | *Rankine* |
| Inlet temperature [°C] | 500 | 250 | 500 | 250 |
| Outlet temperature [°C] | 900 | 750 | 900 | 750 |
| **Thermal power [MW]** | **220** | **250** | **425** | **464** |
| Mean power density [MW/m³] | 2.83 | 3.21 | 5.07 | 5.54 |
| Helium mass flow rate [Kg/s] | 105.9 | 96.24 | 204.5 | 178.6 |
| Pressure drop over the core [bar] | 1.27 | 0.84 | 3.55 | 2.18 |

Ben Said etal, Nucl Eng Des, 236(2006) 648

# Maximum core height



Fission

6.4%

$$^{135}I \xrightarrow[6.6h]{\beta} {}^{135}Xe \xrightarrow[9.2h]{\beta} {}^{135}Cs$$

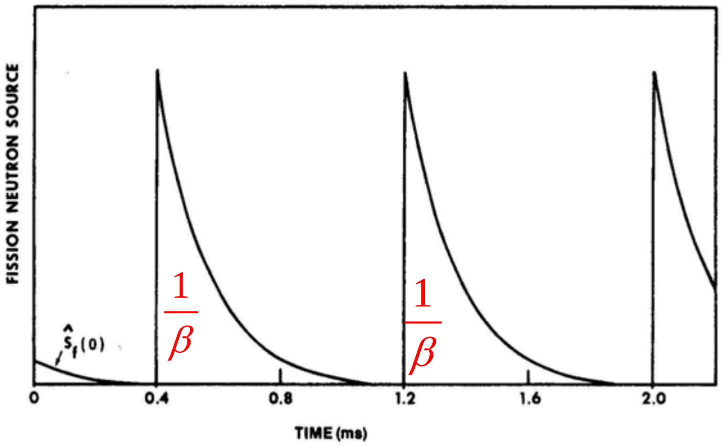
$(n,\gamma)$ $(2.65\cdot10^{6}\ \text{b})$

$^{136}Xe$

$$\Phi\uparrow \;\Rightarrow\; [Xe]\downarrow \;\Rightarrow\; \rho\uparrow \;\Rightarrow\; \Phi\uparrow \quad \text{Positive feedback !}$$

# Xenon oscillations

# Maximum core height



Ott and Neuholt, Introductory Dynamics of Nuclear Reactors

# Maximum core height

Migration area: $M^2 = L^2 + \tau = \frac{1}{6} \langle \ell^2 \rangle$

$(L \text{ diffusion length, } \tau \text{ Fermi age})$

Squared prompt fission chain length: $L_{PFC}^2 = \frac{1}{\beta} \langle \ell^2 \rangle = \frac{1}{\beta} 6M^2$

Prompt fission chain length: $L_{PFC} = \sqrt{\frac{6}{\beta} \langle M^2 \rangle} \approx 30\,M$

Any core larger than 30M is loosely coupled

and therefore sensitive to Xenon oscillations

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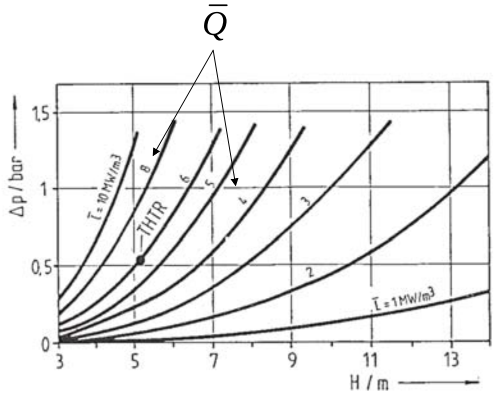
# Maximum core height

Pressure drop: $\Delta p \sim \frac{\bar{Q}^2 H^3}{\rho \Delta T^2}$

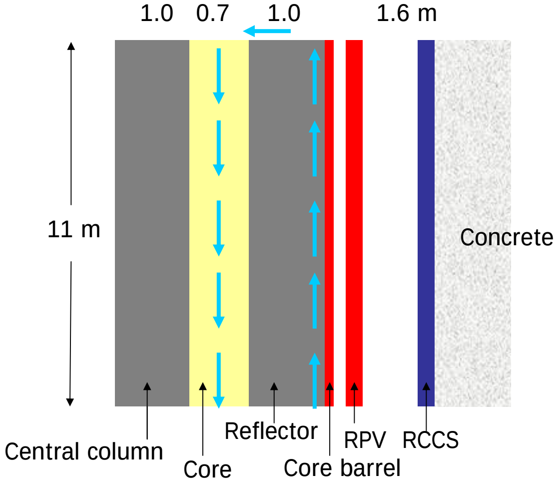
Pumping power: $P_P = \frac{\Delta p}{\rho} \dot{m}$

$(\text{In practice } \Delta p < 0.02\, p)$



Kugeler and Schulten, Hochtemperatur Reaktortechnik, 1989

# Reactor core layout



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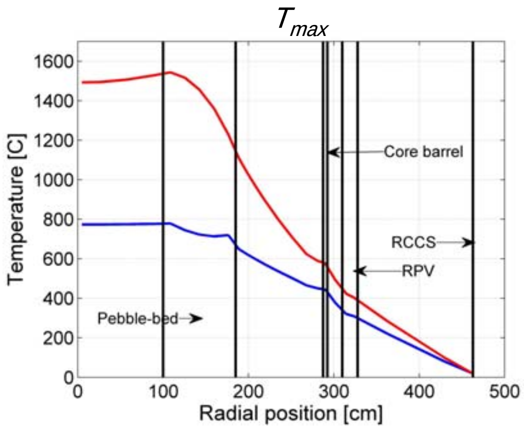
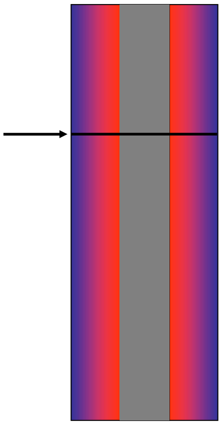
# Temperatures during DLOFC





PBMR results DLOFC, Brian Boer, TU-Delft