Monte Carlo Forward-Adjoint Coupling by Next Event Estimation

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Abstract

Coupling of forward and adjoint collision sites from Monte Carlo calculations can be done by next event estimation. We show that this method is efficient and statistically stable. To do this, we take two steps; 1) a perturbation-invariant technique for the enclosure theorem in generalized contributon response theory, and 2) coupling of the collision sites from a batch of forward histories and another batch of adjoint histories. Step 1 completely eliminates negative scores, and step 2 creates larger numbers of the combinations of forward and adjoint collision sites than simple coupling from one forward and one adjoint history. In addition, the proposed method does not require discretization on an intermediate plane. Numerical results are presented for a simple source-detector problem.

1 Introduction

Monte Carlo (MC) simulations of a small and distant source-detector problem are a challenging task. The target is small for the particles in both forward and adjoint calculations, and the convergence is statistically slow in next event estimation (Pederson 1997). In such cases, forward-adjoint coupling at a large scoring surface is an attractive choice (Serov 1999). Cramer proposed a MC next event coupling as an intermediate device for connecting multiple pairs of forward-adjoint histories in neutron duct-streaming calculations (Cramer 1996). We show that this building component can be an efficient and reliable variance reduction technique for small source-detector problems.

Coupling calculations are based on the enclosure theorem (Williams 1977). Instead of the detector response, one may compute the response flow integral (RFI) on any detector enclosure excluding the source. According to generalized contributon response theory (Williams 1991), the RFI is invariant with respect to any perturbation outside the detector enclosure in the adjoint calculation. Practically, this leads to simple deterministic adjoint calculations by homogenization (Aboughantous 1994) and shorter adjoint histories by black-absorber perturbation (Serov 1999). We utilize a purely absorbing medium for the perturbation-invariant technique in next event coupling across an intermediate plane. This simultaneously eliminates the negative scores and tally discretization on that plane, which was not accomplished by the previous work (Cramer 1996, Serov 1999).

In the actual implementation, we couple the collision sites in a batch of forward and adjoint MC histories. This means that a single statistically-independent unit consists of a collection of forward histories and another collection of adjoint histories. This enables one to create many combinations of forward-adjoint collision sites per coupling from a small number of histories. This effect greatly outweights the additional computational cost, and stabilizes the convergence property of the sample mean and variance. Significant efficiency increase can be attained with no transport biasing.

2 Theoretical background

We consider the following forward and perturbed-adjoint multigroup neutron transport problem for $\underline{r} \in D$ (problem domain) and $1 \leq g \leq G$ (energy group) with the forward source (adjoint detector) S_g^F and forward detector (adjoint source) S_g^A:

$$\underline{\Omega} \cdot \underline{\nabla}\Psi_g(\underline{r}, \underline{\Omega}) + \Sigma_{t,g}(\underline{r})\Psi_g(\underline{r}, \underline{\Omega}) = \sum_{g'=1}^{G} \int_{4\pi} \Sigma_{s,g' \to g}(\underline{r}, \underline{\Omega}' \to \underline{\Omega})\Psi_{g'}(\underline{r}, \underline{\Omega}')d\Omega' + S_g^F(\underline{r}, \underline{\Omega}), \quad (1)$$

$$\Psi_g(\underline{r}, \underline{\Omega}) = 0 \qquad \text{for } \underline{r} \in \partial D \text{ and } \underline{\Omega} \cdot \underline{n} < 0, \quad (2)$$

and:

$$-\underline{\Omega} \cdot \underline{\nabla}\tilde{\Psi}_g^*(\underline{r}, \underline{\Omega}) + \Sigma_{t,g}(\underline{r})\tilde{\Psi}_g^*(\underline{r}, \underline{\Omega}) = \begin{cases} \sum_{g'=1}^{G} \int_{4\pi} \Sigma_{s,g \to g'}(\underline{r}, \underline{\Omega} \to \underline{\Omega}')\tilde{\Psi}_{g'}^*(\underline{r}, \underline{\Omega}')d\Omega' + S_g^A(\underline{r}, \underline{\Omega}), & \underline{r} \in V_A, \\ 0, & \underline{r} \in V_F, \end{cases} \quad (3)$$

$$\tilde{\Psi}_g^*(\underline{r}, \underline{\Omega}) = 0 \qquad \text{for } \underline{r} \in \partial D \text{ and } \underline{\Omega} \cdot \underline{n} > 0, \quad (4)$$

where D is divided into two mutually exclusive regions V_F and V_A by an intermediate surface A (Figure 1). The volumes V_F and V_A contain exclusively the forward source and detector, respectively

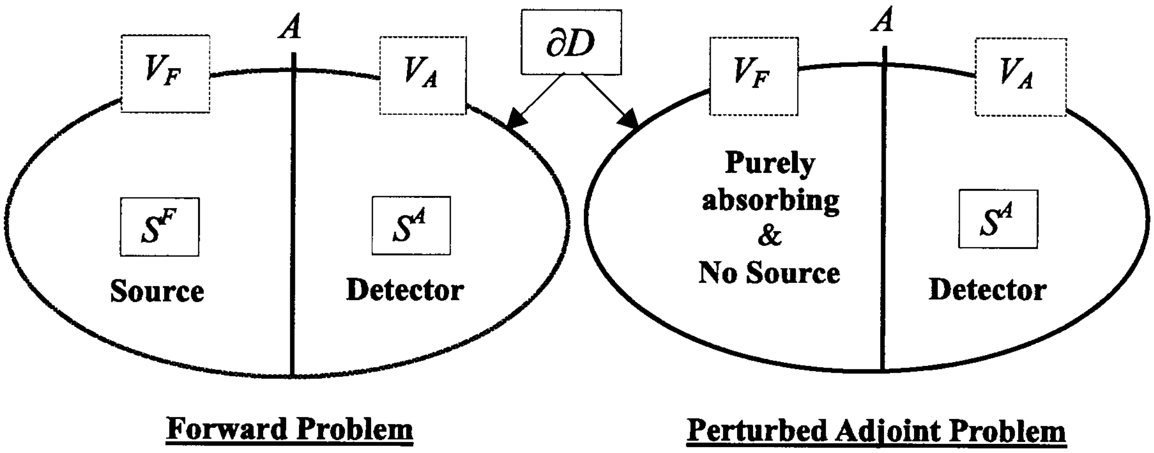

Figure 1: Pictorial Problem Statement

($S_g^A = 0$ in V_F and $S_g^F = 0$ in V_A). Also, the adjoint perturbation is restricted to V_F. According to generalized contributon response theory (Williams 1991), the RFI on the surface A, which is equal to the forward detector response, is invariant with respect to any perturbation in V_F in the adjoint problem. Therefore, by Eqs. (1)-(4), the detector response R is expressed as

$$R = \sum_{g=1}^{G} \int_{V_A} \int_{4\pi} S_g^A(\underline{r}, \underline{\Omega}) \Psi_g(\underline{r}, \underline{\Omega}) d\Omega dV = \sum_{g=1}^{G} \int_{A} \int_{4\pi} \underline{n} \cdot \underline{\Omega} \Psi_g(\underline{r}, \underline{\Omega}) \tilde{\Psi}_g^*(\underline{r}, \underline{\Omega}) d\Omega dA, \qquad (5)$$

where the unit normal vector $\underline{n}$ is pointing toward the forward detector side. Actually, Eq. (5) can be derived by multiplying Eq. (1) by $\tilde{\Psi}_g^*(\underline{r}, \underline{\Omega})$ and Eq. (3) by $\Psi_g(\underline{r}, \underline{\Omega})$, subtracting the latter from the former, integrating the resulting expression over all directions and V_A, using $S_g^F(\underline{r}, \underline{\Omega}) = 0$ in V_A, summing over the energy group indices g, converting the left hand side (LHS) to surface integral, and using the exterior boundary conditions, Eqs. (2) and (4).

Next, multiplying Eq. (1) by $\tilde{\Psi}_g^*(\underline{r}, \underline{\Omega})$ and Eq. (3) by $\Psi_g(\underline{r}, \underline{\Omega})$, subtracting the latter from the former, integrating the resulting expression over all directions and V_F (not V_A!), and summing over the energy group indices, we obtain:

$$\begin{aligned} &\sum_{g=1}^{G} \int_{A} \int_{4\pi} \underline{n} \cdot \underline{\Omega} \Psi_g(\underline{r}, \underline{\Omega}) \tilde{\Psi}_g^*(\underline{r}, \underline{\Omega}) d\Omega dA \\ &\quad = \sum_{g=1}^{G} \int_{V_F} \int_{4\pi} \tilde{\Psi}_g^*(\underline{r}, \underline{\Omega}) \left[\sum_{g'=1}^{G} \int_{4\pi} \Sigma_{s,g' \to g}(\underline{r}, \underline{\Omega}' \to \underline{\Omega}) \Psi_{g'}(\underline{r}, \underline{\Omega}') d\Omega' + S_g^F(\underline{r}, \underline{\Omega}) \right] d\Omega dV \, . \qquad (6) \end{aligned}$$

where, after the conversion from the volume to surface integral in the LHS, we again used the exterior boundary conditions, Eqs. (2) and (4). Eqs. (5) and (6) yield

$$R = \sum_{g=1}^{G} \int_{V_F} \int_{4\pi} \tilde{\Psi}_g^*(\underline{r}, \underline{\Omega}) \left[\sum_{g'=1}^{G} \int_{4\pi} \Sigma_{s,g' \to g}(\underline{r}, \underline{\Omega}' \to \underline{\Omega}) \Psi_{g'}(\underline{r}, \underline{\Omega}') d\Omega' + S_g^F(\underline{r}, \underline{\Omega}) \right] d\Omega dV. \qquad (7)$$

We wish to compute the right hand side (RHS) of Eq. (7) by next-event estimation. To clarify the various terms with different origins, we divide the perturbed-adjoint angular flux into two components:

$$\tilde{\Psi}_g^*(\underline{r}, \underline{\Omega}) = \tilde{\Psi}_{g,c}^*(\underline{r}, \underline{\Omega}) + \tilde{\Psi}_{g,u}^*(\underline{r}, \underline{\Omega}) \qquad (8)$$

where $\tilde{\Psi}_{g,c}^*$ and $\tilde{\Psi}_{g,u}^*$ are the collided and uncollided components, respectively. Eq. (7) then becomes

$$\begin{aligned} R \;=\; & \sum_{g=1}^{G} \int_{V_F} \int_{4\pi} \tilde{\Psi}_{g,c}^*(\underline{r}, \underline{\Omega}) \left[\sum_{g'=1}^{G} \int_{4\pi} \Sigma_{s,g' \to g}(\underline{r}, \underline{\Omega}' \to \underline{\Omega}) \Psi_{g'}(\underline{r}, \underline{\Omega}') d\Omega' \right] d\Omega dV \\ & + \sum_{g=1}^{G} \int_{V_F} \int_{4\pi} \tilde{\Psi}_{g,u}^*(\underline{r}, \underline{\Omega}) \left[\sum_{g'=1}^{G} \int_{4\pi} \Sigma_{s,g' \to g}(\underline{r}, \underline{\Omega}' \to \underline{\Omega}) \Psi_{g'}(\underline{r}, \underline{\Omega}') d\Omega' \right] d\Omega dV \\ & + \sum_{g=1}^{G} \int_{V_F} \int_{4\pi} \tilde{\Psi}_{g,c}^*(\underline{r}, \underline{\Omega}) S_g^F(\underline{r}, \underline{\Omega}) d\Omega dV + \sum_{g=1}^{G} \int_{V_F} \int_{4\pi} \tilde{\Psi}_{g,u}^*(\underline{r}, \underline{\Omega}) S_g^F(\underline{r}, \underline{\Omega}) d\Omega dV \, . \qquad (9) \end{aligned}$$

Now, we explain how to estimate the various terms on the RHS of Eq. (9) through the empirical collision densities based on the realizations of forward and adjoint particle walks. Our key idea is to express these collision densities by the sum of the products of the δ-functions with the arguments $\underline{r}$ and $\underline{\Omega}$ and the Kronecker-δ with the subscript g, convert that sum to emission density (for forward and adjoint terms), and further convert it to angular flux (only for adjoint terms). This enables one to calculate an expectation conditioned on a pair of the realized forward and adjoint particle walks. Since the expectation of a conditional expectation is a true expectation, this approach gives one an unbiased estimation. (Next event estimations collect conditional expectations based on some form of conditioning on a realized particle walk.) We pick up individual contributions to the RHS of Eq. (9), and derive their corresponding scores.

Firstly, we explain how to estimate the first term on the RHS of Eq. (9). Suppose that $\underline{\Omega}^f_{i-1}$, W^f_{i-1} and g^f_{i-1} are the direction of the movement, weight and energy group index of the forward particle that was about to enter the i-th collision at $\underline{r}^f_i$ in V_F. The i-th collided component of the empirically realized g'-group forward collision density at $(\underline{r}, \underline{\Omega}')$ is expressed as:

$$W^f_{i-1}\delta_{g',g^f_{i-1}}\delta(\underline{\Omega}' - \underline{\Omega}^f_{i-1})\delta(\underline{r} - \underline{r}^f_i)\,. \tag{10}$$

This partially characterizes conditioning events in next event estimation. We define the forward kernel at $\underline{r}$ from collision to emission density as:

$$P^f_{g'\to g}(\underline{r}, \underline{\Omega}' \to \underline{\Omega}) = \Sigma_{s,g'\to g}(\underline{r}, \underline{\Omega}' \to \underline{\Omega}) \;/\; \Sigma_{t,g'}(\underline{r})\,, \tag{11}$$

which yields the mean number of emitted particles per collided forward particle if it is integrated over $\underline{\Omega}$ and summed over g. The g-group empirical emission density at $(\underline{r},\underline{\Omega})$ due to the i-th forward collision then becomes

$$\begin{aligned} &\sum_{g'=1}^{G}\int_{4\pi} P^f_{g'\to g}(\underline{r}, \underline{\Omega}' \to \underline{\Omega}) W^f_{i-1}\delta_{g',g^f_{i-1}}\delta(\underline{\Omega}' - \underline{\Omega}^f_{i-1})\delta(\underline{r} - \underline{r}^f_i)d\Omega' \\ =\; & W^f_{i-1}P^f_{g^f_{i-1}\to g}(\underline{r}^f_i, \underline{\Omega}^f_{i-1} \to \underline{\Omega})\delta(\underline{r} - \underline{r}^f_i). \end{aligned} \tag{12}$$

For adjoint quantities, suppose that $-\underline{\Omega}^a_{j-1}$, W^a_{j-1} and g^a_{j-1} are the direction of the movement, weight and energy index of the adjoint particle that was about to enter the j-th collision at $\underline{r}^a_j$ in V_A. The j-th collided component of the empirically realized g'-group adjoint collision density at $(\underline{r}, \underline{\Omega}')$ is expressed as

$$W^a_{j-1}\delta_{g',g^a_{j-1}}\delta(\underline{\Omega}' - \underline{\Omega}^a_{j-1})\delta(\underline{r} - \underline{r}^a_j)\,. \tag{13}$$

Here, we should note that adjoint particles move in the direction opposite to the phase space variable $\underline{\Omega}$. Eqs. (10) and (13) completely characterize conditioning events in the next event estimation for the contribution from the i-th forward and j-th adjoint collisions. We define the adjoint kernel at $\underline{r}$ from collision to emission density as:

$$P^a_{g'\to g}(\underline{r}, \underline{\Omega}' \to \underline{\Omega}) = \Sigma_{s,g\to g'}(\underline{r}, \underline{\Omega} \to \underline{\Omega}') \;/\; \Sigma_{t,g'}(\underline{r}), \tag{14}$$

which yields the mean number of emitted particles per collided adjoint particle if it is integrated over $\underline{\Omega}$ and summed over g. The g-th group empirical emission density at $(\underline{r},\underline{\Omega})$ due to the j-th adjoint collision then becomes

$$\sum_{g'=1}^{G}\int_{4\pi} P^a_{g'\to g}(\underline{r},\underline{\Omega}'\to\underline{\Omega})W^a_{j-1}\delta_{g',g^a_{j-1}}\delta(\underline{\Omega}'-\underline{\Omega}^a_{j-1})\delta(\underline{r}-\underline{r}^a_j)d\Omega'$$
$$= W^a_{j-1}P^a_{g^a_{j-1}\to g}(\underline{r}^a_j,\underline{\Omega}^a_{j-1}\to\underline{\Omega})\delta(\underline{r}-\underline{r}^a_j). \quad (15)$$

By following a conventional treatment for the conversion from emission density to angular flux as can be seen in (Lux, 1991), the g-group empirical adjoint angular flux at $(\underline{r},\underline{\Omega})$ due to the j-th adjoint collision becomes

$$\int_{V_A}\Bigg[\exp\left(-\int_0^{|\underline{r}'-\underline{r}|}\Sigma_{t,g}(\underline{r}+s\underline{\Omega})ds\right)$$
$$W^a_{j-1}P^a_{g^a_{j-1}\to g}(\underline{r}^a_j,\underline{\Omega}^a_{j-1}\to\underline{\Omega})\delta(\underline{r}'-\underline{r}^a_j)\delta\left(\underline{\Omega}-\frac{\underline{r}'-\underline{r}}{|\underline{r}'-\underline{r}|}\right)\frac{1}{|\underline{r}'-\underline{r}|^2}\Bigg]dV'$$
$$= \exp\left(-\int_0^{|\underline{r}^a_j-\underline{r}|}\Sigma_{t,g}(\underline{r}+s\underline{\Omega})ds\right)$$
$$W^a_{j-1}P^a_{g^a_{j-1}\to g}(\underline{r}^a_j,\underline{\Omega}^a_{j-1}\to\underline{\Omega})\,\delta\left(\underline{\Omega}-\frac{\underline{r}^a_j-\underline{r}}{|\underline{r}^a_j-\underline{r}|}\right)\frac{1}{|\underline{r}^a_j-\underline{r}|^2}, \quad (16)$$

where we should again note that the adjoint particle moves in the direction $-\underline{\Omega}$, and the δ function involving $\underline{\Omega}$ was introduced to pick up the contribution only from the adjoint particles emitted toward $\underline{r}$ along the line $\underline{r}+s\underline{\Omega}$. Finally, using the expressions in Eqs. (12) and (16), the first term on RHS of Eq. (9) due to the i-th forward and j-th adjoint collisions is estimated as

$$\sum_{g=1}^{G}\int_{V_F}\int_{4\pi}\Bigg[W^f_{i-1}P^f_{g^f_{i-1}\to g}(\underline{r}^f_i,\underline{\Omega}^f_{i-1}\to\underline{\Omega})\delta(\underline{r}-\underline{r}^f_i)$$
$$\exp\left(-\int_0^{|\underline{r}^a_j-\underline{r}|}\Sigma_{t,g}(\underline{r}+s\underline{\Omega})ds\right)W^a_{j-1}P^a_{g^a_{j-1}\to g}(\underline{r}^a_j,\underline{\Omega}^a_{j-1}\to\underline{\Omega})\,\delta\left(\underline{\Omega}-\frac{\underline{r}^a_j-\underline{r}}{|\underline{r}^a_j-\underline{r}|}\right)\frac{1}{|\underline{r}^a_j-\underline{r}|^2}\Bigg]d\Omega dV$$
$$=\sum_{g=1}^{G}W^f_{i-1}P^f_{g^f_{i-1}\to g}\left(\underline{r}^f_i,\underline{\Omega}^f_{i-1}\to\frac{\underline{r}^a_j-\underline{r}^f_i}{|\underline{r}^a_j-\underline{r}^f_i|}\right)\exp\left[-\int_0^{|\underline{r}^a_j-\underline{r}^f_i|}\Sigma_{t,g}\left(\underline{r}^f_i+s\frac{\underline{r}^a_j-\underline{r}^f_i}{|\underline{r}^a_j-\underline{r}^f_i|}\right)ds\right]$$
$$W^a_{j-1}P^a_{g^a_{j-1}\to g}\left(\underline{r}^a_j,\underline{\Omega}^a_{j-1}\to\frac{\underline{r}^a_j-\underline{r}^f_i}{|\underline{r}^a_j-\underline{r}^f_i|}\right)\frac{1}{|\underline{r}^a_j-\underline{r}^f_i|^2} \quad (17)$$

Since $\underline{r}^f_i\in V_F$ by Eq. (9) and $\underline{r}^a_j\in V_A$ by Eq. (3), this coupling occurs only across the intermediate surface A and always yields non-negative scores. (Coupling does not occur if $\underline{r}^f_i\in V_A$, and $\underline{r}^a_j$ can not be in V_F, or in other words $P^a_{g^a_{j-1}\to g}$ is zero for $\underline{r}^a_j\in V_F$.) The former aspect in this estimation virtually eliminates the extremely close encounter of forward and adjoint particles as in the previous work (Cramer 1996). The latter aspect was not accomplished in that work and is validated in this work based on the perturbation-invariant technique in generalized contributon response theory (Williams

1991). In addition, the estimation procedures do not require the spatial and angle discretization on the intermediate surface A. This was not accomplished in other previous work (Serov 1999). We show a schematic picture for this estimation in Figure 2.

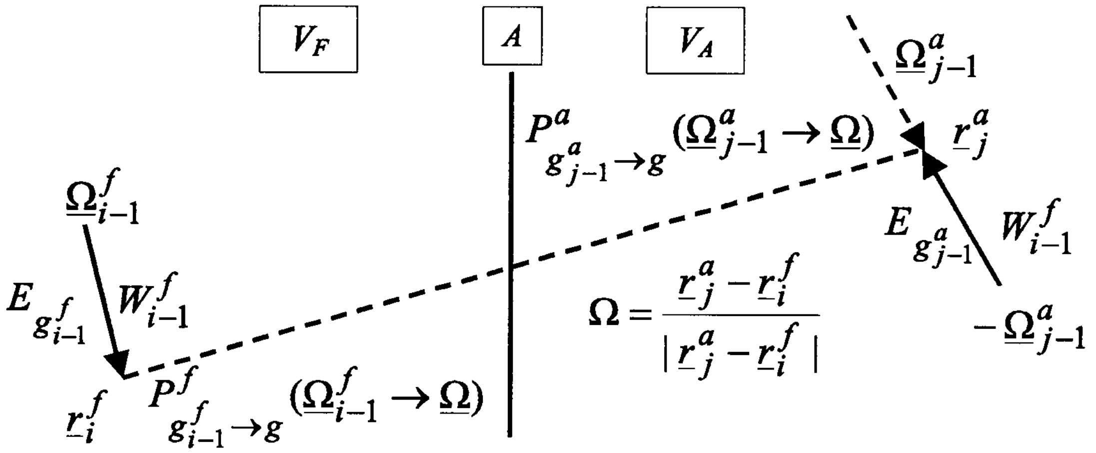

Figure 2: Schematic Picture of Estimation

Next, we explain how to estimate the second term on the RHS of Eq. (9). Suppose that an adjoint particle was born at $\underline{r}^a_0$ where $\int_{4\pi} S^A_g(\underline{r}^a_0, \underline{\Omega})d\Omega \neq 0$. By defining

$$P^{S^A}_g(\underline{r}^a_0, \underline{\Omega}) = S^A_g(\underline{r}^a_0, \underline{\Omega}) \;/\; \sum_{g'=1}^{G} \int_{4\pi} S^A_{g'}(\underline{r}^a_0, \underline{\Omega}')d\Omega' \,, \tag{18}$$

we express the empirical emission density conditioned on the birth at $\underline{r}^a_0$ as $\delta(\underline{r} - \underline{r}^a_0)P^{S^A}_g(\underline{r}^a_0, \underline{\Omega})$. Here we consider a birth event as zero-th collision. This is compatible with the foregoing treatment if the scattering probability of the zero-th collision is considered to be unity. We then obtain the adjoint uncollided angular flux at $(\underline{r}, \underline{\Omega})$ as before. Combining it with Eq. (12) and taking the steps similar to the previous derivation, the second term on the RHS of Eq. (9) is estimated as:

$$\sum_{g=1}^{G} W^f_{i-1} P^f_{g^f_{i-1}\to g}\left(\underline{r}^f_i, \underline{\Omega}^f_{i-1} \to \frac{\underline{r}^a_0 - \underline{r}^f_i}{|\underline{r}^a_0 - \underline{r}^f_i|}\right)$$
$$\exp\left[-\int_0^{|\underline{r}^a_0 - \underline{r}^f_i|} \Sigma_{t,g}\left(\underline{r}^f_i + s\frac{\underline{r}^a_0 - \underline{r}^f_i}{|\underline{r}^a_0 - \underline{r}^f_i|}\right) ds\right] P^{S^A}_g\left(\underline{r}^a_0, \frac{\underline{r}^a_0 - \underline{r}^f_i}{|\underline{r}^a_0 - \underline{r}^f_i|}\right)\frac{1}{|\underline{r}^a_0 - \underline{r}^f_i|^2} \,. \tag{19}$$

For the third and fourth terms on RHS of Eq. (9), we define

$$P^{S^F}_g(\underline{r}^f_0, \underline{\Omega}) = S^F_g(\underline{r}^f_0, \underline{\Omega}) \;/\; \sum_{g'=1}^{G} \int_{4\pi} S^F_{g'}(\underline{r}^f_0, \underline{\Omega}')d\Omega' \,, \tag{20}$$

where it was supposed that a forward particle was born at $\underline{r}_0^f$ for which $\int_{4\pi} S_g^F(\underline{r}_0^f, \underline{\Omega})d\Omega \neq 0$. By expressing the empirical emission density conditioned on the birth at $\underline{r}_0^f$ as $P_g^{S^F}(\underline{r}_0^f, \underline{\Omega})\delta(\underline{r} - \underline{r}_0^f)$, and following the steps similar to the above derivations, we can estimate these terms by next event estimation.

The total score per coupling is the sum of all of the above contributions multiplied by the total source weights [the denominator of Eqs. (18) and (20) integrated over space].

3 Coupling of batched histories

In the actual implementations, we couple collision sites from a pair of batches of forward and adjoint histories, and repeat this coupling by independently replicating such pairs. No coupling occurs between collision sites in different pairs. For a fixed number of total forward and adjoint histories, such coupling creates more forward-adjoint collision combinations than simply coupling collision sites in one-to-one forward-adjoint history-combination. This improves the statistics. For example, one hundred history-combinations are created for the collision combinations from ten forward and ten adjoint histories. (In this case, the normalization factor is $1/10 \times 10$.) This effect greatly outweighs the increase of computational time, and brings a significant gain in efficiency. In a distant source-detector problem, a rare realization of histories having approached the intermediate surface A is combined with many histories in the other mode of the calculation. Therefore, rare collisions in an important region, which often cause instability in next event estimation, can be overcome.

4 Statistical considerations

We monitor the index α of an extreme value distribution of Frechet type model (Reiss 1989). When $\alpha > 0$, this model contains the asymptotic form of Pareto distribution. We employ Hill's estimator which converges to $\max(\alpha, 0)$ (Dekkers 1989). We define $slope = 1/\alpha + 1$ (Pederson 1997). If *slope* is less than 3, the second moment may be infinite based on observed scores, because the score s from coupling is distributed as $1/s^{1/\alpha+1}$ at the upper tail. It should decrease faster than $1/s^3$, which implies that *slope* must be larger than 3. Also, we monitor VOV, the scaled variance of sample variance. It should decrease as $1/N$ over the last half of a run, where N is the number of statistically-independent units in a calculation (Pederson 1997). Since the estimations in Section 2 contain the $1/r^2$ and exponential factors, the above monitoring is important to statistical convergence.

5 Numerical results

We show numerical results for a simple source-detector problem with finite-sized cylindrical geometry and two energy groups (Figure 3 and Table 1). Particles are generated uniformly and isotropically inside the spherical source. Implicit capture with Russian roulette was employed. Its lower weight limit is uniform and the same for both standard forward and coupling calculations.

To show the correctness of the calculation, we calculate the average second group collision rate at the detector site for a near source-detector problem. The results in Table 2 show that mean values in the

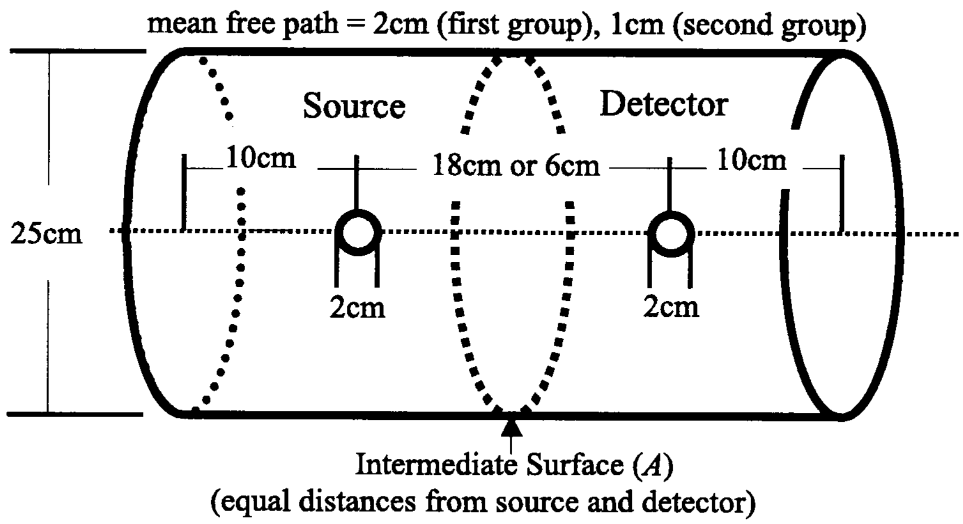

Figure 3: Problem geometry (not scaled)

Table 1: Macro Cross Section Data

Group	Macro Cross Section (cm^{-1})
First	$\Sigma_{t,1} = 0.5$, $\Sigma_{s,1} = 0.4$, $\Sigma_{s,1\to 2} = 0.3$
Second	$\Sigma_{t,2} = 1.0$, $\Sigma_{s,2} = 0.7$

coupling and the standard forward calculations agree within one standard deviation (68% confidence intervals contains common values). The conditions stated in the previous section are satisfied. Figure of merit (FOM) in the coupling calculation increases by about eight times.

Numerical results for a distant source-detector problem are shown in Table 3. FOM in the coupling calculation increases by about two orders of magnitude compared with the standard forward calculation. This efficiency increase was attained without transport biasing. VOV satisfies the condition in the previous section. We observed $3 < slope < 4$, which indicates that the third moment may not exist based on observed scores. The slow convergence of the third and fourth moments was observed in many other problems. We also observed that the number of histories per coupling affected such property. For example, *slope* is 3.6 and 5.0 for 200/200 and 500/500 forward/adjoint histories per coupling, respectively, when the total number of forward-adjoint history-combinations per run is 10^9 as in Table 3. Although extensive analysis of this aspect must await a future study, we can say that there is stability which allows one to form a confidence interval.

6 Conclusions and discussions

We have shown that MC forward-adjoint coupling by next event estimation is efficient and statisti-

Table 2: Second Group Collsion Rate per Source Particle for a Near Source Detector Problem (Average source-detector distance = 6 cm)

Standard Forward Calculation with Track Length Estimator				
History 20,000,000	Mean	1.449×10^{-3}	FSD	0.00454
FOM ($\min^{-1}$)	1.11×10^3			
Next-Event Coupling (10/10 forward/adjoint histories per coupling)				
Number of Coupling	Mean	FSD	VOV	Slope
25,000	1.449×10^{-3}	0.00458	0.010	4.1
30,000	1.450×10^{-3}	0.00417	0.008	4.2
35,000	1.454×10^{-3}	0.00380	0.006	4.1
40,000	1.456×10^{-3}	0.00364	0.005	4.2
45,000	1.455×10^{-3}	0.00349	0.005	4.2
50,000	1.455×10^{-3}	0.00330	0.004	4.2
FOM ($\min^{-1}$)	8.70×10^3			

Table 3: Second Group Collision Rate per Source Particle for a Distant Source Detector Problem (Average source-detector distance = 18 cm)

Standard Forward Calculation with Track Length Estimator				
History 2,000,000,000	Mean	4.481×10^{-7}	FSD	0.0237
FOM ($\min^{-1}$)	0.299			
Next-Event Coupling (100/100 forward/adjoint histories per coupling)				
Number of Coupling	Mean	FSD	VOV	Slope
50,000	4.562×10^{-7}	0.00322	0.020	3.1
60,000	4.560×10^{-7}	0.00288	0.016	3.1
70,000	4.554×10^{-7}	0.00271	0.018	3.1
80,000	4.561×10^{-7}	0.00257	0.016	3.0
90,000	4.559×10^{-7}	0.00238	0.013	3.0
100,000	4.550×10^{-7}	0.00222	0.012	3.1
FOM ($\min^{-1}$)	65.4			

cally stable in the framework of multigroup calculations. Significant efficiency gains can be attained without any attempt of transport biasing.

The presented method is limited to multigroup calculations. This is because we couple forward collision sites with adjoint collision sites and there is one-to-one correspondence between energy change and scattering angle in continuous energy calculations. The significance of this work is the simultaneous elimination of negative scores and discretized spatial-angle tally bins. Since Eq. (7) can be generalized to continuous energy representation, future work should develop a scheme which removes the restriction of multigroup calculations.

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