NEUTRON NOISE MEASUREMENTS AT THE DELPHI SUBCRITICAL ASSEMBLY

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ABSTRACT

The paper presents the results and evaluations of a comprehensive set of neutron noise measurements on the Delphi subcritical assembly of the Delft University of Technology. The measurements investigated the effect of different source distributions (inherent spontaneous fission and $^{252}$Cf) and the position of the detectors applied (both radially and vertically). The evaluation of the measured data has been performed by the variance-to-mean ratio (VTMR, Feynman-$\alpha$), the autocorrelation (ACF, Rossi-$\alpha$) and the cross-correlation (CCF) methods. The values obtained for the prompt decay constant show a strong bias, which depends both on the detector position and on the source distribution. This is due to the presence of higher modes in the system. It has been observed that the $\alpha$ value fitted is higher when the detector is close to the boundary of the core or to the $^{252}$Cf point-source. The higher alpha-modes have also been observed by fitting functions describing two alpha-modes. The successful set of measurement also provides a good basis for further theoretical investigations including the Monte Carlo simulation of the noise measurements and the calculation of the alpha-modes in the Delphi subcritical assembly.

Key Words: neutron noise, subcritical system, Feynman-$\alpha$, Rossi-$\alpha$, cross-correlation

1. INTRODUCTION

In a cooperation between the Institute of Nuclear Techniques of the Budapest University of Technology and Economics and the Delft University of Technology a set of measurements have been performed on the Delphi subcritical assembly of the Reactor Institute Delft during the period between October to November 2010. The objective of the measurements was to examine the influence of the source distribution and the detector position on the prompt decay constant ($\alpha$) obtained with the different neutron noise methods. Earlier experiments and theoretical investigations indicated the existence of multiple $\alpha$-modes in subcritical systems. One example of this was the measurements performed at the Yalina-Booster facility [1] where multiple $\alpha$-modes were observed and a theory has been developed for the interpretation [2]. In contrast to the Yalina-Booster, which was a highly heterogeneous fast system, Delphi is a thermal system. In

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such thermal systems the multiple $\alpha$-modes have not been observed and used for the evaluation and interpretation of neutron noise measurements.

A further objective of the measurements was to provide a validation basis for Monte Carlo simulations. The Monte Carlo method is suitable for the simulation of neutron noise measurements and several codes have been developed for this purpose[3–5]. Co-authors of this paper have also developed variance reduction methods for neutron noise simulations, which can improve the efficiency of the calculations[6,7]. Detailed Monte Carlo simulations can help the understanding of the nature of the multiple $\alpha$-modes by providing results, which are not possible to obtain from measurements. Simulations offer much more freedom in the selection of detector material, position, size, time resolution etc. compared to real measurements. However, the reliability of such computational results needs validated methods and models.

This paper gives a description of the measurement set-up and the evaluation methods and summarizes the most important results and observations.

2. THE DELPHI SUBCRITICAL ASSEMBLY

The Delphi subcritical assembly is located in the Reactor Institute Delft. For safety reasons it was installed inside the reactor hall of the institute, where a research reactor (with 2 MW thermal power) is also located. The Delphi was built for training and research purposes after the previous subcritical system was decommissioned. It consists of two vessels, one being upon the other (see Figure 1). The lower vessel is made of stainless steel and is filled with de-mineralized water before the start of an experiment. The upper acrylic glass air-filled container is used to store 168 fuel pins that can be lowered one by one using a special handling tool. Below the steel vessel, a shielding box is positioned containing a $^{252}$Cf-neutron source that can be pneumatically inserted to its experimental position (2.5 cm below the bottom plane of the active fuel) in the steel vessel. The $^{252}$Cf-neutron source contained in a plastic capsule had an initial activity of 18.5 MBq (on 1 December 2003) corresponding to a neutron source emission rate of $2.4 \times 10^6$ s$^{-1}$ and a gamma-ray emission rate of $1.3 \times 10^7$ s$^{-1}$.

The fuel pins are positioned in a square lattice of 13x13 positions, where the central position is occupied by a water-filled steel tube (see Figure 2). The pitch of the fuel pin lattice is 23 mm. Each fuel pin contains 43-45 pellets made of 3.8% enriched UO$_2$ fuel (total mass $\approx 365.5$ g/pin). The pellets are stacked in an aluminum tube with an outer diameter of 12 mm and wall thickness of 0.95 mm. The total length of a fuel pin is 66.5 cm (44 cm of which is the active length of the fuel). The maximal $k_{eff}$ of the assembly (according to MCNP calculations) reaches 0.92, which is a conservative overestimate, with fuel enrichment of 3.9% and the omission of some structural materials.

3. MEASUREMENT SET-UPS

3.1. Detector Positions

10-bar $^3$He proportional counter tubes with diameter of 6 mm and active length of 76 mm (General Electric, RS-P4-0203-212) were used for the measurements. The pulses from the detectors were amplified and converted to TTL format (standardized ‘square box’ pulses with a
Figure 1. Picture of the Delphi subcritical assembly. At the top the 168 fuel pins can be seen in their storage container made of acrylic glass. It is fixed at the top of the stainless steel water-filled vessel. Below the vessel the shielding box of the $^{252}$Cf neutron source is situated.
height of 5 V) by PDT amplifiers (PDT20A-SHV), which also supply the high voltage. The pulses were subsequently recorded in a PC equipped with a pulse-counter card (National Instruments PCI-6602 counter/timer-card) and LabView software to control the measurements. The instrumentation of the Delphi made the parallel data acquisition from two independent channels possible, which offered the opportunity to measure the cross-correlation between two detectors.

In order to investigate the effect of the detector position, several positions had been selected for measurements in the Delphi core. A first set of positions aimed at the investigation of the effect of the radial position of the detector. For this purpose four positions were identified in the four quadrant of the Delphi core with different radial distances (see Table I and the left-hand side of Figure 2). Vertically all detectors were positioned in the midplane of the core. The positions are marked by the letters of the neighboring columns and the numbers of the neighboring rows of the fuel rods. In the second set of measurements axial traverses were performed in positions HI67 and DE89 in four steps (see the right-hand side of Figure 2) in order to investigate the effect of the vertical position of the detectors. The detectors in LM23 and FG23 remained in the midplane of the core as did during the first set of measurements.

### Table I. Radial distances of the measurement positions from the center of the core. Positions are marked by the letters of the neighboring columns and the numbers of the neighboring rows of the fuel rods

<table>
<thead>
<tr>
<th>Position</th>
<th>Radial distance from center [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM23</td>
<td>163</td>
</tr>
<tr>
<td>HI67</td>
<td>36</td>
</tr>
<tr>
<td>DE89</td>
<td>67</td>
</tr>
<tr>
<td>FG23</td>
<td>104</td>
</tr>
</tbody>
</table>

3.2. Neutron sources

As it was mentioned above, a $^{252}$Cf source can serve as a neutron source in Delphi. Its source strength during the measurements was estimated to be $\approx 350,000$ neutrons/s. Besides this external source the spontaneous fission of $^{238}$U provides an inherent neutron source for Delphi. Based on the $^{238}$U content of the core its source strength is estimated to be $\approx 685$ neutrons/s. This difference of several orders of magnitude means that when the $^{252}$Cf source is inserted into the core, the effect of the inherent source can practically be neglected. In this way measurements can be performed with two completely different spatial source distributions: a point source 2.5 cm below the bottom plane of the active fuel and a volumetric source evenly distributed in the fuel rods.
Figure 2. Left-hand side: Delphi core map with detector positions. Cross-correlation measurements have been performed between detectors in the connected positions. Right-hand side: vertical measurement positions with distances from core midplane.
4. THEORETICAL BACKGROUND

Neutron noise methods are based on the measurement of the fluctuations in the number of neutrons in a neutron multiplying system. Let random variable \( n \) be the number of neutrons in a certain volume of the phase space (e.g.: a detector) and \( p(n = n|t, \vec{r}_0, \vec{v}_0) \) be the probability that \( n \) neutron is present at time \( t \) with the condition that one neutron was present at \( t = 0 \) at position \( \vec{r}_0 \) with velocity \( \vec{v}_0 \). A complete and general description of this probability distribution can be obtained with the help of the Pál-Bell equation, which is an integro-differential equation reminiscent to the Boltzmann-type kinetic equation describing the \( g(t, \vec{r}_0, \vec{v}_0, z) \) probability generation function (pgf), which is defined as

\[
g(t, \vec{r}_0, \vec{v}_0, z) = \langle z^n \rangle = \sum_{n=0}^{\infty} p(n = n|t, \vec{r}_0, \vec{v}_0)z^n. \tag{1}
\]

Its actual form, a derivation and other details about the Pál-Bell equation can be found in [8]. As it follows from the definition (1), the \( k \)th derivatives of the pgf gives the \( k \)th factorial moments of the probability distribution:

\[
\left[ \frac{\partial^{(k)}}{\partial z^k} g(t, \vec{r}_0, \vec{v}_0, z) \right]_{z=1} = \sum_{n=0}^{\infty} p(n = n|t, \vec{r}_0, \vec{v}_0) \frac{n!}{(n-k)!} = \left\langle \frac{n!}{(n-k)!} \right\rangle. \tag{2}
\]

Since from the factorial moments all the other moments can be determined, the Pál-Bell equation determines all the quantities which are needed to describe the neutron fluctuations. An important characteristic of the Pál-Bell equation is that it contains the same transport operator \( L^+ \) as the adjoint Boltzmann neutron-transport equation:

\[
\left( -\frac{\partial}{\partial t} - L^+ \right) \Phi^+(\vec{r}, \vec{v}, t) = S^+(\vec{r}, \vec{v}, t), \tag{3}
\]

where \( \Phi^+(\vec{r}, \vec{v}, t) \) is the adjoint flux, \( S^+(\vec{r}, \vec{v}, t) \) is the adjoint source depending on the detector parameters and adjoint transport operator \( L^+ \) is defined in the same way as in [9, Chapter 6]. It is known that the solution of (3) can be obtained as

\[
\Phi^+(\vec{r}, \vec{v}, t) = \sum_{i=0}^{\infty} C_i e^{-\alpha_i t} \phi_i^+(\vec{r}, \vec{v}), \tag{4}
\]

where \( \alpha_i \) and \( \phi_i^+(\vec{r}, \vec{v}) \) are the corresponding eigenvalues and eigenfunctions of the following equation:

\[
-\frac{\alpha_i}{v} \phi_i^+(\vec{r}, \vec{v}) = L^+ \phi_i^+(\vec{r}, \vec{v}) \tag{5}
\]

and the amplitudes \( C_i \) can be derived from the initial adjoint flux distribution \( \Phi^+(\vec{r}, \vec{v}, 0) \) and the adjoint source \( S^+(\vec{r}, \vec{v}, t) \). The same holds for the forward flux \( \Phi(\vec{r}, \vec{v}, t) \) applying the forward source \( S(\vec{r}, \vec{v}, t) \), the forward transport operator \( L \) and its \( \phi_i(\vec{r}, \vec{v}) \) eigenfunctions. However, the \( \alpha_i \) eigenvalues are the same for both the direct and the adjoint case.

(4) is called the \( \alpha \)-modes expansion. In the point-kinetics approximation only the so-called fundamental mode corresponding to the smallest eigenvalue \( \alpha_0 \) is kept because the others die out.
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...sooner. In this approach, the $\alpha_0 = \alpha$ is called the prompt decay constant, which is directly related to the reactivity:

$$\alpha = \frac{\rho - \beta_{\text{eff}}}{\Lambda},$$

(6)

where $\rho$ is the reactivity, $\beta_{\text{eff}}$ is the effective delayed neutron fraction and $\Lambda$ is the neutron generation time. However, this is often not applicable to source driven subcritical systems where the higher modes are also present. As it is shown in [2] the solution of the Pál-Bell equation can also be expanded in $\alpha$-modes and in this way the $\alpha$-modes appear in the moments of the neutron fluctuation. This explains the appearance of higher $\alpha$-modes in the noise measurements. In the following sections formulas are given for the different noise techniques, both in the point-kinetic approach and with $\alpha$-modes expansion.

### 4.1. Variance-to-mean Ratio (VTMR, Feynman-$\alpha$) Method

The Feynman-$\alpha$ method basically measures the deviation of the neutron distribution from the Poisson-distribution by calculating the variance-to-mean ratio of the number of counts $c(\Delta T)$ in a detector for different counting times $\Delta T$:

$$Y(\Delta T) = \frac{\sigma^2(c(\Delta T))}{\langle c(\Delta T) \rangle} = \frac{\langle c(\Delta T)^2 \rangle - \langle c(\Delta T) \rangle^2}{\langle c(\Delta T) \rangle}.$$  

(7)

Based on the $\alpha$-modes expansion the following formula can be derived for this quantity[2]:

$$Y(\Delta T) = \frac{\nu(\nu - 1)}{(S^+, \Phi)} \sum_{k,l} C_{k,l} \frac{\alpha_k^{\nu} \alpha_l}{\alpha_k \alpha_l} \left[ 1 + \frac{\alpha_k^2 (1 - e^{-\alpha_k \Delta T}) + \alpha_l^2 (1 - e^{-\alpha_l \Delta T})}{\alpha_k \alpha_l (\alpha_k + \alpha_l) \Delta T} \right],$$

(8)

where $\nu$ is the number of neutrons produced in a fission, scalar product $(S^+, \Phi)$ gives the detection rate and the $C_{k,l}$ coefficients can be determined from the fission source and the adjoint source $S^+$. In the point-kinetics approach, by omitting the higher $\alpha$-modes and the spatial effects, (8) simplifies to:

$$Y(\Delta T) = \frac{D \epsilon F}{(\Lambda \alpha)^2} \left( 1 + \frac{1 - e^{-\alpha \Delta T}}{\alpha \Delta T} \right),$$

(9)

where Diven-factor $D = \frac{\langle \nu(\nu - 1) \rangle}{\langle \nu \rangle^2}$, $\epsilon$ is the detector efficiency in counts per fission in the system and $F$ is the fission rate in the system.

### 4.2. Auto-correlation (ACF, Rossi-$\alpha$) and Cross-correlation (CCF) Methods

The Rossi-$\alpha$ method measures the correlation between neutrons originating from the same source event in a multiplying medium. Originally, the time interval distribution of the subsequent counts were measured but recently rather the auto-correlation function is calculated for the number of counts $c(t)$ in a the time interval $[t, t + \Delta T]$:

$$ACF(\tau) = \frac{\langle c(t) c(t + \tau) \rangle}{\langle c(t) \rangle^2}.$$  

(10)

Theoretical considerations based on the $\alpha$-modes expansion result in the following formula[2]:

$$ACF(\tau) = \langle \nu(\nu - 1) \rangle (S^+, \Phi) \sum_{k,l} \frac{C_{k,l}}{\alpha_k + \alpha_l} e^{-\alpha_l \tau} + (S^+, \Phi)^2.$$  

(11)
Again the point-kinetics approach results in a single $\alpha$-mode formula:

$$ACF(\tau) = \frac{D\epsilon^2 F^2 e^{-\alpha \tau}}{2\alpha^2} + (\epsilon F)^2.$$  \hspace{1cm} (12)

In the cross-correlation measurement two detectors are applied with count rates $c_1(t)$ and $c_2(t)$:

$$CCF(\tau) = \frac{\langle c_1(t) c_2(t + \tau) \rangle}{\langle c_1(t) c_2(t) \rangle}.$$  \hspace{1cm} (13)

No derivation of the $\alpha$-modes expansion can be found in the literature for the cross-correlation case. However, based on the derivation of (11) one can assume that it will differ only in the adjoint source $S^+$ applied, as it is determined by the detector parameters. In the point-kinetics, CCF is not different from ACF as no spatial effects are considered.

5. DATA EVALUATION WITH THE NEUTRON NOISE METHODS

The output signal of the detectors were recorded by the counter/timer-card (with sampling frequency $f = 20$ GHz) along with their arrival time. The measurement files contain the time intervals between detections, in a sequence of 64 bit real binary format numbers. These files were evaluated with the different noise techniques.

5.1. Variance-to-mean-ratio (VTMR, Feynman-$\alpha$) Method

For the VTMR evaluation the measurement time $T$ was split to $M = \frac{T}{\Delta T}$ consecutive time intervals of $\Delta T$ length and the number of counts $N_i$ belonging to each time interval were determined. In this way, the variance-to-mean ratio $Y$ as a function of $\Delta T$ can be easily determined:

$$Y(\Delta T) = \frac{\sum_{i=1}^{M} N_i^2}{\sum_{i=1}^{M} N_i} - \frac{1}{M} \sum_{i=1}^{M} N_i - 1$$  \hspace{1cm} (14)

The $\Delta T$ values were selected according to a quasi logarithmic scale as in this way better parameter estimation can be achieved at lower $\Delta T$. The use of a fully logarithmic scale was limited by the time resolution. The quasi logarithmic scale contained logarithmically interpolated points between 1 $\mu$s and 5 ms and maximum 20 linearly interpolated points between these logarithmic points. All values were rounded to 1 $\mu$s to fit the resolution.

Based on the theoretical considerations in Section 4.1, non-linear least-squares parameter fits were performed for all VTMR curves using the Origin (OriginLab, Northampton, MA) data analysis software, which applies the Levenberg-Marquardt algorithm [10,11], with both of the following functions:

$$Y(\Delta T) = \tilde{c} \left( 1 - \frac{1 - e^{-\tilde{\alpha} \Delta T}}{\tilde{\alpha} \Delta T} \right)$$  \hspace{1cm} (15)

$$Y(\Delta T) = \tilde{c}_0 \left( 1 - \frac{1 - e^{-\tilde{\alpha}_0 \Delta T}}{\tilde{\alpha}_0 \Delta T} \right) + \tilde{c}_1 \left( 1 - \frac{1 - e^{-\tilde{\alpha}_1 \Delta T}}{\tilde{\alpha}_1 \Delta T} \right),$$  \hspace{1cm} (16)

where $\tilde{c}, \tilde{\alpha}$ and $\tilde{c}_0, \tilde{\alpha}_0, \tilde{c}_1, \tilde{\alpha}_1$ were the estimated parameters, respectively.
An example of the measured and fitted Feynman-$\alpha$ curves can be seen if Fig. 3. Fit residuals at the bottom show that fitting function (16) provides a better fit.

![Graph showing Feynman-$\alpha$ curves and residuals](image)

**Figure 3.** Measured and fitted Feynman-$\alpha$ curve for the measurement in position DE89 ($r = 67$ mm) at the mid-plane of the core ($h = 0$ mm). Fits were done assuming single and dual $\alpha$-modes. Residuals of the fits (bottom) suggest that a second $\alpha$-mode is present.

### 5.2. Auto-correlation (ACF, Rossi-$\alpha$) Method

In order to evaluate the measured data set according to the auto-correlation method $\Delta T = 0.01$ ms was applied and the number of counts in each time interval were divided by the
total number of counts:
\[ n_i = \frac{N_i}{\sum_{j=1}^{M} N_j}. \] (17)

From the time series of the normalized number of counts \( n_i \) the auto-correlation function can be calculated:
\[ ACF(\tau) = \sum_{i=1}^{M-K} n_i n_{i+K}, \text{ where } K = \frac{\tau}{\Delta T}. \] (18)

Based on the theoretical considerations in Section 4.2, non-linear least-squares parameter fits were performed for all ACF curves in the same way as that shown in Section 5.1, with both of the following functions:
\[ ACF(\tau) = \tilde{c} e^{-\tilde{\alpha} \tau} + \tilde{a} \] (19)
\[ ACF(\tau) = \tilde{c}_0 e^{-\tilde{\alpha}_0 \tau} + \tilde{c}_1 e^{-\tilde{\alpha}_1 \tau} + \tilde{a}_{01} \] (20)

where \( \tilde{c}, \tilde{\alpha}, \tilde{a} \) and \( \tilde{c}_0, \tilde{\alpha}_0, \tilde{c}_1, \tilde{\alpha}_1, \tilde{a}_{01} \) were the estimated parameters, respectively. Since \( ACF(0) \) is a singular point it was skipped from the fittings.

An example of the measured and fitted ACF curves can be seen in Fig. 4. Fit residuals at the bottom show that fitting function (20) provides a better fit. One can also observe that the auto-correlation function seems to have worse statistics than the Feynman-\( \alpha \) curve from the same measurement (see Fig. 3). However, the apparent smoothness of the Feynman-\( \alpha \) curve is only due to the fact that a strong covariance exists between the points of the curve as all of them were calculated from the complete data set. On the other hand, to a given \( \tau \) point of the auto-correlation function only those time intervals contribute which contain counts correlated with time shift \( \tau \).

### 5.3. Cross-correlation (CCF) Method

The calculation of the cross-correlation function requires two synchronized measurement files. Both have to be processed in the same way as in the case of the auto-correlation technique (see Section 5.2) to obtain the time series of the normalized number of counts \( n_{1,i} \) and \( n_{2,i} \). From these the cross-calculation function can be determined:
\[ CCF(\tau) = \sum_{i=1}^{M-K} n_{1,i} n_{2,i+K}. \] (21)

Although according to the theory \( CCF(\tau) = CCF(-\tau) \), due to the two independent measurement files \( CCF(\tau) \) and \( CCF(-\tau) \) are independent data points. In order to make use of this fact, the fitting functions in this case were defined as:
\[ CCF(\tau) = \tilde{c} e^{-s(t)\tilde{\alpha} \tau} + \tilde{a} \] (22)
and
\[ CCF(\tau) = \tilde{c}_0 e^{-s(t)\tilde{\alpha}_0 \tau} + \tilde{c}_1 e^{-s(t)\tilde{\alpha}_1 \tau} + \tilde{a}_{01}, \] (23)
where
\[ s(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases} \] (24)
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Figure 4. Measured and fitted ACF curve for the measurement in position DE89 ($r = 67$ mm) at the mid-plane of the core ($h = 0$ mm). The fit was done assuming single and dual $\alpha$-modes. Residuals of the fits (bottom) suggest that a second $\alpha$-mode is present.
and $\tilde{c}, \tilde{\alpha}, \tilde{a}$ and $\tilde{c}_0, \tilde{\alpha}_0, \tilde{c}_1, \tilde{\alpha}_1, \tilde{a}_{01}$ were the estimated parameters, respectively. It is worth noting that the above definitions of the fitting functions proved to be also useful when desynchronized measurement files had to be handled. By substituting $t - \tilde{t}_0$ in place of $t$ in (22) and assuming $\tilde{t}_0$ as a free parameter, the synchronization can be reconstructed.

An example of the measured and fitted CCF curves can be seen in Fig. 5. This typical case shows that in the case of the cross-correlation function the presence of the higher $\alpha$-mode is less obvious.

6. RESULTS

In Fig. 6-8 the $\tilde{\alpha}, \tilde{\alpha}_0$ and $\tilde{\alpha}_1$ parameters obtained from the different measurements were plotted according to the detector position. Results of the cross-correlation measurements are plotted at the average radius of the two detector positions involved in the radial case (Fig. 6), while the same results are given in the two axial cases (Fig. 7-8) since two detectors at the same height but in different radial positions were used. With the inherent source, measurements were performed at the height of 180 mm and -90 mm, the results of which are reflected to the mid-plane and plotted also at -180 mm and 90 mm, respectively, as the system is assumed to be symmetrical.

The analysis showed that in the case of assuming one $\alpha$-mode (using functions (15), (19) and (22) in the fitting process) the absolute value of the fitted $\tilde{\alpha}$ parameter

- increases as the detector position gets closer to the boundary of the core either radially (see in Fig. 6) or vertically (see in Fig. 7 and 8),
- increases as the detector gets closer to the $^{252}$Cf point-source (in the case of the measurements where the $^{252}$Cf source were used, see the left-hand side of Fig. 6-8).

In the case of assuming two $\alpha$ modes (using functions (16), (20) and (23) in the fitting process) the fitted $\tilde{\alpha}_0$ values show much less spatial variance than in the previous case. This fact confirms the assumption that the variance of the fitted $\alpha$ values is due to the biasing effect of the higher $\alpha$ modes. The observation that this bias is stronger near the source and near the boundaries of the geometry can be explained by the assumption that the higher modes gain more importance compared to the fundamental mode in these regions. Unfortunately $\tilde{\alpha}_1$ values could be fitted with large errors only.

One can assume that the best estimate for the prompt decay constant $\alpha$ can be obtained from the measurement where the less bias is expected from the higher modes. This is the measurement where the fitted $\tilde{\alpha}$ and $\tilde{\alpha}_0$ values are the closest to each other. Based on the above observations the inherent source and the position closest to the center of the core (i.e. HI67, $r = 36$ mm, $h = 0$ mm, see in Fig. 6 and Fig. 8 on the left-hand side) were chosen to give an estimation for the $\alpha$ value of Delphi. Concerning the measurement method the cross-correlation is excluded as it always involves a detector farther from the center of the core (i.e. DE98, $r = 67$ mm, $h = 0$ mm). Since among the Feynman- and Rossi-$\alpha$ methods the later seems to give less biased $\tilde{\alpha}_0$ values a best estimate can be expected from that measurement which is $\alpha_0 = 1704 \pm 53$ s$^{-1}$. The Feynman-$\alpha$ evaluation of the same data gives a very close value: $\alpha_0 = 1696 \pm 19$ s$^{-1}$. The lower standard deviation is attributable to the strong covariance of the Feynman-$\alpha$ data points as mentioned in Section 5.2.
Figure 5. Measured and fitted CCF curve for the measurement in position DE89 and HI67 ($r = 67$ mm and 36 mm, respectively) at the mid-plane of the core ($h = 0$ mm). Fits were done assuming single and dual $\alpha$-modes, but as it can be observed from the residuals of the fits (bottom) the presence of the second $\alpha$-mode is less striking.
Figure 6. Fitted $\tilde{\alpha}_1$, $\tilde{\alpha}_0$ (top plots) and $\tilde{\alpha}_1$ (bottom plots) parameters from the different noise measurements performed at the mid-plane positions at different radial distances from the axis with the inherent spontaneous fission source (left-hand side) and the $^{252}\text{Cf}$ source (right-hand side). Results of the cross-correlation measurements are plotted at the average radius of the two detector positions involved in the measurements.
Figure 7. Fitted $\tilde{\alpha}$, $\tilde{\alpha}_0$ (top plots) and $\tilde{\alpha}_1$ (bottom plots) parameters from the different noise measurements performed at the DE89 position at different heights with the inherent spontaneous fission source (left-hand side) and the $^{252}$Cf source (right-hand side). With the inherent source, measurements were performed at the height of 180 mm and -90 mm, the results of which are reflected to the mid-plane as the system is assumed to be symmetrical. Cross-correlation measurements were performed with another detector at the same height in position HI67.
Figure 8. Fitted $\tilde{\alpha}$, $\tilde{\alpha}_0$ (top plots) and $\tilde{\alpha}_1$ (bottom plots) parameters from the different noise measurements performed at the HI67 position at different heights with the inherent spontaneous fission source (left-hand side) and the $^{252}$Cf source (right-hand side). With the inherent source, measurements were performed at the height of 180 mm and -90 mm, the results of which are reflected to the mid-plane as the system is assumed to be symmetrical. Cross-correlation measurements were performed with another detector at the same height in position DE89.
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It is much harder to decide which is the best estimation for $\alpha_1$. Following the considerations above one could choose the measurement where the influence of the higher mode is the highest. This is the position closest to the $^{252}$Cf source i.e. HI67, $r = 36$ mm, $h = -180$ mm (see in Fig. 8 on the right-hand side), where $\tilde{\alpha}_1 = 9090 \pm 1854$ s$^{-1}$. However, at this position the statistical uncertainty is very high both for $\tilde{\alpha}_0$ and $\tilde{\alpha}_1$. It is probable that even $\tilde{\alpha}_1$ is biased by higher $\alpha$-modes. From the same measurement which was chosen above as best estimate for $\alpha_0$ an estimation with lower uncertainty can be obtained: $\tilde{\alpha}_1 = 8370 \pm 1041$ s$^{-1}$. Due to the high statistical errors from these measurements one can only conclude that $\alpha_1$ is in the range of 8-9000 s$^{-1}$.

7. CONCLUSIONS

This paper has presented the results of a comprehensive set of neutron noise measurements performed on the Delphi subcritical assembly. The measurements investigated the effect of the different source distributions (inherent spontaneous fissions and $^{252}$Cf) and the radial and vertical position of the detectors. The evaluation of the measured data has been performed by the variance-to-mean ratio (VTMR, Feynman-$\alpha$), the autocorrelation (ACF, Rossi-$\alpha$) and the cross-correlation (CCF) method. Non-linear least squares fits have been applied in order to obtain the prompt decay constant ($\alpha$) from the evaluated data. The $\alpha$ values obtained from a single $\alpha$-fit show strong bias depending both on the detector position and on the source distribution. This is due to the presence of higher modes in the system. It has been observed that the fitted $\alpha$ is higher when the detector is close to the boundary of the core or to the $^{252}$Cf point-source. The higher $\alpha$-modes have been observed also by fitting functions describing dual $\alpha$-modes ($\alpha_0$ and $\alpha_1$). The fundamental mode ($\alpha_0$) showed much less variance in this case, but due to the insufficient time resolution the higher mode could not be determined accurately enough. Based on the set of measurement the $\alpha_0$ in Delphi can be estimated as $1704 \pm 53$ s$^{-1}$, while $\alpha_1$ appears to be in the range of 8-9000 s$^{-1}$. A successful set of measurements also gives a good basis for further theoretical investigations, including Monte Carlo simulations of the noise measurements and the calculation of the $\alpha$-modes in the Delphi subcritical assembly.

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REFERENCES


