

ESTIMATION OF COINCIDENCE AND CORRELATION IN NON-ANALOGOUS MONTE CARLO PARTICLE TRANSPORT

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ABSTRACT

The conventional non-analogous Monte Carlo methods are optimized to preserve the mean value of the distributions and therefore they are not suited for non-Boltzmann problems like the estimation of coincidences or correlations. This paper presents a general method called history splitting for the non-analogous estimation of such quantities. The basic principle of the method is that a non-analogous particle history can be interpreted as a collection of analogous histories with different weights according to the probability of their realization. Calculations with a simple Monte Carlo program for a pulse-height-type estimator prove that the method is feasible and provides unbiased estimation. Different variance reduction techniques have been tried with the method and Russian roulette turned out to be ineffective in high multiplicity systems. An alternative history control method is applied instead. Simulation results of a Feynman- α measurement shows that even the reconstruction of the higher moments is possible with the history splitting method, which makes the simulation of neutron noise measurements feasible.

Key Words: Monte Carlo, non-Boltzmann, variance reduction, pulse height estimator, neutron noise

1. INTRODUCTION

During the investigation of particle transport with Monte Carlo methods one often faces tasks which need the estimation of coincidence of events or correlations between events. A coincidence is when separate events (e.g. detection) occur during a given time period. Some examples for such problems are the additive peaks in detectors or the dead-time effect. Correlation measures the deviation from independency

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of certain events and its estimation involves higher moments. The general definition of correlation is the following:

$$\rho(\xi, \eta) = \frac{\sigma_{\xi\eta}}{\sigma_{\xi}\sigma_{\eta}} \quad (1)$$

where ξ and η are random variables, and ρ denotes the correlation while σ is the (co)variance. In the case of particle transport problems the random variables refer to the contribution in a certain detector in a given time interval. The different noise measurement techniques (e.g. Feynman- α , Rossi- α , etc.) uses different ways to quantify correlations, but all involves the variance or other higher moments. The problem becomes a transport problem when several detection events can originate from the same source event either due to a non-stopping detection event (e.g. scattering) or a multiple source event (e.g. spontaneous fission, positron annihilation), or because the particle is transported in a multiplicative media between the source and the detection (e.g. fissile material for neutrons, pair production for photons).

Such problems are often referenced as non-Boltzmann estimators as they depend on the collective effects of particles not described in the Boltzmann transport equation. The conventional non-analogous Monte Carlo methods are optimized to preserve the mean value of the distributions and therefore they are not suited for non-Boltzmann problems. By using different variance reduction techniques it introduces artificial coincidences by splitting particle trajectories and biases the higher moments by the weighting of the events. Analogous Monte Carlo simulation avoids these problems but the computer time needed to arrive at acceptable statistics in full-scale problems may be overwhelming.

This problem has already been addressed by Booth[1,2] with the motivation of using variance reduction techniques for photon pulse height tally (total energy deposition in a detector). Pulse height tally falls into the category of non-Boltzmann estimators because it collects the energy deposition from several collisions of a single particle. Furthermore photons may undergo pair production or double fluorescence which results in multiplication. Booth suggests three possible approaches: the *deconvolution* approach applies single particle variance reduction methods to each particle of a collection and then analyzes (deconvolutes) how the distribution of the collection of particles has been modified and weights the tallies appropriately. The *supertrack* approach applies variance reduction to collections of tracks (supertracks) and requires redefinition of standard Monte Carlo terms. For example, the individual particle tracks would no longer carry any weight; the variance reduction is applied to the supertracks, and thus the weights are associated with the supertracks. The *corrected single particle* approach is perhaps the most difficult. In this approach, the tracks are first treated as single particles with the traditional single particle weights, and then the collective effects are introduced by estimating the difference between transporting the particles as a collection and transporting the particles individually.

The deconvolution approach based on Booth's method has been implemented in MCNP5[3,4] and MCNPX[5]. Similar method has been introduced in MCBEND[6] for splitting and Russian roulette also for photon pulse height tally. In the GATE medical imaging and simulation application based on GEANT4 a method has been implemented to make the geometrical importance sampling technique compatible with the pulse height tally for single photon emission computed tomography (SPECT) simulation[7].

The above methods are all focused on the photon pulse height tally which is a problem containing low or no multiplicity and involves only coincidences of events. This paper presents a general method, which is similar in its philosophy to the deconvolution approach and therefore easy to integrate into the conventional Monte Carlo program flow. Furthermore we will demonstrate its applicability to pulse

height estimation and show the problems arising in systems with higher multiplicity (e.g. neutron simulation in a source driven subcritical assembly). Methods are presented for the simulation of such systems and also for the estimation of correlation between events which requires the estimation of higher moments.

2. CALCULATIONAL METHODS IN MONTE CARLO SIMULATIONS

2.1. Analogous Monte Carlo Simulation

In Monte Carlo particle transport problems particles start from a source event at $t=0$, spend some t_j time with transport in the media (transport time) while a detection event occurs. In case of simulation of coincidence events from the same source event, the time and the detector contribution of each event has to be registered. The detector response of a given source event (i) which resulted in n_i detection event with d_i^j detector contributions at time t_i^j in a given time interval from t_0 to $t_0 + T$ can be described by the following integral:

$$r_i = \int_{t_0}^{t_0+T} \sum_j^{n_i} d_i^j \delta(t - t_i^j) dt \quad (2)$$

which in case of $t_0 = 0$ and $T = \infty$ gives the total detector response for a single source event, and (2) simplifies to:

$$r_i = \sum_j^{n_i} d_i^j \quad (3)$$

A Monte Carlo estimation of the detector response function can be obtained by the simulation of a large number of source events (N) and collecting the detector responses into a bin structure (R_k) to estimate the probability distribution of the response (R):

$$P(R_k < R < R_{k+1}) = \left\langle \frac{1}{N} \sum_{R_k < r_i < R_{k+1}} 1 \right\rangle \quad (4)$$

In the case of the pulse height estimator (3) can be used and the d_i^j detector contributions equal to the energy deposition in the given collision in the detector, while the probability distribution in (4) gives the pulse height spectrum measured by the detector.

For the estimation of correlation one needs to determine the higher moments of the detector response for a certain T interval. In the case of the analogous simulation the l^{th} moment of R can be calculated on a straight-forward way:

$$\langle R^l \rangle = \left\langle \frac{1}{N} \sum_{i=1}^N r_i^l \right\rangle \quad (5)$$

Analogous simulation makes possible the estimation of coincidences and correlations without any major problem, but in order to arrive to acceptable statistics in a reasonable time in a full-scale problem one has to use non-analogous simulation with variance reduction methods.

2.2. Non-Analogous Monte Carlo Simulation with History Splitting

2.2.1. The history splitting method

The above scheme cannot be applied when non-analogous Monte Carlo simulation is used. The particle weight w can change between the detection events which makes it impossible to simply sum the detector contributions as in (2) and (3). Furthermore, the weight change implies a splitting event which may result in artificial coincidences. There cannot be physical coincidence between events belonging to different tracks coming from the same splitting event, because such events are not possible to be present at the same analogous history. The change in the weight is a common characteristic of the variance reduction events and the ratio of the weight change gives the probability of the track followed. Therefore a non-analogous particle history can be interpreted as a collection of analogous histories with different weights according to the probability of their realization. A so-called history splitting method has been developed in order to generate these analogous histories. This method is similar in its philosophy to Booth's deconvolution approach as it leaves the Monte Carlo transport unchanged and collects only information about the variance reduction events, in order to generate (deconvolute) the analogous histories when the non-analogous transport is finished.

One can consider the particle history from a non-analogous transport as a tree having physical multiplicity nodes (eg. fission) and variance reduction multiplicity nodes. This tree can be simplified by collecting all the physical events not separated by variance reduction nodes into a single node. (see Fig. 1.) The resulting tree has two kind of nodes with different properties. A *variance reduction node* is connected to physical nodes only (one parent and at least two children) while a *physical node* is connected only with variance reduction nodes. The source (root of the tree) and the end points are always physical nodes. The weight of the particle changes only as we pass through a variance reduction node. From the non-analogous history one can create an analogous one by selecting at every variance reduction node only one of the successive physical nodes. An analogous history is a single physical node with a weight of one.

During the transport certain data need to be saved about the nodes in order to describe the tree (see in Fig. 2.):

i is the index of a physical node. The source node (the root of the tree) has the index of 1;

n_i is the number of child nodes;

m_i^j , where $j = 1, \dots, n_i$ is the number of child nodes of the j^{th} variance reduction node of the physical node i ;

$b_i^j + k - 1$, where $k = 1, \dots, m_i^j$ is the index of the k^{th} child physical node of the j^{th} variance reduction node of the physical node i ;

w_i is the weight of the physical node i , i.e. the ratio of weight change as passing through the parent variance reduction node.

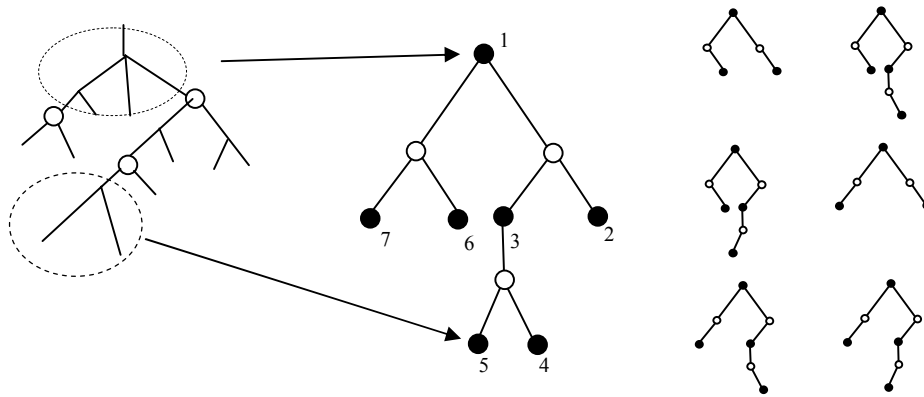


Figure 1. Simplification of a non-analogous history. Physical branches from the non-analogous history (on the left) are united into a physical node. The simplified history-tree (in the middle) contains only physical (●) and variance reduction nodes (○). From the non-analogous history in the example one can generate 6 different analogous ones (on the right)

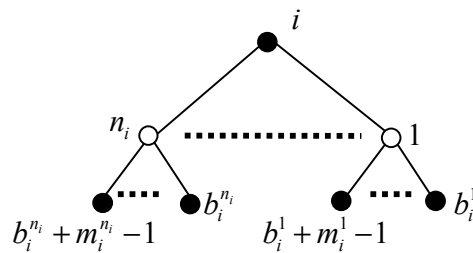


Figure 2. Indexing of the nodes in the simplified history-tree (see Fig. 1.)

The weights of every physical node originating from the same variance reduction node has to sum up to one, as they are interpreted as the probabilities of selecting the given physical node*:

$$\sum_{k=b_i^j}^{b_i^j+m_i^j+1} w_k = 1 \quad i = 1, \dots, n \quad j = 1, \dots, n_i \quad (6)$$

Detector events have to be also recorded with their time, detector contribution and physical node.

A recursive formula can be derived for the total number of subhistories ($M = M_I$), where M_i is the total number of subhistories for a tree having its root in physical node i :

$$M_i = \prod_{j=1}^{n_i} \sum_{k=b_i^j}^{b_i^j+m_i^j-1} M_k \quad (7)$$

* This condition is not met in this form in the case of the geometrical splitting as it is discussed later in Section 2.2.2.

For the practical description of the possible subhistories the subhistory matrix $\underline{\mathbf{S}}$ is introduced which has the same number of columns as physical nodes in the history and every row describe a subhistory by putting 1 in a column if the corresponding physical node is present in the subhistory and 0 if not. The generation of the possible subhistories can be done with a recursive algorithm if all the required data (see above) has been recorded during the transport. Once the subhistory matrix $\underline{\mathbf{S}}$ has been prepared, one can calculate the vector $\underline{\mathbf{W}}$, which contains the weight (W_i) of each subhistory, as the product of the weights (w_j) of the physical branches present in the given subhistory:

$$W_i = \prod_{s_{ij}=1} w_j \quad (8)$$

Due to condition (6) it can be shown that the sum of weights of the subhistories from a single history equals to one:

$$\sum_{i=1}^M W_i = 1 \quad (9)$$

This makes possible to interpret the W_i subhistory weight as the probability of the selection of the given subhistory.

Since the generated subhistories are analogous particle histories, the detector response r_i defined by (2) or (3) can be calculated for each of them. However, in the estimation of the probability distribution of the responses one has to take into account the subhistory weights:

$$P(R_k < R < R_{k+1}) = \left\langle \frac{1}{N} \sum_{j=1}^N \sum_{R_k < r_i^j < R_{k+1}} W_i^j \right\rangle = \left\langle \frac{1}{N} \sum_{j=1}^N W_k'^j \right\rangle \quad (10)$$

where the upper index j runs through the N original non-analogous histories simulated, i goes from 1 to M^j , which is the number of subhistories generated from history j , and $W_k'^j$ is the total weight of all the subhistories of history j that have contribution to bin k . One has to keep in mind that for the calculation of the variance of the estimation independent estimator contributions has to be used to avoid the calculation of the covariance between correlating contributions. As the subhistories generated from the same history are not completely independent one has to use again the W' total contributing weight from a history. Hence the relative error of the estimation in (10) can be estimated as:

$$s_k = \frac{\sum_{j=1}^N (W_k'^j)^2}{\left(\sum_{j=1}^N W_k'^j \right)^2} - \frac{1}{N} \quad (11)$$

For the correct estimation of the higher moments one has to interpret again the weight of the subhistories as the probability of selecting the given subhistory. Assuming that the l^{th} moment of the number of detections ($d_i^l \equiv 1$ in (2)) in a time interval is in question, it can be derived:

$$\langle R^l \rangle = \sum_{k=0}^{\infty} P(R = k) k^l = \sum_{k=0}^{\infty} \left\langle \frac{1}{N} \sum_{i=1}^{N'} W_i \delta_{r_i, k} \right\rangle k^l = \left\langle \frac{1}{N} \sum_{i=1}^{N'} W_i r_i^l \right\rangle, \quad (12)$$

where $\delta_{i,j}$ symbolizes the Kronecker- δ function and $N' > N$ is the total number of subhistories generated from N non-analogous histories. From (9) it is obvious that:

$$\sum_{i=1}^{N'} W_i = N \quad (13)$$

(12) emphasizes again that one cannot use the subhistory weights as the weight of single contributions but as the probability of the realization of the given contributions.

2.2.2. The application of history splitting for different variance reduction methods

Russian roulette (RR) is a basic method to control the spread and the weights in a history. Under certain conditions (e.g.: weight below cutoff) a track is removed ($w=0$) with a probability of $p < 1$ or it survives with an increased weight ($w'=w/p$)[8]. In the case of the above described approach the RR game is played on every subhistory to which the given branch belongs. This means that several Russian roulette games can be played on a subhistory and each of them can kill it. This results in a very low survival probability (p^n , where n is the number of Russian roulette games) and high survival weight. The resulting badly sampled high importance contributions can destroy the advantages gained from the application of variance reduction. Because of this problem an alternative method is proposed to control the histories. After certain limits is reached (e.g.: number of subhistories) the variance reduction techniques are switched off, and the history will soon terminate due to absorption.

In the case of the *implicit capture* at every collision the particle is split into an absorbed ($w=\sigma_a/\sigma_t$) and an unabsorbed ($w=1-\sigma_a/\sigma_t$) part[8]. Only the unabsorbed track is followed further. As the weight decreases continuously normally weight cutoff with Russian roulette is needed. Alternatively when weight drops below the cutoff, implicit capture is switched off. In order to control the number of subhistories the same happens when the number of implicit captures exceeds a certain limit.

Geometrical splitting means that a particle is split into n pieces when it enters a region with n times higher importance[8]. The weight is set to $w'=w/n$. In the reverse direction RR should be played with probability $1/n$. In case RR needs to be avoided, in order to prevent successive particle splittings on the same surface, geometrical splitting is made only when the particle enters a region for the first time. The optimisation of this method is an open question, but it avoids the problem related to the RR.

A special issue rises in the usual case when n (the ratio of the importances on the two sides of the splitting surfaces) is not an integer. Then most Monte Carlo codes (e.g. MCNP) split the particle into n' pieces which is randomly sampled from the integers neighboring n from a distribution having a mean of n :

$$P(n') = \begin{cases} n - [n] & n' = [n] + 1 \\ [n] + 1 - n & n' = [n] \end{cases}, \quad (14)$$

but uses the factor of $1/n$ for the the weight change of each particle. Obviously in this case (6) and consequently (9) can be true only for the expected values:

$$\left\langle \sum_{k=b_i^j}^{b_i^j+m_i^j+1} w_k \right\rangle = 1 \quad i = 1, \dots, n \quad j = 1, \dots, n_i \quad \text{and} \quad \left\langle \sum_{i=1}^M W_i \right\rangle = 1. \quad (15)$$

However, this change does not effect the results as the estimators defined in (10) and (12) already include the expected value.

Time splitting is similar to geometrical splitting but the importances are ordered to time intervals instead of geometry regions. If the importance always increases with time, there is no need for Russian roulette.

3. CALCULATIONS FOR THE VERIFICATION OF THE HISTORY SPLITTING METHOD

The subhistory generating algorithm has been implemented in Fortran90 subroutines. The subroutines have been written in a way that they can easily be integrated into the general Monte Carlo program flow without substantial modification. Special subroutines are called at the variance reduction events to record the above specified data, and an other subroutine is called when the transport of a history is finished to process the recorded data, generate the subhistories (i.e. prepare matrix $\underline{\mathbf{S}}$) and write their data into an output file for further processing. This structure allowed the integration of the method even into the MCNP4C3[9] Monte Carlo code[10]. However, for the verification calculations a very simple Monte Carlo code has been developed in order to be able to compare with analytical solution and fully analogous Monte Carlo calculation. This is a one dimensional, monoenergetic Monte Carlo code, which works in slab or infinite homogeneous geometries. Capture, fission, backscatter and detection cross-sections can be specified in the different regions of the geometry. The particles have velocity of unity: they cover unit of track length in a unit of time. Therefore the model is dimensionless and no time, length or cross-section units are given in the followings.

3.1. Simulation of the Pulse-Height Estimator

For the verification of the correct calculation of a pulse-height-type estimator a detection (Σ_d) and a capture (Σ_c) cross-section were specified in an infinite homogeneous medium ($\Sigma_t = \Sigma_d + \Sigma_c$). The detection does not stop the particle, it follows its track without any change. The estimator was the total number of detection from one source event, i.e. the $d_i^j \equiv 1$ in (3). It is straight-forward to calculate analytically the probability distribution (P) of the number of detections k :

$$P(k) = \left(\frac{\Sigma_d}{\Sigma_t} \right)^k \left(\frac{\Sigma_c}{\Sigma_t} \right), \quad (16)$$

where $\Sigma_t = \Sigma_d + \Sigma_c$. Simulations have been done both in a fully analogous way and with the application of a special variance reduction technique: after a certain number of detections (k_s) the particles were split into two with halved weight. To have the most efficient variance reduction k_s was chosen in a way that $P(k_s) \sim 0.5$. Simulations have been performed with different limits (n) for the number of variance reduction nodes.

In Fig. 3. one can see the theoretical results and the calculated ones with different n values. $n = 0$ means the analogous simulation. The fact that the variance reduction cases are in complete agreement with the analogous simulation and the analytical calculation verifies the correctness of the method. The computer time was the same for all calculations so that the efficiency of the calculations could be compared simply

by the comparison of the statistical error of the result. This comparison can be seen in Fig. 4. In the analogous case the relative error (s) exponentially deteriorates with the number of detections (k). This is in agreement with the result expected from theoretical considerations:

$$s_0(k) = \frac{1}{\sqrt{P(k)N}}, \quad (17)$$

where N is the number of histories started. In the non-analogous cases the relative error behaves differently:

$$s_n(k) = \frac{1}{\sqrt{P(k)2^{\min(n, [k/k_s])} N}}, \quad (18)$$

as splitting into two occurs after k_s detections while the number of splittings reaches the maximum (n).

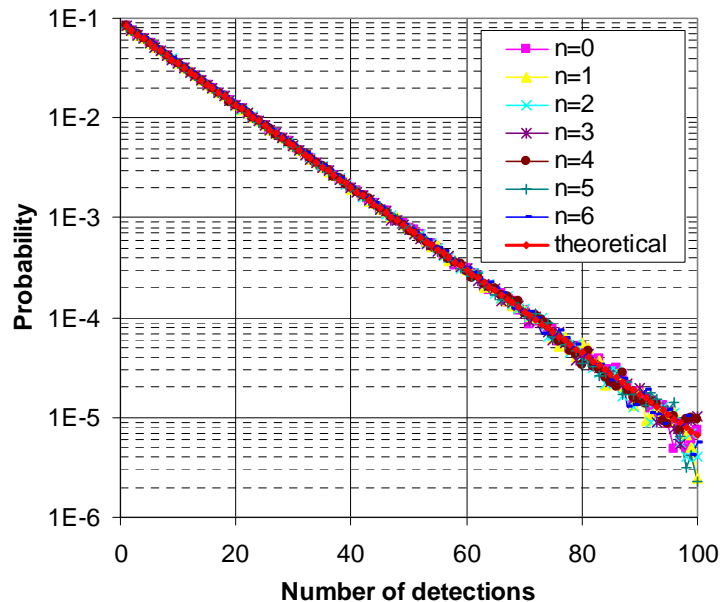


Figure 3. Probability distribution of the number of detections in an infinite, homogenous media with non-stopping detections cross-section $\Sigma_d = 0.1$ and capture cross-section $\Sigma_c = 0.01$ from analytical calculation and Monte Carlo estimations with different number of particle splittings allowed (n)

These predicted reductions in the error can be observed in Fig 4. when the given number of detections is reached ($k_s = 5$). Obviously the number of histories (N) simulated during the fixed calculation time is different in every case as the simulation of non-analogous histories and the generation of the subhistories afterward needs more effort. This is the reason why the relative error for the low number of detection is higher in the variance reduction cases then in the analogous case. The number of histories simulated together with the number of subhistories can be seen in Fig. 5. as a function of the number of variance reduction nodes (n). One can observe that the maximum number of subhistories is reached around $n = 3$

which explains why this case gives the lowest relative error for the higher number of detections (above 15 detections). It can be seen in Fig. 5. that as the number of variance reduction nodes is increased above $n = 3$ the number of subhistories decreases due the increasing effort needed for the simulation of the history and for the subhistory generation. This very simple case raises the attention that due to the complicated task of subhistory generation one has to be careful with the application of variance reduction methods. Obviously in a more complicated problem which needs more calculational effort in the transport part, more variance reduction nodes can be used to get optimal result.

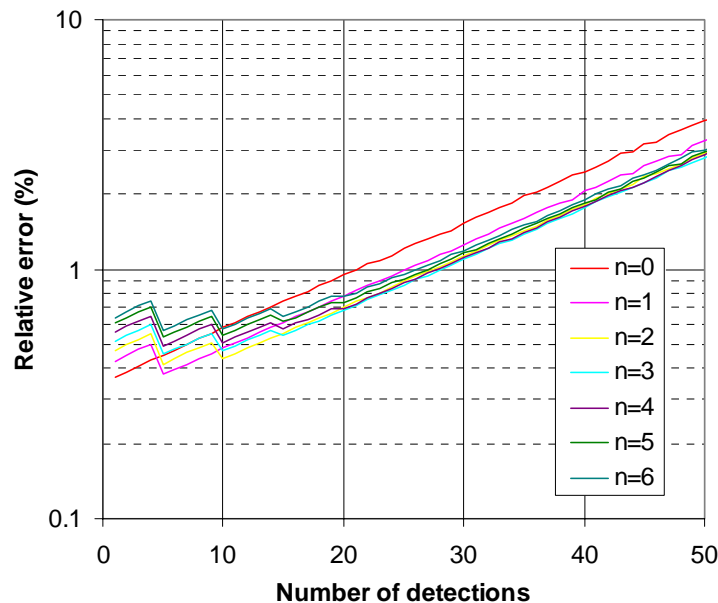


Figure 4. Relative error of the estimated distribution of the number of detections (see Fig. 3.) after the same computer time in each cases

3.2. Investigation of the Russian Roulette and the Alternative History Control Methods

For the investigation of the problem with the Russian roulette game mentioned above a more complicated one dimensional problem has been defined. The geometry can be seen in Fig. 6. while the cross-sections of the different materials in Table I. The main difference compared to the previous case is the presence of the multiplication (fission). The average number of particles from a fission is two. This results in more complicated subhistories as a physical node can have more then one child node and as it can be see from (7) this results in an exponential growth in the number of subhistories. The geometry is divided into regions where different importances of the particles can be defined. If all the imporances are equal, the simulation is analogous, otherwise geometrical splitting is applied as described in Section 2.2.2. For the sake of comparison both the conventional way of geometrical splitting (including Russian roulette) and the above proposed alternative solution for the replacement of Russian roulette has been used in different calculations. A limit was also set for the maximum number of variance reduction events in a history (n). The computer time for the different cases was again the same.

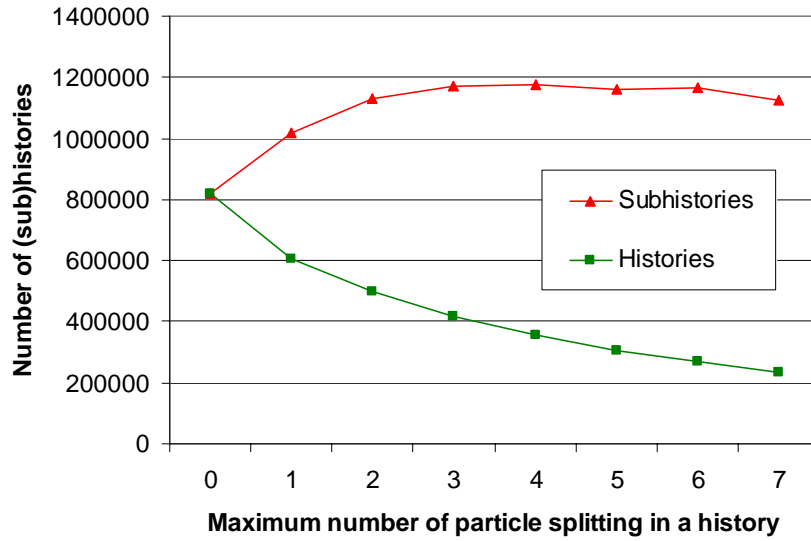


Figure 5. Number of simulated histories and subhistories during the same computer time for the cases on Fig. 3. with different number of variance reduction nodes allowed (n)

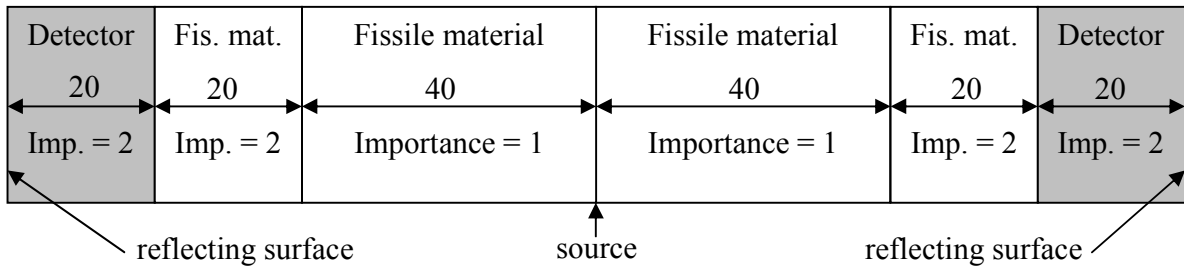


Figure 6. Problem geometry for the investigation of geometrical splitting and Russian roulette

Table I. Cross-section data for the materials in the geometry described in Fig. 6.

Material	Cross-section			
	Fission	Capture	Detection	Backscatter
Fissile material	0.05	0.07	0.0	0.05
Detector	0.0	0.01	0.1	0.05

Fig 7. shows the number-of-detections distributions and one can see that there is a good agreement between the different calculations, although in the case when RR is applied with max. 10 variance reduction events allowed a systematic underestimation can be observed for higher (>50) number of detections. In Fig 8. the relative error can be seen and the surprising result is that both RR case gives worse statistics then the analogous case. Furthermore the RR case with more variance reduction event (and therefore RR events) allowed gives the highest error of the results. On the other hand the alternative

method without RR provides a clear advantage over the analogous calculation. A reason for the bad performance of the RR can be found if one looks at the statistics of the RR game. It can be observed in both cases that there are a couple of subhistories which suffer many RR game (10-14) resulting in a huge weight multiplication (factor of 2^{10} - 2^{14}). Such extremely rare and big contributions cause the poor statistics of the results and explain the underestimation.

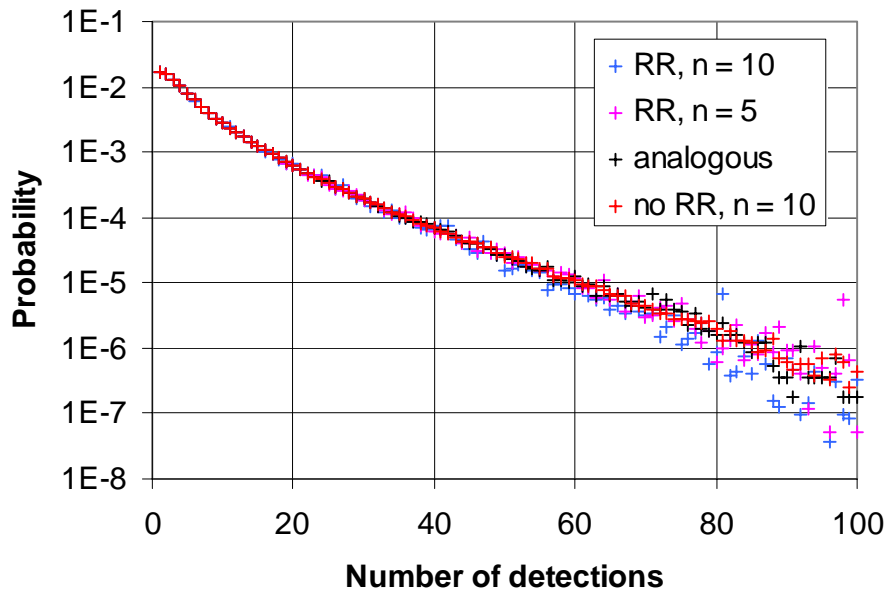


Figure 7. Probability distribution of the number of detections in the system described in Fig. 6. and Table I. estimated with analogous Monte Carlo and using geometrical splitting with or without Russian roulette (RR) and with different number of particle splittings allowed (n)

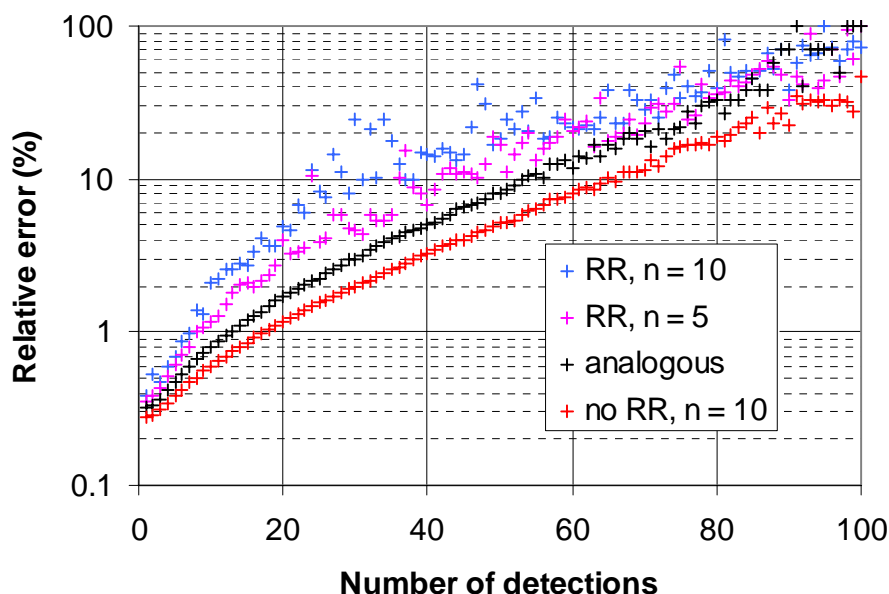


Figure 8. Relative error of the estimated number-of-detections distribution (see Fig. 7.) after the same computer time in each cases

3.3. Simulation of the Feynman- α Measurement

In order to verify that the method is capable for the estimation of correlations, the simulation of the Feynman- α measurement in an infinite, homogenous multiplicative medium has been performed. The model is again dimensionless with a velocity of unity, as described at the beginning of Section 3. Fission, capture and detection cross-sections have been defined in the system, where the detection reaction is a capture, as well. In such a simple case the variance-to-mean ratio of the number of count (C) during a time interval (T) can be analytically calculated from the well known formula[11]:

$$\frac{\sigma^2(C)}{\langle C \rangle} = 1 + \frac{\varepsilon}{(\rho - \beta)^2} \frac{\bar{\eta}(\bar{\eta} - 1)}{\bar{\eta}^2} \left(1 - \frac{1 - e^{-\alpha T}}{\alpha T} \right) = 1 + Y_\infty \left(1 - \frac{1 - e^{-\alpha T}}{\alpha T} \right), \quad (19)$$

where η is the number of neutron produced in one fission and ε is the efficiency of the detector defined as the ratio the number of detections over the number of fissions in the system. As the detection cross-section is homogenized also in the system, the efficiency can be obtained as:

$$\varepsilon = \frac{\Sigma_d}{\Sigma_f} \quad (20)$$

Delayed neutrons are neglected in the simulation ($\beta = 0$) and the reactivity (ρ) can be calculated from infinite medium multiplication factor (k_∞):

$$\rho = \frac{k_\infty - 1}{k_\infty}, \text{ where } k_\infty = \frac{v\Sigma_f}{\Sigma_t} \quad (21)$$

Based on the reactivity and the generation time (Λ) the prompt decay constant (α) can be obtained:

$$\alpha = -\frac{\rho}{\Lambda}, \text{ where } \Lambda = \frac{1}{v\Sigma_f v}, \quad (22)$$

where v is the velocity of the particle, which in this dimensionless model equals one. Table II. summarizes the parameters used in the model and the analytically calculated parameters of (19).

Table II. Parameters of the infinite, homogeneous model for the Feynman- α simulation

Cross-sections		Distribution of particles from fission (η)	
		η	$P(\eta)$
Fission	0.0425	0	0.1
Capture	0.0475	1	0.2
Detection	0.01	2	0.4
Backscatter	0.0	3	0.2
Total	0.1	4	0.1
α	0.005	Y_∞	60.8

In the case of the non-analogous simulation based on (12) one can reconstruct only the higher moments for the distribution of the detector contribution from one single source event in a time interval T . The moments of the detector contribution from the total source (i.e. the count rate) can be calculated from these, assuming that the source events are independent. For the variance the following is true:

$$\sigma^2(C) = N\sigma^2(R) = N(\langle R^2 \rangle - \langle R \rangle^2), \quad (23)$$

as the contributions from the N different histories are independent.

Simulations have been performed both on analogous and non-analogous way. In the non-analogous case implicit capture and time splitting has been used in order to sample more efficiently the correlations over longer time.

In order to produce independent source events with Poisson distribution, which is assumed in the Feynman- α method, the source time for all the subhistories are sampled uniformly for a certain “measurement time” which is several order of magnitudes longer, than the largest T what one wants to evaluate. Then the detector contributions are collected into time bins of T length along the complete measurement time similarly to the real measurement. In the analogous case from this the mean and the variance of the count rate C can be directly calculated. However, in the non-analogous case the

contributions from the same subhistory have to be collected to estimate the second moment of R based on (12). Then the variance of the count-rate can be calculated from (23). The calculation of the mean of the count-rate is straight-forward in both cases.

The evaluated variance-to-mean curves for the two simulations can be seen in Fig. 9. A nonlinear fit has been done to obtain the α and Y_∞ parameters of (19). The fitted parameters are in Table III. together with the number of histories run. One can observe that both simulation provided a very good estimation of the analytically determined parameters, however the statistics in the analogous case is apparently worse despite the approximately same running time. The results prove that the reconstruction of the higher moment is possible with the history splitting method and the applied variance reduction techniques make the simulation faster.

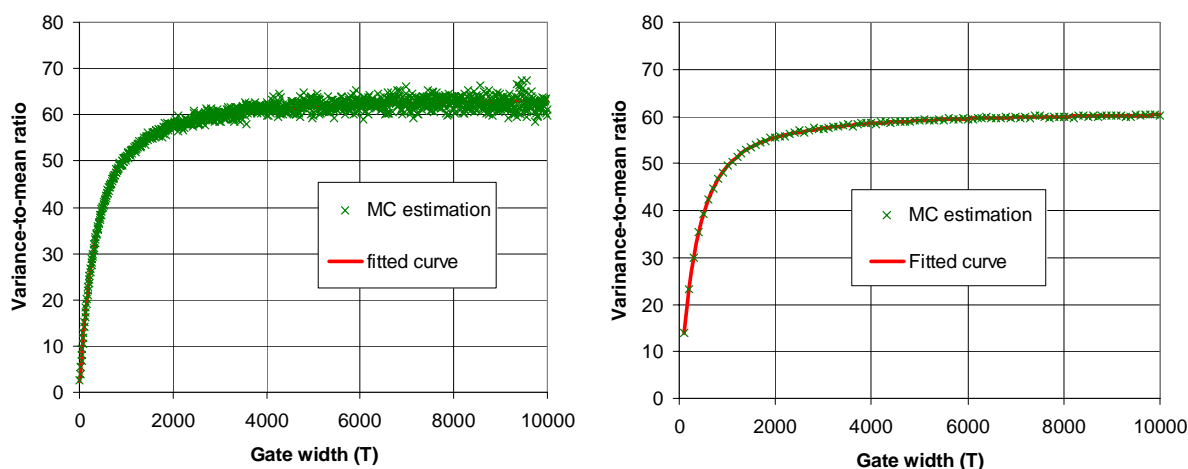


Figure 9. Variance-to-mean ratio curves obtained from analogous (left) and non-analogous (right) simulations for a infinite, homogeneous system with parameters in Table II. Parameters of the fitted curves can be found in Table III. The model is dimensionless with a velocity of unity.

Table III. Summary of the Feynman- α simulation results shown in Fig. 9.

Simulation	Number of histories	Fitted α	Fitted Y_∞
Analogous	2 000 000	0.00472	63.35
Non-analogous	100 000	0.005	60.48
Analytical	-	0.005	60.8

4. CONCLUSIONS

A general algorithm has been presented to provide variance reduction methods to non-Boltzmann problems. The history splitting algorithm generates analogous subhistories from a non-analogous history. The calculations performed prove its feasibility and applicability for pulse-height-type estimators and also for simulation of neutron noise measurements. Due to the calculational effort needed for the history splitting, careful optimization is needed for the efficient application of these variance reduction methods. It has also been shown that the Russian roulette is inefficient in high multiplicity problems and that this

can be overcome by using an alternative history control method. This method makes it possible to reconstruct the higher moments of the distribution. This has been demonstrated by the simulation of a Feynman- α measurement.

The history splitting method has already been implemented in MCNP4C3 and calculations are in progress for the simulation of neutron noise measurement on different source driven subcritical systems. The precise simulation of such measurement can help the development of methods for the monitoring of the subcriticality in a future ADS. Since such full-scale simulations are not feasible with fully analogous Monte Carlo, the method presented can provide a valuable contribution to these investigations.

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