

STOCHASTIC TRANSPORT THEORY OF THE DETERMINISTIC AND STOCHASTIC PULSING FEYNMAN- α METHODS: VALIDATION WITH MUSE EXPERIMENTS

D. Ballester* and J. L. Muñoz-Cobo

Department of Chemical and Nuclear Engineering
Polytechnic University of Valencia, Cami de Vera s/n, 46022 Valencia, Spain
dabalber@mat.upv.es; jlcobos@iqn.upv.es

J. L. Kloosterman

Interfaculty Reactor Institute, Department of Reactor Physics
Delft University of Technology, Mekelweg 15, 2629 JB Delft, The Netherlands
kloosterman@iri.tudelft.nl

ABSTRACT

Stochastic neutron transport theory is applied to the derivation of the Feynman- Y function for subcritical assemblies when external pulsed sources are used. We obtain a general relationship between the probability generating functions of the kernel and the source considering the contribution to the detector statistics of both, the pulsed source and the intrinsic neutron source. Expressions corresponding to the fundamental mode approach are obtained.

In addition, these expressions are used to fit the system prompt neutron time constant, $1/(-\alpha_0)$, with experimental data gathered during the MUSE-4 European project for different nuclear assembly conditions.

Experiments show that, under certain circumstances, the deterministic character of the external pulsed source makes the system to behave as a sub-Poissonian one. In addition, the stochastic pulsing method seems to be more adequate than the deterministic one because the number of fitting parameters is lower, and also due to its better statistical behavior for the given experimental conditions.

KEYWORDS: Neutron stochastic transport theory, Feynman-alpha method

1. INTRODUCTION

Following the study of the neutron fluctuations in a multiplying medium, several static and dynamic neutron noise methods appeared concerning their applicability for the determination of some nuclear reactor physics parameters.

Within the group of dynamic methods, a classical neutron noise technique is the Feynman- α method [1]. As in other similar techniques, neutron detector counting rates related to individual fission-chain events must be discerned from the total counting rate, therefore these methods are

*Present address: Dept. of Applied Mathematics, Polytechnic University of Valencia, Cami de Vera s/n, 46022 Valencia, Spain

applicable to subcritical systems near delayed critical conditions.

Recently the interest on these kind of nuclear methods has been recovered from both the experimental and the theoretical point of view, due to the increasing interest shown by researchers on the conceptual development of accelerator-driven systems (ADS) for nuclear waste transmutation and energy production purposes. An important issue regarding its future industrial applicability is the development of a periodically subcriticality level measurement and monitoring technique, since both the operation safety and performance as a part of the nuclear fuel cycle, will be seriously affected by this variable.

Further, in the 1960's, dynamic methods based on a time-dependent external neutron source were proposed [2]. Recently, the use of these methods has increased during the MUSE European experimental studies carried out at the MASURCA facility (Cadarache, France) due to their applicability in order to investigate ADS kinetic parameters. In these experiments D-D and D-T neutron sources running in pulsed mode have been used. Anyway, measurements can be done in two different ways [3, 4]: in the first one, the neutron detector time gate is synchronised with the external neutron pulse injection, thus this method is referred to as deterministic pulsing method, whereas, in the second case, the relative delay between the neutron pulse injection and the beginning of the neutron counting time is uniformly sampled between zero and the pulsed source period, which is known as stochastic pulsing method. For the latter case, the neutron source can be assumed to have the form

$$S(t) = k \sum_{m=-\infty}^{\infty} \delta(t - (\xi + mT)), \quad (1)$$

where k is the number of particles injected per accelerator pulse, T is the pulsed source period, and ξ is uniformly sampled within the time interval $[0, T]$. Obviously, for the deterministic pulsing method we will put $\xi = 0$.

Some recent works have correctly described the non-Poissonian character of a periodic pulsed source, also including the effect of the intrinsic source and deduced the expressions for the Feynman- Y function for the deterministic and the stochastic pulsing methods, within the framework of stochastic neutron transport theory [4, 5].

In this paper we have neglected the contribution of delayed neutrons, therefore all quantities appearing here can be interpreted as prompt variables for short time-scales, in comparison with the delayed neutron precursors lifetimes.

2. THE DETERMINISTIC PULSING METHOD

We will just focus on the applicability of the deterministic pulsing method, assuming that the detector time gate is synchronised at the beginning (instant $t_f - \tau_c$, t_f and τ_c being the final detection time and the inspection interval length, respectively) with the injection of one external source pulse [6, 7]. In this case the variance is lower than the corresponding one when the synchronisation is done at the end of the time gate, t_f , [5]. Also, from a practical point of view this method is easier to apply.

Under these circumstances, the Feynman-Y function can be recast as

$$Y^{\text{dpm}}(\tau_c) = \frac{\text{Var}^{\text{dpm}}[n]}{\text{E}^{\text{dpm}}[n]} - 1 = \frac{\mu_2 - \mu_1^2}{\mu_1}, \quad (2)$$

where $\text{E}[n]$ and $\text{Var}[n]$ are the expected value and variance operators, and μ_1 and μ_2 are the first and second factorial moments of the number of counts gathered by the neutron detector, which are, obviously, functions of the inspection time length. The superscript dpm applies for the deterministic pulsing method.

Assuming the number of detections induced by both the intrinsic source and the external pulsed source, to be mutually independent discrete random variables, we find that the first factorial moment of the number of detections in the fundamental mode approximation (subscript 0) can be expressed as [5]

$$\mu_1 = \langle \bar{n} | S_1 \rangle = \mu_1^{\text{sf}} + \mu_1^{\text{sp}}, \quad (3)$$

where the contribution of the spontaneous fission source to the average number of counts is

$$\mu_1^{\text{sf}} = (S_1^{\text{sf}}, \Phi_0^+) \frac{(S_1^+, \Phi_0)}{(\frac{1}{v}\Phi_0^+, \Phi_0)} \left(-\lambda_{\text{sf}} \frac{\tau_c}{\alpha_0} \right), \quad (4)$$

λ_{sf} being the spontaneous fission disintegration time constant, with the phase-space inner product $(a, b) = \int d\mathbf{r} d\mathbf{v} d\Omega a(\vartheta) b(\vartheta)$, $\vartheta \equiv (\mathbf{r}, v, \Omega)$, and

$$\mu_1^{\text{sp}} = (S_1^{\text{sp}}, \Phi_0^+) \frac{(S_1^+, \Phi_0)}{(\frac{1}{v}\Phi_0^+, \Phi_0)} \left(-\frac{1}{\alpha_0} \right) \left(\ell + \frac{1 - \exp(\alpha_0(\tau_c - \ell T))}{1 - \exp(\alpha_0 T)} \right), \quad (5)$$

ℓ being the integer part of τ_c/T , is the average number of counts coming from the pulsed source. In these equations we have used the α -modes neutron flux expansion

$$\phi(\vartheta, t) = \phi^{\text{sf}}(\vartheta, t) + \phi^{\text{sp}}(\vartheta, t) = \sum_j \varphi_j(\vartheta) \left[\zeta_j^{\text{sf}}(t) + \zeta_j^{\text{sp}}(t) \right], \quad (6)$$

where the α -eigenfunctions φ_j satisfy the α -eigenvalue equation

$$L\varphi_j(\vartheta) = \frac{\alpha_j}{v}\varphi_j(\vartheta), \quad (7)$$

L being the classical time-independent transport operator [8]. As usually, the superscript + applies for adjoint functions.

In addition, in Eqs. 4 and 5 we have defined the time-dependent intrinsic and spallation neutron sources

$$S_1^{\text{sf}}(\vartheta, t) = S_1^{\text{sf}}(\vartheta) \times S_1^{\text{sf}}(t), \quad (8)$$

$$S_1^{\text{sp}}(\vartheta, t) = S_1^{\text{sp}}(\vartheta) \times S_1^{\text{sp}}(t), \quad (9)$$

$$S_1^{\text{sf}}(\vartheta) = \bar{v}_{\text{sf}} N_0 \rho_{\text{sf}}(\mathbf{r}) \frac{\chi_S(v)}{4\pi}, \quad (10)$$

where N_0 is the initial number of nuclei forming part of the spontaneous fission source, $\rho_{\text{sf}}(\mathbf{r})$ is its shape probability distribution function, and $\chi_S(v)$ is the spectrum of neutrons isotropically emitted by the intrinsic source,

$$S_1^{\text{sf}}(t) = \lambda_{\text{sf}} N(t) / N_0 \equiv \lambda_{\text{sf}}, \quad (11)$$

assuming the initial number of nuclei corresponding to the spontaneous disintegration neutron source to be constant in our time-scale,

$$S_1^{\text{sp}}(\vartheta) = \bar{v}_{\text{pp}} \bar{v}_{\text{sp}} \rho_{\text{sp}}(\mathbf{r}) f_{\text{sp}}(v, \Omega), \quad (12)$$

where $\rho_{\text{sp}}(\mathbf{r})$ is the shape probability distribution of the external pulsed source, $f_{\text{sp}}(v, \Omega)$ is the spectral and angular probability distribution function corresponding to those neutrons produced following an external particles interaction,

$$S_1^{\text{sp}}(t) = \sum_{m=-\infty}^{\infty} \delta(t - (\xi + mT)), \quad (13)$$

with $\bar{v}_w = \sum_1^{I_w} j \epsilon_j^w$, ϵ_j^w , $w = \text{sf, sp, pp}$, being the probability to emit j neutrons following a spontaneous fission event, the probability to emit j neutrons per external particle introduced following a collision event, and the probability for the accelerator to introduce j external particles in the system in each source pulse, respectively, and I_w the maximum number of particles produced after a given event.

Equivalently, the adjoint first factorial moment source, S_1^+ , corresponds to the effective macroscopic neutron detection cross section [9, 10]:

$$S_1^+(\vartheta, t) = S_1^+(\vartheta) \times S_1^+(t) = [\eta_c \Sigma_c^D + \eta_s \Sigma_s^D + \eta_f \Sigma_f^D] \times [H(t - (t_f - \tau_c)) - H(t - t_f)]. \quad (14)$$

This magnitude¹ will be non-zero only for $\mathbf{r} \in V_D$ and $t \in (t_f - \tau_c, t_f]$.

On the other hand, for the second factorial moment of the number of neutron detections we will have:

$$\mu_2 = \langle \bar{n} | S_1 \rangle^2 + \langle \overline{n(n-1)} | S_1 \rangle + \Delta\mu_2, \quad (15)$$

where the second term is given by

$$\langle \overline{n(n-1)} | S_1 \rangle = \langle \overline{n(n-1)} | S_1^{\text{sf}} \rangle + \langle \overline{n(n-1)} | S_1^{\text{sp}} \rangle, \quad (16)$$

$$\begin{aligned} \langle n(n-1) | S_1^{\text{sf}} \rangle &= \bar{v}^2 D(\Sigma_f \Phi_0, \Phi_0^+, \Phi_0^+) \frac{(S_1^{\text{sf}}, \Phi_0^+)}{(\frac{1}{v} \Phi_0, \Phi_0^+)} \frac{(S_1^+, \Phi_0)^2}{(\frac{1}{v} \Phi_0^+, \Phi_0)^2} \\ &\times \left(-\frac{\lambda_{\text{sf}}}{\alpha_0^3} \right) \left(\tau_c + \frac{(1 - \exp(\alpha_0 \tau_c))}{\alpha_0} \right), \end{aligned} \quad (17)$$

$D = \overline{v(v-1)}/\bar{v}^2$ being the nuclear system Diven's factor, with $\bar{v} = \sum_1^I j \epsilon_j$, $v(v-1) = \sum_2^I j(j-1) \epsilon_j$, where ϵ_j is the probability for the nuclear system to emit j neutrons after a fission event, and where

$$\begin{aligned} (\Sigma_f \Phi_0, \Phi_0^+, \Phi_0^+) &= \int_{V_{\text{SYS}+V_D}} d\mathbf{r} \int dv \int d\Omega \Sigma_f(\mathbf{r}, v) \Phi_0(\mathbf{r}, v, \Omega) \\ &\times \left[\int dv' \int d\Omega' \frac{\chi(\mathbf{r}, v')}{4\pi} \Phi_0^+(\mathbf{r}, v', \Omega') \right]^2, \end{aligned} \quad (18)$$

where $\chi(\mathbf{r}, v')$ is the spectrum of neutrons isotropically emitted after a fission event within the fissile and detector volume.

Similarly, for the term corresponding to the spallation source in Eq. 16:

$$\begin{aligned} \langle \overline{n(n-1)} | S_1^{\text{sp}} \rangle &= \bar{v}^2 D(\Sigma_f \Phi_0, \Phi_0^+, \Phi_0^+) \frac{(S_1^{\text{sp}}, \Phi_0^+)}{(\frac{1}{v} \Phi_0, \Phi_0^+)} \frac{(S_1^+, \Phi_0)^2}{(\frac{1}{v} \Phi_0^+, \Phi_0)^2} \\ &\times \left\{ \frac{(1 - 2 \exp(\alpha_0 \tau_c) + \exp(2\alpha_0(\tau_c - \ell T)))}{\alpha_0^3 (1 - \exp(2\alpha_0 T))} - \frac{2(1 - \exp(\alpha_0 \tau_c))}{\alpha_0^3 (1 - \exp(\alpha_0 T))} \right\} \end{aligned}$$

¹ Σ_x^D , $x = c, s, f$ is the detector volume, V_D , capture, scattering, and fission macroscopic cross section and η_x its corresponding efficiency.

$$\begin{aligned}
& + \frac{2T}{\alpha_0^2} \left[\frac{\ell \exp(-\alpha_0 \ell T)}{(1 - \exp(\alpha_0 T))} - \frac{(1 - \exp(-\alpha_0 \ell T))}{(2 - \exp(-\alpha_0 T) - \exp(\alpha_0 T))} \right] \\
& \times \left. \exp(\alpha_0 \tau_c) - \frac{2\tau_c \exp(\alpha_0 (\tau_c - \ell T))}{\alpha_0^2 (1 - \exp(\alpha_0 T))} - \frac{\ell}{\alpha_0^3} \right\}. \tag{19}
\end{aligned}$$

Eq. 16 represents the second factorial moment contribution due to fission processes within the fissile material. We have neglected multiplicative effects corresponding to scattering and fission events within the detector volume leading to a variance positive contribution [5, 10].

Further, for the third term in Eq. 15 we obtain:

$$\Delta\mu_2 = \Delta\mu_2^{\text{sf}} + \Delta\mu_2^{\text{sp}}, \tag{20}$$

$$\Delta\mu_2^{\text{sf}} = \bar{v}_{\text{sf}} D_{\text{sf}} \lambda_{\text{sf}} \left(S_1^{\text{sf}}, \Phi_0^+, \Phi_0^+ \right) \frac{(S_1^+, \Phi_0)^2}{\left(\frac{1}{v} \Phi_0^+, \Phi_0 \right)^2} \frac{1}{\alpha_0^2} \left(\tau_c + \frac{(1 - \exp(\alpha_0 \tau_c))}{\alpha_0} \right), \tag{21}$$

$D_{\text{sf}} = \overline{v_{\text{sf}}(v_{\text{sf}} - 1)}/\bar{v}_{\text{sf}}^2$ being the intrinsic neutron source Diven's factor, and

$$\left(S_1^{\text{sf}}, \Phi_0^+, \Phi_0^+ \right) = \bar{v}_{\text{sf}} N_0 \int d\mathbf{r} \rho_{\text{sf}}(\mathbf{r}) \left[\int dv' \int d\Omega' \chi_S(v') / 4\pi \Phi_0^+(\mathbf{r}, v', \Omega') \right]^2, \tag{22}$$

whereas for the spallation source:

$$\begin{aligned}
\Delta\mu_2^{\text{sp}} & = \left[\bar{v}_{\text{sp}} D_{\text{sp}} (S_1^{\text{sp}}, \Phi_0^+, \Phi_0^+) + (D_{\text{pp}} - 1) (S_1^{\text{sp}}, \Phi_0^+) (S_1^{\text{sp}}, \Phi_0^+) \right] \\
& \times \frac{(S_1^+, \Phi_0)^2}{\left(\frac{1}{v} \Phi_0^+, \Phi_0 \right)^2} \frac{1}{\alpha_0^2} \left\{ 2 \frac{(1 - \exp(-\alpha_0 \ell T))}{(1 - \exp(\alpha_0 T))} \exp(\alpha_0 \tau_c) \right. \\
& \left. + \frac{(1 - 2 \exp(\alpha_0 \tau_c) + \exp(2\alpha_0 (\tau_c - \ell T)))}{(1 - \exp(2\alpha_0 T))} + \ell \right\}, \tag{23}
\end{aligned}$$

where, as before, $D_w = \overline{v_w(v_w - 1)}/\bar{v}_w^2$, $w = \text{sp}, \text{pp}$, is Diven's factor for the spallation neutron production process, and for the pulsed external source, respectively, and where

$$\left(S_1^{\text{sp}}, \Phi_0^+, \Phi_0^+ \right) = \bar{v}_{\text{pp}} \bar{v}_{\text{sp}} \int d\mathbf{r} \rho_{\text{sp}}(\mathbf{r}) \left[\int dv' \int d\Omega' f_{\text{sp}}(v', \Omega') \Phi_0^+(\mathbf{r}, v', \Omega') \right]^2. \tag{24}$$

Therefore, Eq. 2 can be written as

$$Y^{\text{dpm}}(\tau_c) = \frac{\langle \overline{n(n-1)} | S_1^{\text{sf}} \rangle + \langle \overline{n(n-1)} | S_1^{\text{sp}} \rangle + \Delta\mu_2^{\text{sf}} + \Delta\mu_2^{\text{sp}}}{\mu_1^{\text{sf}} + \mu_1^{\text{sp}}}, \quad (25)$$

where terms in the numerator corresponding to the spontaneous fission source have the same time-dependent factor.

3. THE STOCHASTIC PULSING METHOD

When the stochastic pulsing method (spm) is applied, the relative delay between the neutron pulse injection and the beginning of the neutron counting time is uniformly sampled between zero and the pulsed source period [3, 6]). In this case, the Feynman-Y function can be written as

$$Y^{\text{spm}}(\tau_c) = \frac{\langle \mu_2 \rangle_\xi - \langle \mu_1^2 \rangle_\xi}{\langle \mu_1 \rangle_\xi}, \quad (26)$$

where $\langle \circ \rangle_\xi = 1/T \int_0^T d\xi \circ$.

This expression can be shortly recast as [11]

$$Y^{\text{spm}}(\tau_c) = \gamma_1 \frac{\kappa(\pi\tau_c/T; (2\pi)^{-1}\alpha_0 T)}{\tau_c} + \gamma_2 \left(1 + \frac{(1 - \exp(\alpha_0\tau_c))}{\alpha_0\tau_c} \right), \quad (27)$$

where the time-independent factors are given by

$$\gamma_1 = \frac{(-\alpha_0)T^2}{2\pi^4} \frac{(S_1^{\text{sp}}, \Phi_0^+)^2}{(S_1^{\text{sf+sp}}, \Phi_0^+)} \frac{(S_1^+, \Phi_0)}{(\frac{1}{v}\Phi_0^+, \Phi_0)}, \quad (28)$$

$$\begin{aligned} \gamma_2 = & \left\{ \bar{v}^2 D(\Sigma_f \Phi_0, \Phi_0^+, \Phi_0^+) (S_1^{\text{sf+sp}}, \Phi_0^+) \left(\frac{1}{v} \Phi_0, \Phi_0^+ \right)^{-1} \frac{1}{(-\alpha_0)} \right. \\ & + \left\{ \bar{v}_{\text{sf}} D_{\text{sf}} \lambda_{\text{sf}} (S_1^{\text{sf}}, \Phi_0^+, \Phi_0^+) + 1/T [\bar{v}_{\text{sp}} D_{\text{sp}} (S_1^{\text{sp}}, \Phi_0^+, \Phi_0^+) \right. \\ & \left. \left. + (D_{\text{pp}} - 1) (S_1^{\text{sp}}, \Phi_0^+) (S_1^{\text{sp}}, \Phi_0^+) \right] \right\} \left. (S_1^{\text{sf+sp}}, \Phi_0^+)^{-1} \frac{(S_1^+, \Phi_0)}{(\frac{1}{v}\Phi_0^+, \Phi_0)} \frac{1}{(-\alpha_0)} \right\}, \quad (29) \end{aligned}$$

$$\kappa(h; a) = \sum_{s=1}^{\infty} \frac{\sin^2(sh)}{s^2(s^2 + a^2)} = \frac{(h - m\pi)(\pi - (h - m\pi))}{2a^2} - \frac{\pi \coth(\pi a)}{4a^3} \left(1 - \frac{\cosh(2(h - m\pi)a - \pi a)}{\cosh(\pi a)} \right), \quad (30)$$

with $h \geq 0$, m being the integer part of h/π .

This second time-independent factor takes into account the contribution stemming from the non-Poissonian nature of the external pulsed source and the intrinsic spontaneous source as well [5]. Owing to the stochastic-pulsing-method randomising effect, the time-dependence of Feynman-Y function, within the fundamental mode approach, has a very familiar form. From a practical point of view, this means that when the stochastic pulsing method is used we need to fit three parameters: γ_1 , γ_2 , and the eigenvalue α_0 , whereas for the deterministic pulsing method we will have five parameters.

As before, for a coherent external pulsed γ_2 can be positive or negative, depending on the solution of the complete transport problem.

4. EXPERIMENTAL VALIDATION

Under the 5th Euratom Framework Programme of the European Commission, the MUSE-4 experiments have been carried out within the framework of a large international collaboration. One of the main objectives of this program is the definition of experimental methods allowing the determination of subcriticality levels in support of the operation of an ADS [12].

Among the methods studied, experiments with D-D and D-T neutron sources running in pulsed mode have been used, although the utilisation of an external spallation proton source has also been thought. In both cases an external deuteron accelerator (GENEPI) is coupled with a fast subcritical assembly (MASURCA facility, Cadarache). This accelerator injects deuteron pulses that impinge on a TiD or TiT target. The beam peak intensity is ~ 50 mA with a width of less than $1 \mu s$. The repetition rate can vary from a few hertz up to 5 kHz [12].

The on-line monitoring of the neutron production for both the deuterium and the tritium targets, is based on the detection by Si detectors placed up-stream of the target of: recoil protons induced by the D(d,p)T reaction, which occurs about as often as the D(d,n)³He reaction on the deuterium target, or, recoil α particles produced by the T(d,n)⁴He reactions on the tritium target.

Two different experimental subcritical conditions are investigated in the present work:

1. SC0 configuration (1108 fuel cells) with the pilot rod (PR) not inserted (down or d), and $k_{\text{eff}} = 0.9933$,
2. SC0 configuration (1108 fuel cells) with PR inserted (up or u), and $k_{\text{eff}} = 0.9946$,

Those different configurations are achieved by replacing some peripheral fuel cells by stainless steel-sodium cells. The PR consists of a polyethylene-made rod which can be inserted (up) acting as a neutron moderator.

Fig. 1 shows the experimental Feynman- Y curves and their corresponding fittings using Eqs. 25 and 27 for the deterministic and the stochastic pulsing method, respectively.

Two different behaviors can be clearly distinguished: in Figs. 1 (a) and (b), both corresponding to the same experiment, we can appreciate the fact that, when the TiD target is used, positive terms in the expression applicable for the pulsing Feynman- α method, Eqs. 25 and 27, have higher contributions than the negative term due to the non-Poissonian character of the pulsed source. Indeed, the two main differences in the experimental set-ups used in Fig. 1 (a) and (b) and in Figs. 1 (c) and (d) are the insertion of the PR and the variation of the target nature.

The insertion of the PR only provokes a modest change in the subcriticality level due to the neutron moderation introduced by the polyethylene, as it has been mentioned before. In addition, notice that the influence that the neutron moderation must have on the Feynman- Y function is the opposite one. For a higher neutron thermalisation we can expect two effects: the fission rate might increase, and the absolute value of α_0 will be lower. Hence the relative importance of the fission multiplicity term, that proportional to $v(v-1)$, might be higher because of a higher fission rate and because this term is proportional to $1/(-\alpha_0)^3$, whereas terms arising from the non-Poissonian nature of the pulsed source and the intrinsic source are proportional to $1/(-\alpha_0)^2$.

However, owing to the limited amount of measurements available it has not been possible to compare both experimental conditions for the same PR position, what might permit us to discard its effect on the qualitative behavior of the Feynman- Y function for our experimental conditions.

On the other hand, our transport problem can be affected by the use of a TiD or a TiT target in the following way. In any case, for the neutron production processes within the target material we will have $\bar{v}_{sp} < 1$ and $\overline{v_{sp}(v_{sp}-1)} = 0$. Then, from Eqs. 25 and 27 it is clear that the external pulsed source can behave as a sub-Poissonian neutron source if the variance of the number of deuterons introduced by the accelerator in each pulse, $\overline{v_{pp}^2} - \bar{v}_{pp}^2$, is lower than the one corresponding to a Poissonian source, as it would be the case of a disintegration source. According to neutron production calibrations [13, 14], the neutron strength of the external source when the TiD target was used during MUSE-4 experiments was 3.0×10^4 neutrons per pulse, whereas for the TiT target this quantity changed along time: for the calibration corresponding to conditions of Figs. 1 (c) and (d) a value of 3.3×10^6 neutrons per pulse was obtained. Neutron production calibrations were always made for the optimal accelerator deuteron pulse and based on its reproducibility [13, 14]. Therefore, assuming that the number of deuterons introduced in the accelerator was more or less constant for the given experimental conditions, there is an important quantitative difference in the values of \bar{v}_{sp} for the TiD and TiT targets. In both the deterministic pulsing method and the stochastic one, this effect can explain why the negative terms appearing in Eqs. 25 and 27, and stemming from the pulsed periodic nature of the external source, is higher enough to compensate the fission multiplicity term when a TiT target is used, but not for the case of the TiD target.

It should be noticed that Figs. 1 (a) and (b) are similar to Figs. 5 and 6 obtained in Ref. [15]. In fact, they correspond to the same experimental conditions (also with TiD target), but those presented here were obtained with a different pulsed period and also have better statistics. That means that experiments reported in Ref. [15] and our Figs. 1 (a) and (b) were carried out using

exactly the same instrumentation, the same core set-up and with neutron detectors at the same positions. For the cited reference, the graph corresponding to the deterministic pulsing method presents a short initial part where $Y < 0$, for small values of τ_c , whereas that one applicable for the stochastic pulsing method $Y \geq 0$ for any τ_c , which means that the fission multiplicity term is dominant in those particular conditions, as in our Figs. 1 (a) and (b).

Further, comparing both measurement methods, we can see how the amount of data available does not suffice in order to apply the deterministic pulsing method to fit the eigenvalue α_0 . Indeed, the same data have been used for both pulsing methods, hence the order of magnitude of the experimental error is exactly the same for the deterministic and the stochastic pulsing method. However, it can be easily observed that the scale of the Y^{spm} curve is about one or two orders of magnitude higher than that of Y^{dpm} . Therefore, it can be concluded that the stochastic pulsing method behaves better from a statistical point of view. Further, if Fig. 1 (b) (TiD target) is compared with Fig. 1 (d) (TiT target) it seems that this nice behavior increases when the magnitude of the Feynman- Y curve increases, what is achieved by increasing the number of neutrons introduced per external particle, $\bar{\nu}_{\text{sp}}$. The only term proportional to the external source strength, $\bar{\nu}_{\text{pp}}$, (or nearly proportional, if we consider the contribution of the intrinsic source) in Eq. 27 is the time correlation term, hence, under some circumstances, the statistical efficiency of the stochastic pulsing method can be improved by increasing the number of external particles injected by the accelerator in each pulse.

The fitted values of α_0 using the stochastic pulsing method are comparable with other values obtained for the same conditions using other neutron noise methods [11].

The deterministic pulsing method has not been applied to fit the eigenvalue α_0 due to insufficient experimental data shown in Fig. 1 and the fact that the number of parameters to be fitted is higher than for the stochastic pulsing method. Taking into account the eigenvalue obtained by fitting the expression Y^{spm} , we have tried to fit the different time-independent factors appearing in Eq. 27 (four parameters, because one of them can be taken as a renormalisation factor). In Figs. 1 (a) and (c) we can see how the qualitative behavior of those fits agrees satisfactorily with the experimental curves.

5. CONCLUSIONS

In the present work we have dealt with the applicability of the pulsing Feynman- α method as subcritical level monitoring technique for a future subcritical ADS.

In Sections 2 and 3 we have made use of Feynman- Y expressions for both the deterministic and the stochastic pulsing method. The different contributions stemming from the non-Poissonian nature of the periodic pulsed source and the intrinsic source due to the presence of spontaneous fission events within the fissile material have been discussed.

In Section 4 we have reported graphs corresponding to the Feynman- α method for some of the experimental conditions studied during the MUSE-4 European project. We have shown that under certain circumstances, the coherent nature of the external source provokes the system to behave as a sub-Poissonian one ($Y < 0$).

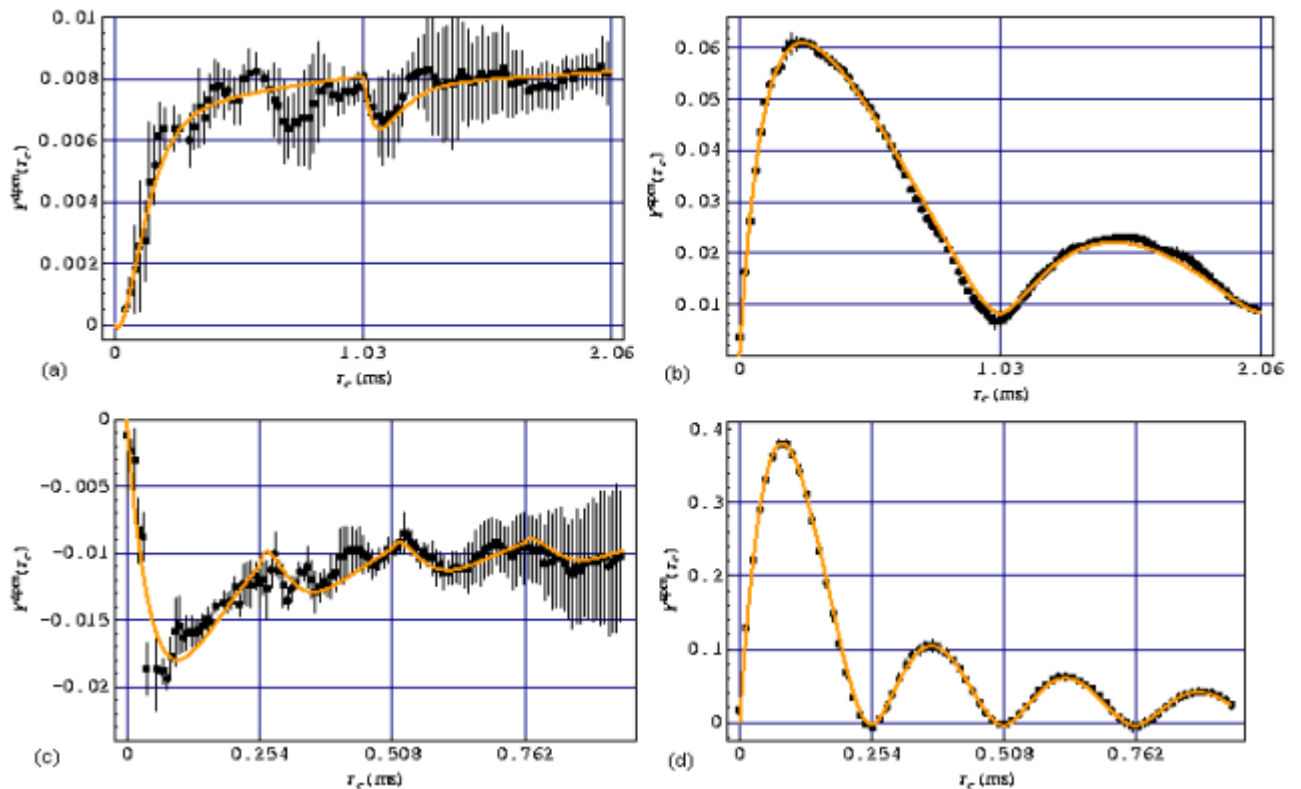


Figure 1. Feynman-Y function in the fundamental mode approach as a function of the detector time gate when the deterministic and the stochastic pulsing methods are applied: experimental and fitted curves. Figs. (a)-(b): SC0, PR down, TiD target; Figs. (c)-(d): SC0, PR up, TiT target.

From a statistical point of view, the stochastic pulsing method seems to be more adequate than the deterministic one, at least for fast subcritical assemblies. Owing to the fact that the wavy term in Eq. 27 is nearly proportional to the external source strength, it is possible to get a more flexible method for the determination of the subcriticality in a future ADS. For a stronger external source, the statistical problem of fitting the prompt neutron time constant will be more independent on other particular conditions. Furthermore, owing to the randomising effect of the stochastic pulsing method, the number of fitting parameters is lower for this technique.

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