A high-order discontinuous Galerkin solver for the incompressible RANS equations coupled to the $k – \epsilon$ turbulence model

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Abstract

Accurate methods to solve the Reynolds-Averaged Navier-Stokes (RANS) equations coupled to turbulence models are still of great interest, as this is often the only computationally feasible approach to simulate complex turbulent flows in large engineering applications. In this work, we present a novel discontinuous Galerkin (DG) solver for the RANS equations coupled to the $k – \epsilon$ model (in logarithmic form, to ensure positivity of the turbulence quantities). We investigate the possibility of modeling walls with a wall function approach in combination with DG. The solver features an algebraic pressure correction scheme to solve the coupled RANS system, implicit backward differentiation formulae for time discretization, and adopts the Symmetric Interior Penalty method and the Lax-Friedrichs flux to discretize diffusive and convective terms respectively. We pay special attention to the choice of polynomial order for any transported scalar quantity and show it has to be the same as the pressure order to avoid numerical instability. A manufactured solution is used to verify that the solution converges with the expected order of accuracy in space and time. We then simulate a stationary flow over a backward-facing step and a Von Kármán vortex street in the wake of a square cylinder to validate our approach.

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1. Introduction

Engineering applications often require the simulation of complex turbulent flows via accurate Computational Fluid Dynamics (CFD) methods. Direct Numerical Simulation (DNS) and Large Eddy Simulation (LES) constitute superior approaches in this regard, as they are able to resolve the stochastic fluctuations (though only the large-scale ones in case of LES) of any turbulent flow quantity [1]. However, nowadays they are still very computationally expensive and often unaffordable for large engineering applications. For this reason, the Reynolds-Averaged Navier-Stokes (RANS) equations coupled to turbulence closure models often remain the preferred approach, even if it only allows for the modeling of the time-averaged flow quantities [2]. Accurate and efficient numerical methods to solve the RANS equations are therefore still of great relevance.

In this perspective, discontinuous Galerkin Finite Element methods (DG-FEM) are particularly attractive, due to their flexibility, high accuracy, and robustness. The characteristic feature of DG is that unknown quantities are approximated with polynomial basis functions discontinuous across the mesh elements. This requires the use of numerical fluxes to deal with the discretization of face integrals, as in Finite Volume methods. However, thanks to the lack of continuity constraints, conservation laws are satisfied locally on each element, and the resulting algorithm stencil is compact, making the method suitable for efficient parallelization. As in continuous Galerkin FEM, the accuracy of the solution can be improved by increasing the order of the polynomial discretization. Moreover, DG methods can easily handle complex geometries, structured or unstructured meshes, and non-conformal local mesh and/or order refinement.

This class of Finite Element methods was originally developed in the early ‘70s to solve radiation transport problems [3]. However, it has become increasingly popular for CFD applications only in the past three decades, after the development of effective DG schemes for hyperbolic and elliptic problems. We refer to the reviews by Cockburn and Shu [4] and Arnold et al. [5] for a complete overview of these methods.

Nowadays, substantial experience has been gained in the DG-FEM discretization of the incompressible Navier-Stokes equations, and a variety of approaches can be found in literature. An early research effort in the field is the work of Cockburn et al. [6], who proposed a locally conservative Local DG (LDG) method for the incompressible Oseen equations. The authors later extended the...